ENEE 380 Spring 2003. Homework #2, 2/18/03

Due 2/25/03

(1) Cheng Problem (3.17)

(2) A charge of 1C is uniformly distributed in the xy plane between $-1 \leq x \leq 1$ and $-1 \leq y \leq 1$. Calculate and plot the electric field in the z-direction as z varies from 0 to 1000m.

(3) In Question (2) How far up the z-axis do you need to be before the field can be calculated within 1% accuracy by treating all the charge as if it were at the point(0,0,0).

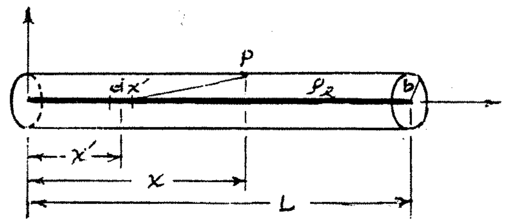
(4) Cheng Problem (3.20)

(5) One type of quadrupole is an arrangement of 4 charges of magnitudes +q. -q, +q, -q arranged at the corners of a square. The spacing of the charges is negligible compared to the distance to an observation point where the field is measured. Derive expressions for the potential distribution from a quadrupole and thereby the various field components. Plot the equipotential surfaces in the plane of the quadrupole, and in a plane perpendicular to the plane of the quadrupole passsing through the center of the square where the charges are located. Plot the radial electric field variation in the plane of the quadrupole.

(6) Repeat (5) for a linear quadrupole, where the charges are +q, -2q, +q arranged in a straight line

ENEE 380 Problem Set # 2.

(1) P. 3-17



$$dV_p(x) = \frac{\rho_\ell\, dx'}{4\pi\epsilon_0\sqrt{(x-x')^2+b^2}}$$

$$V_p(x) = \frac{\rho_\ell}{4\pi\epsilon_0}\int_0^L \frac{dx'}{\sqrt{(x-x')^2+b^2}}$$

$$= \frac{\rho_\ell}{4\pi\epsilon_0}\left[\sinh^{-1}\left(\frac{L-x}{b}\right) + \sinh^{-1}\left(\frac{x}{b}\right)\right].$$

(4) P. 3-20 Positive charge Ne uniformly distributed over a sphere of radius R_0: $\rho = \frac{Ne}{\frac{4}{3}\pi R_0^3} = \frac{3Ne}{4\pi R_0^3}$.

Inside the sphere (applying Gauss's law): $\bar{E} = \bar{a}_R E_R = \bar{a}_R \frac{Ner}{4\pi\epsilon_0 R_0^3}$.

a) Force experienced by an electron $-e$: $\bar{F} = -\bar{a}_R \frac{Ne^2 r}{4\pi\epsilon_0 R_0^3}$.

b) Equation of motion for an electron with mass m:

$$m\frac{d^2r}{dt^2} = -\frac{Ne^2 r}{4\pi\epsilon_0 R_0^3}, \quad \text{or} \quad \frac{d^2r}{dt^2} + \left(\frac{Ne^2}{4\pi\epsilon_0 m R_0^3}\right) r = 0,$$

$$\text{or} \quad \frac{d^2r}{dt^2} + \omega_e^2 r = 0, \quad \text{where} \quad \omega_e = \sqrt{\frac{Ne^2}{4\pi\epsilon_0 m R_0^3}}.$$

Hence the electrons would undergo a simple harmonic motion with an angular frequency ω_e.

c) The oscillating electrons would lose power through radiation and lead to an unstable atomic model.

(2)&(3) The field in the z-direction is

$$E_{z.} = \int_{-1}^{1}\int_{-1}^{1} \frac{z}{4\cdot\left(x^2+y^2+z^2\right)^{\frac{3}{2}}}\,dx\,dy$$

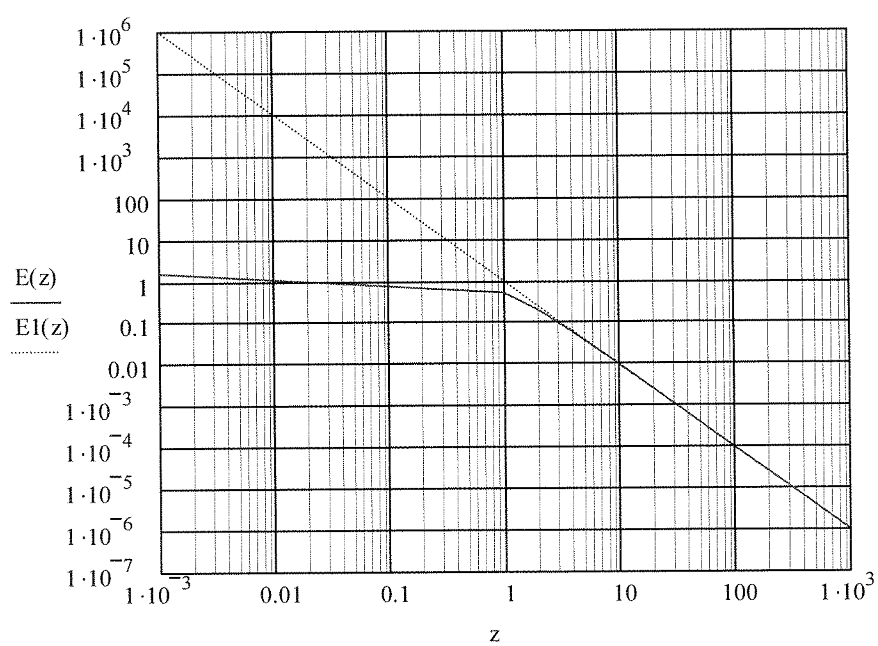
The field only points in the z-direction because of the symmetry of the problem, but during the integration the z-component of each contribution to the total field must be found by using the cosine of the vertical angle between the line to the charge element at dxdy and the z-axis

$$z := 0.001, 1 .. 1000$$

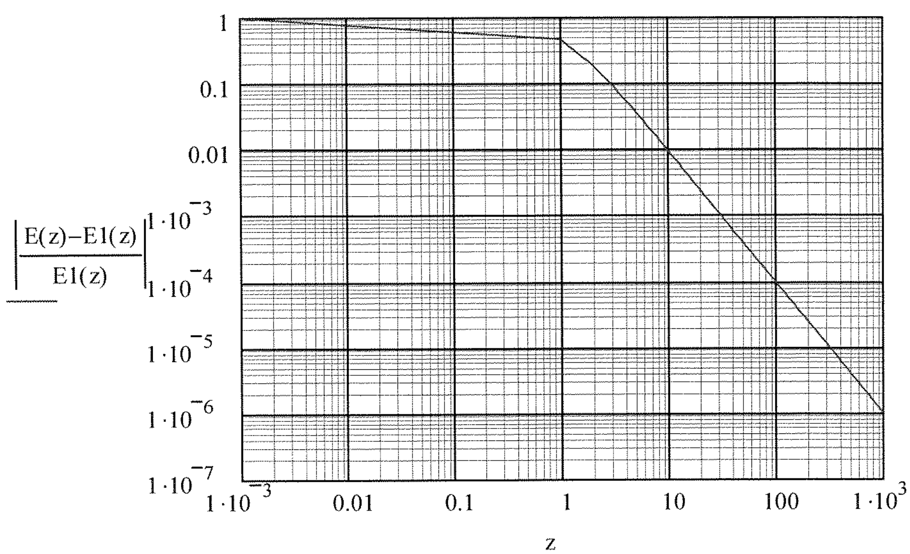
$$E(z) := \int_{-1}^{1}\int_{-1}^{1} \frac{z}{4\cdot\left(x^2+y^2+z^2\right)^{\frac{3}{2}}}\,dx\,dy$$

Assuming that the sheet of charge can be treated as a point charge

$$E1(z) := \frac{1}{z^2}$$



Test different values of z



For z>10 the result is within 1% of the correct value if the charge is assumed to be a point charge of value 1C

ENEE 380 Problem Set #2, Problem (5)
In plane potential surfaces of a square quadrupole

$\theta := 0, 0.01 .. 2\cdot\pi$

$V = \dfrac{3\cdot qdd\cdot\sin(\theta)\cdot\cos(\theta)}{4\cdot\pi\cdot\varepsilon_0\cdot r^3}$ qdd is strength of quadrupole

$\varepsilon_0 := 1$

Change to a scaled system of units to avoid big numbers

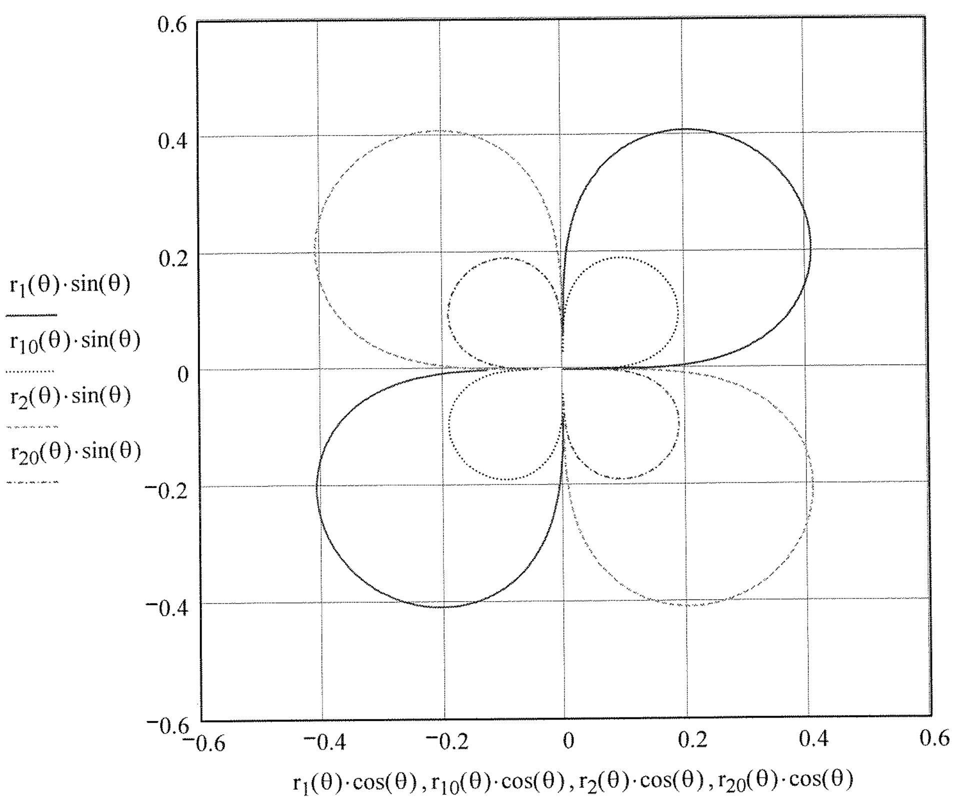
$qdd := 1$

$i := 1, 2 .. 10$

$V_i := i \qquad VV_i := -i$ Choose both positive and negative numbers to see full equipotential pattern

$r_1(\theta) := \left(\dfrac{3\cdot qdd\cdot\sin(\theta)\cdot\cos(\theta)}{4\cdot\pi\cdot\varepsilon_0\cdot V_1}\right)^{\frac{1}{3}} \qquad r_{10}(\theta) := \left(\dfrac{3\cdot qdd\cdot\sin(\theta)\cdot\cos(\theta)}{4\cdot\pi\cdot\varepsilon_0\cdot V_{10}}\right)^{\frac{1}{3}}$

$r_2(\theta) := \left(\dfrac{3\cdot qdd\cdot\sin(\theta)\cdot\cos(\theta)}{4\cdot\pi\cdot\varepsilon_0\cdot VV_1}\right)^{\frac{1}{3}} \qquad r_{20}(\theta) := \left(\dfrac{3\cdot qdd\cdot\sin(\theta)\cdot\cos(\theta)}{4\cdot\pi\cdot\varepsilon_0\cdot VV_{10}}\right)^{\frac{1}{3}}$



Some examples of equipotentials in the plane

$$Er = \frac{9 \cdot qdd \cdot \sin(\theta) \cdot \cos(\theta)}{4 \cdot \pi \cdot \varepsilon_0 \cdot r^4}$$ Radial Electric Field Strength

In terms of x and y coordinates

$i := 1, 2 .. 100$

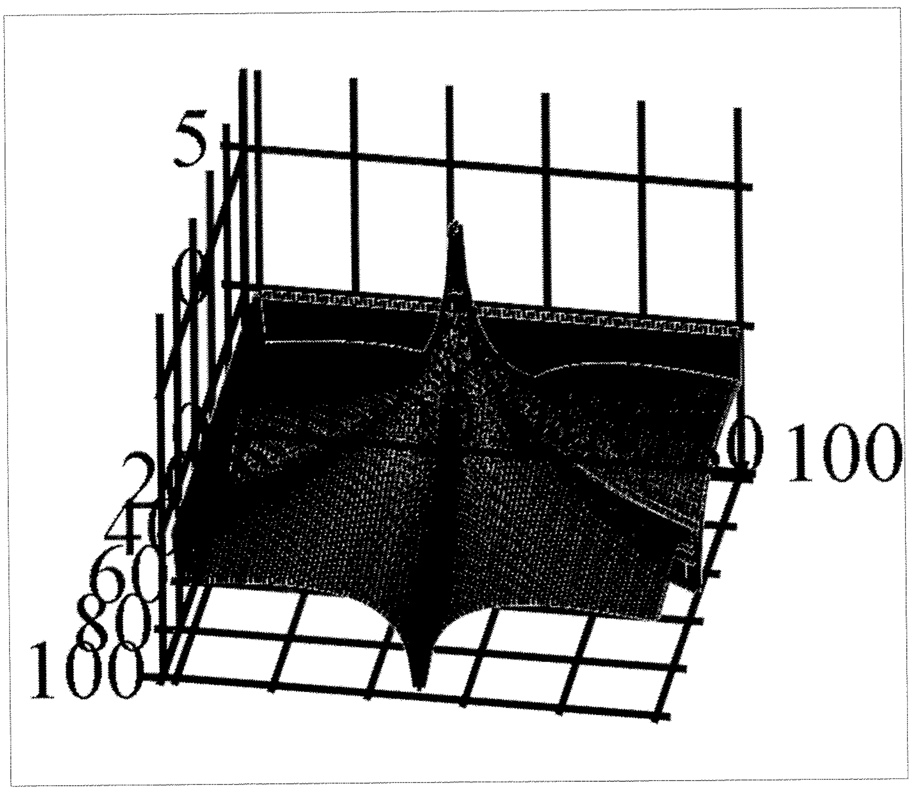
$j := 1, 2 .. 100$

$x_i := -1 + (i - 1) \cdot \frac{2}{99}$ $\qquad$ $y_j := -1 + (j - 1) \cdot \frac{2}{99}$

$$Er_{i,j} := \frac{9 \cdot qdd \cdot x_i \cdot y_j}{4 \cdot \pi \cdot \varepsilon_0 \cdot \left[\left(x_i\right)^2 + \left(y_j\right)^2 \right]^2}$$

$Er_{i,j} := \ln\left(Er_{i,j}\right)$ Take log to make plots look better

Log of Radial electric strength in the plane of the quadrupole



Er

Numerical Approach

$x_1 := 10^{-4}$ $\quad$ $y_1 := 10^{-4}$

$x_2 := 10^{-4}$ $\quad$ $y_2 := -10^{-4}$

$x_3 := -10^{-4}$ $\quad$ $y_3 := -10^{-4}$

$x_4 := -10^{-4}$ $\quad$ $y_4 := 10^{-4}$

$q_1 := 10^{8}$ $\quad$ $q_2 := -10^{8}$

$q_3 := 10^{8}$ $\quad$ $q_4 := -10^{8}$

$x := 0.1, 0.11 .. 1$ $\quad$ $y := 0.1, 0.11 .. 1$

Charge magnitudes and locations

$$V_1(x,y,z) := \frac{q_1}{4\cdot\pi\cdot\varepsilon_0\cdot\left[(x-x_1)^2+(y-y_1)^2+(z)^2\right]^{\frac{1}{2}}}$$

$$V_2(x,y,z) := \frac{q_2}{4\cdot\pi\cdot\varepsilon_0\cdot\left[(x-x_2)^2+(y-y_2)^2+(z)^2\right]^{\frac{1}{2}}}$$

$$V_3(x,y,z) := \frac{q_3}{4\cdot\pi\cdot\varepsilon_0\cdot\left[(x-x_3)^2+(y-y_3)^2+(z)^2\right]^{\frac{1}{2}}}$$

$$V_4(x,y,z) := \frac{q_4}{4\cdot\pi\cdot\varepsilon_0\cdot\left[(x-x_4)^2+(y-y_4)^2+(z)^2\right]^{\frac{1}{2}}}$$

The four contributions to the total potential

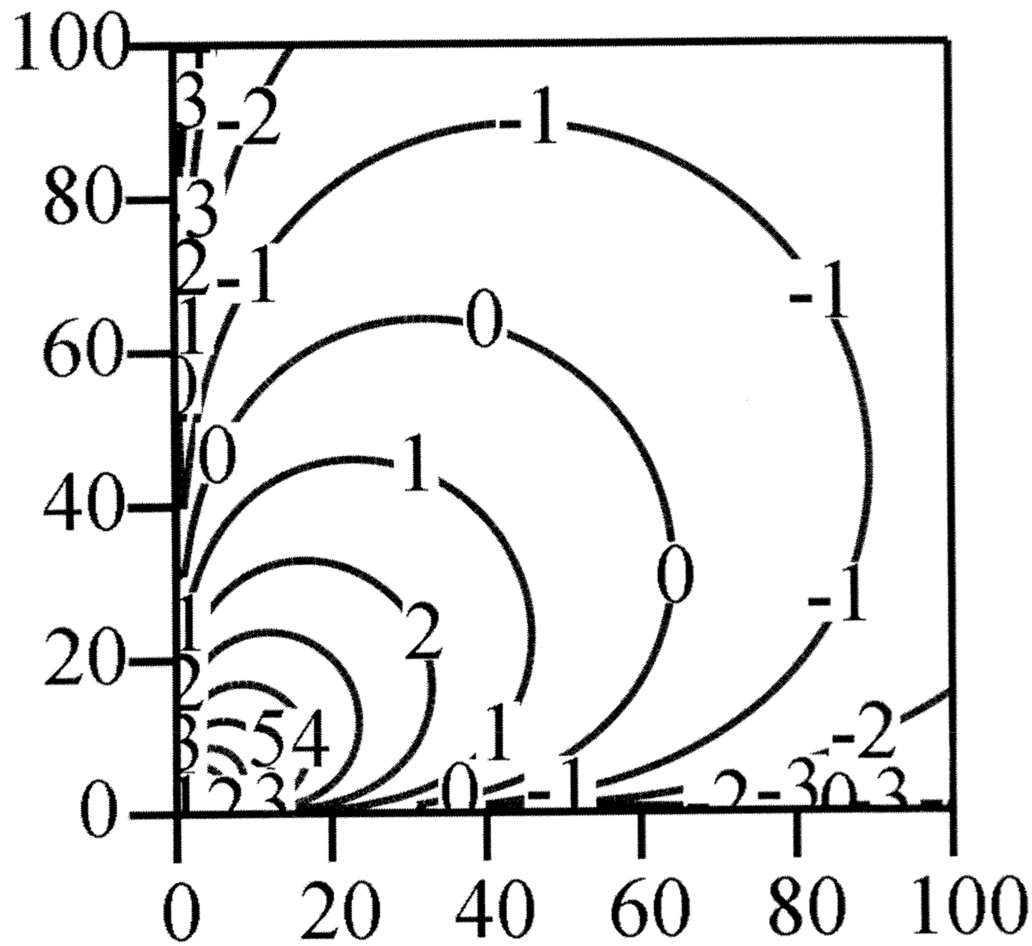
$$V(x,y,z) := V_1(x,y,z) + V_2(x,y,z) + V_3(x,y,z) + V_4(x,y,z)$$

$$x_i := 0 + (i)\cdot\frac{1}{99}$$

$$y_j := 0 + (j)\cdot\frac{1}{99}$$

$$\phi_{i,j} := \ln\left(V\left(x_i, y_j, 0\right)\right)$$

Equipotentials in xy plane



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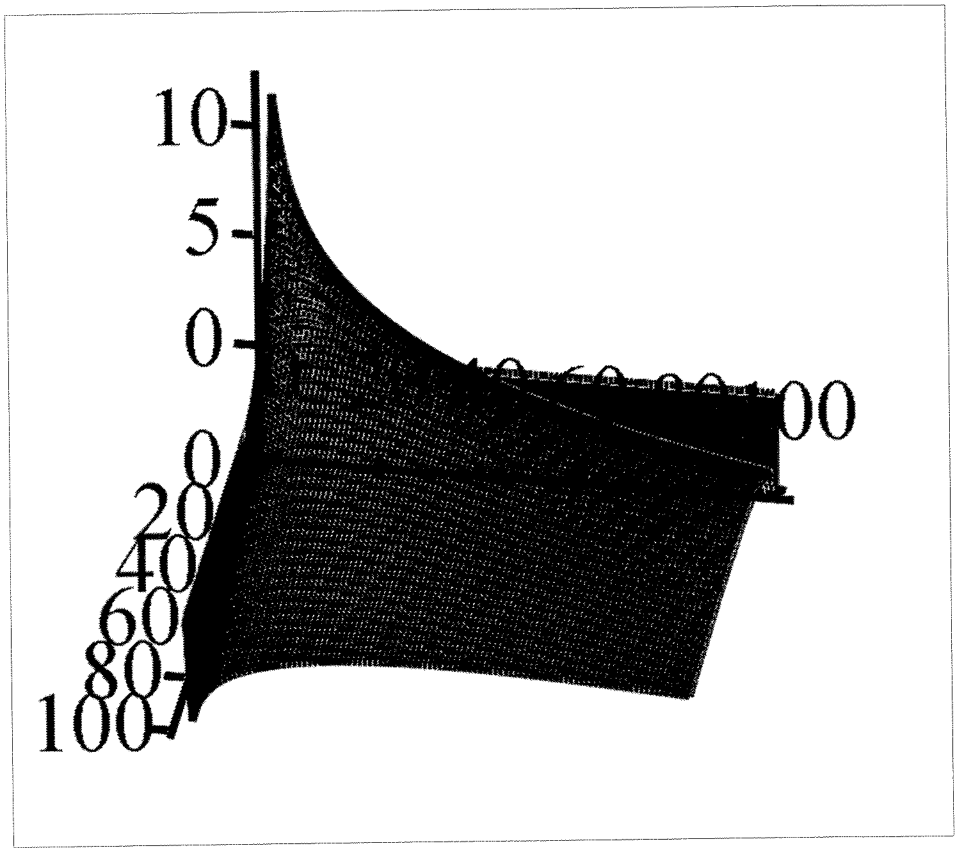

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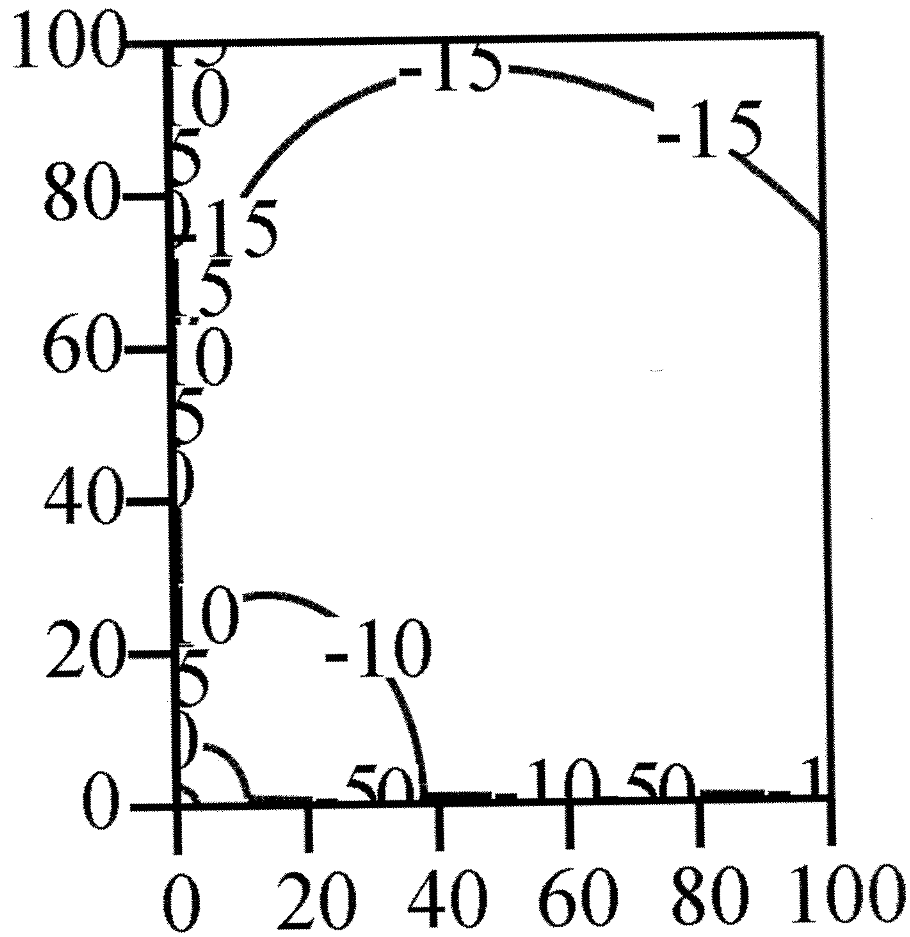
Log of Potential Function in first quadrant of xy plane

$$x_i := 0 + (i)\cdot\frac{1}{99}$$

$$z_j := 0 + (j)\cdot\frac{1}{99}$$

$$\phi_{i,j} := \left(V\left(x_i, 10^{-6}, z_j\right)\right)$$

$$\phi_{i,j} := \ln\left(\phi_{i,j}\right)$$



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Equipotentials in the xz plane

(6) Linear Quadrupole

$\theta := 0, 0.01 .. 2\cdot\pi$

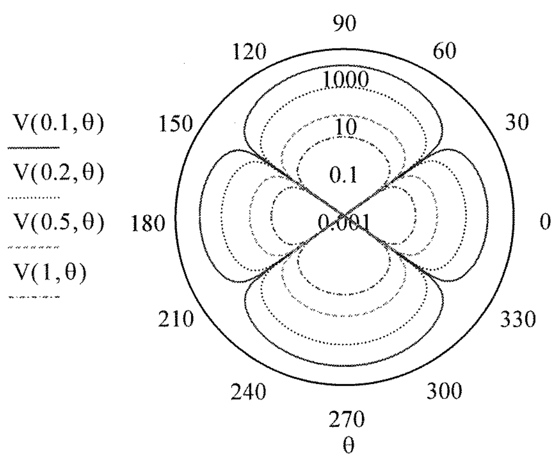
$R := 0.01, 0.02 .. 1$

$$V = \frac{q}{4\cdot\pi\cdot\varepsilon_0}\cdot\left(\frac{3}{R^3}\cdot d^2\cdot\sin(\theta)^2 - \frac{d^2}{R^3}\right)$$

Change to different scaled unit so that

$$V(R,\theta) := \frac{3\cdot\sin(\theta)^2 - 1}{R^3}$$

polar plot of potential function

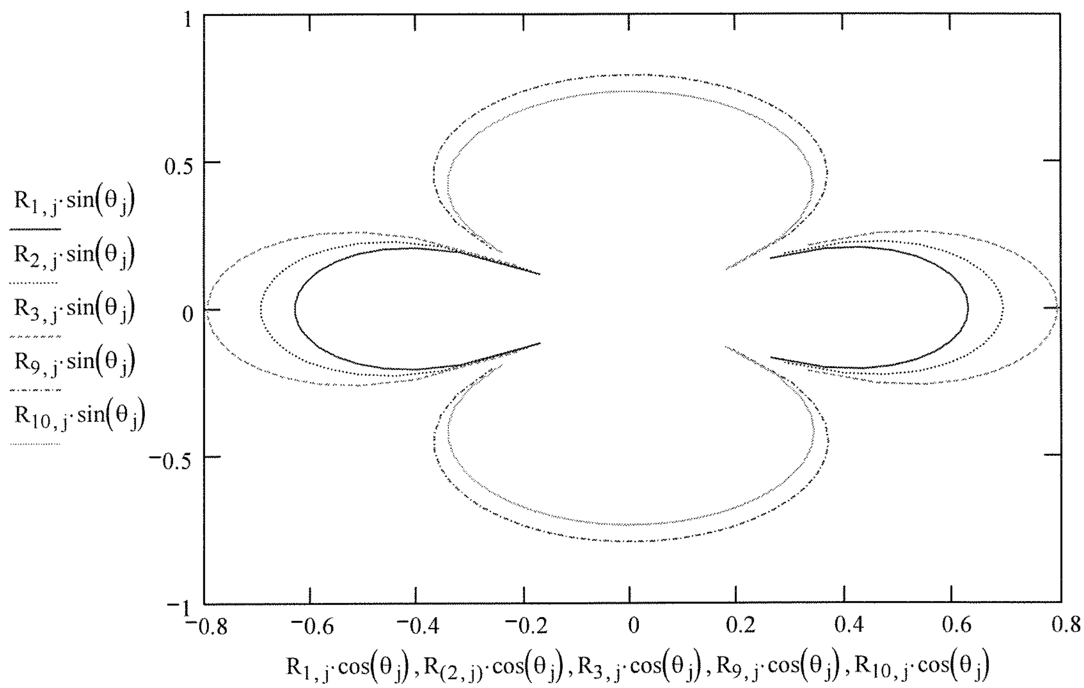


$i := 1, 2 .. 10$ $\qquad v_i := -4.999 + i$

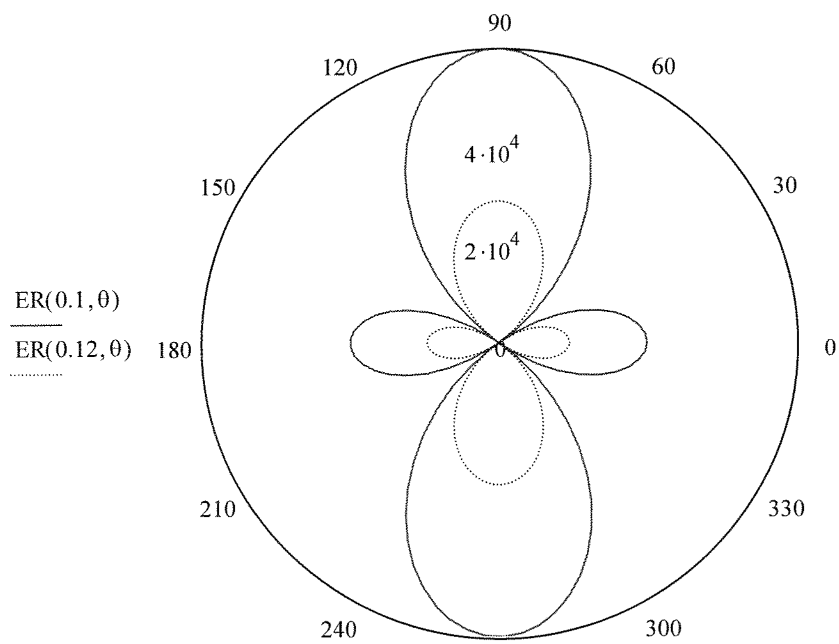
$j := 1, 2 .. 100$

$$R_{i,j} := \left[\frac{3\cdot\left(\sin(\theta_j)\right)^2 - 1}{v_i}\right]^{\frac{1}{3}}$$

Examples of some equipotentials



Radial electric field $\quad ER(R,\theta) := \frac{-3}{R^4} \cdot \left(3 \cdot \sin(\theta)^2 - 1\right)$



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Alternative Numerical approach: set up 4 charges and draw equipotentials

$i := 1, 2 .. 100$

$j := 1, 2 .. 100$

$y1 := 10^{-6}$

Location of charges

$y2 := 0$

$y3 := -10^{-6}$

$x_i := \frac{i}{100} - \frac{1}{1.999}$ $y_j := \frac{j}{100} - \frac{1}{1.999}$ different measurement locations

$R1_{i,j} := \sqrt{(x_i)^2 + (y_j - y1)^2}$

$x_0 := x_1$

$y_0 := y_1$

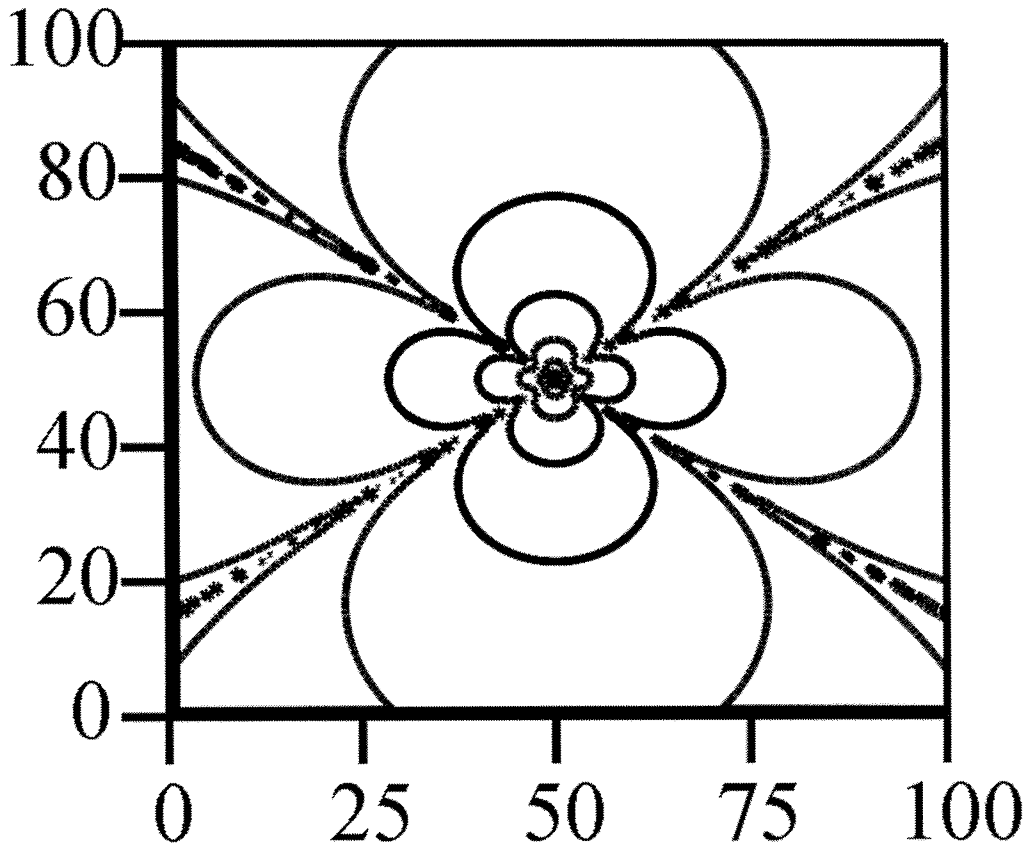
$R2_{i,j} := \sqrt{(x_i)^2 + (y_j - y2)^2}$

$R3_{i,j} := \sqrt{(x_i)^2 + (y_j - y3)^2}$

$V_{i,j} := \left(\frac{1}{R1_{i,j}} - \frac{2}{R2_{i,j}} + \frac{1}{R3_{i,j}}\right)$ normalized units

$\phi_{i,j} := \log(V_{i,j})$

Equipotentials in xy plane



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