ENEE 380 Spring 2003 Homework #3

Due March 11, 2003

(1) Cheng Problem (3.21)

(2) Cheng Problem (3.22)

(3) Cheng Problem (3.28)

(4) Cheng Problem (3.30)

(5) Cheng Problem (3.33)

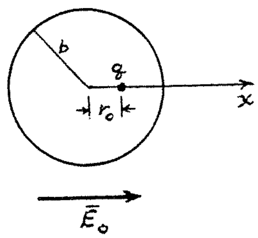
(6) Cheng Problem (3.46)

(7) Cheng Problem (3.47)

(8) Cheng Problem (3.48)

ENEE 380 PROBLEM SET # 3 SOLUTIONS

(1) P.3-21



Applied E_0 causes a displacement r_0.
Force of separation: qE_0
Restoring force (attraction): qE_x

E_x at q due to spherical volume of electrons of radius r_0 is (by Gauss's law)

$$E_x = \frac{\rho r_0}{3\epsilon_0} = -\frac{r_0}{3\epsilon_0}|\rho|, \text{ where } |\rho| = \frac{q}{\frac{4}{3}\pi b^3} = \frac{3Ne}{4\pi b^3}.$$

At equilibrium: $E_0 = |E_x| = \frac{r_0 Ne}{4\pi\epsilon_0 b^3}$, or $r_0 = \frac{4\pi\epsilon_0 b^3}{Ne} E_0$.

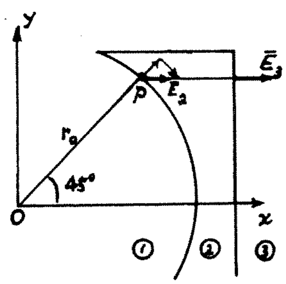
(2) P.3-22 a) $\rho_{ps} = \bar{P}\cdot\bar{a}_n = P_0 \frac{L}{2}$ on all six faces of the cube.

$\rho_p = -\bar{\nabla}\cdot\bar{P} = -3P_0$.

b) $Q_s = (6L^2)\rho_{ps} = 3P_0 L^3$; $Q_v = (L^3)\rho_p = -3P_0 L^3$.

Total bound charge $= Q_s + Q_v = 0$.

(3) P.3-28



Assume $\bar{E}_2 = \bar{a}_r E_{2r} + \bar{a}_\phi E_{2\phi}$.

Boundary condition: $\bar{a}_n \times \bar{E}_1 = \bar{a}_n \times \bar{E}_2$
$\longrightarrow E_{2\phi} = -3$.

For $\bar{E}_3$, and hence $\bar{E}_2$, to be parallel to the x-axis, $E_{2\phi} = -E_{2r}$
$\longrightarrow E_{2r} = 3$.

Boundary condition: $\bar{a}_n\cdot\bar{D}_1 = \bar{a}_n\cdot\bar{D}_2$
$\longrightarrow \epsilon_1 E_{r1} = \epsilon_2 E_{r2} \longrightarrow \epsilon_0 5 = \epsilon_0\epsilon_{r2} 3$
$\longrightarrow \epsilon_{r2} = \frac{5}{3} = 1.667$.

(4) P.3-30 $\epsilon = \frac{\epsilon_2 - \epsilon_1}{d} y + \epsilon_1$.

Assume Q on plate at $y=d$. $\bar{E} = -\bar{a}_y \frac{\rho_s}{\epsilon} = \frac{}{S(\frac{\epsilon_2-\epsilon_1}{d}y + \epsilon_1)}$.

$$V = -\int_{y=0}^{y=d} \bar{E}\cdot d\bar{l} = \frac{Qd\ \ln(\epsilon_2/\epsilon_1)}{S(\epsilon_2 - \epsilon_1)}.$$

$$C = \frac{Q}{V} = \frac{S(\epsilon_2 - \epsilon_1)}{d\ \ln(\epsilon_2/\epsilon_1)}.$$

(5) P.3-33 Gauss's law: $\oint_S \bar{D}\cdot d\bar{s} = \rho_\ell L$.

$$\bar{E}_1=\bar{E}_2=\bar{a}_r E_r, \qquad \pi r L(\epsilon_0\epsilon_{r1}+\epsilon_0\epsilon_{r2})E_r=\rho_\ell L$$

$$\longrightarrow E_r = \frac{\rho_\ell}{\pi r\epsilon_0(\epsilon_{r1}+\epsilon_{r2})}; \quad V=-\int_{r_o}^{r_i} E_r\, dr = \frac{\rho_\ell}{\pi\epsilon_0(\epsilon_{r1}+\epsilon_{r2})}\ln\left(\frac{r_o}{r_i}\right).$$

$$\therefore\ C=\frac{\rho_\ell L}{V} = \frac{\pi\epsilon_0(\epsilon_{r1}+\epsilon_{r2})L}{\ln(r_o/r_i)}.$$

(6) P.3-46 Denoting the plate separation by y and using Eq.(3-192):

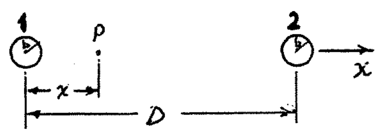
$$\bar{F}_V = \bar{a}_y \frac{\partial W_e}{\partial y} = \bar{a}_y\frac{\partial}{\partial y}\left(\frac{1}{2} C V_0^2\right) = \bar{a}_y \frac{V_0^2}{2}\frac{dC}{dy},$$

where C is the capacitance of the series connection of C_d and C_a: $\quad C_d=\frac{\epsilon S}{d_1}; \qquad C_a=\frac{\epsilon_0 S}{y-d_1}$.

$$C=\frac{C_d C_a}{C_d+C_a}=\frac{\epsilon\epsilon_0 S}{\epsilon(y-d_1)+\epsilon_0 d_1}$$

$$\therefore\ \bar{F}_V = -\bar{a}_y\frac{\epsilon^2\epsilon_0 S V_0}{2[\epsilon(d-d_1)+\epsilon_0 d_1]^2} \quad \text{(attractive force).}$$

(7) P.3-47



Assume line charge densities ρ_ℓ and $-\rho_\ell$ on conductors 1 and 2 respectively. At point P,

$$\bar{E}_P=\bar{E}_1+\bar{E}_2=\bar{a}_x\left[\frac{\rho_\ell}{2\pi\epsilon_0 x}+\frac{\rho_\ell}{2\pi\epsilon_0(D-x)}\right].$$

$$V_0 = V_1-V_2=\int_b^{D-b}\bar{E}_P\cdot d\bar{x} = \frac{\rho_\ell}{2\pi\epsilon_0}\int_b^{D-b}\left(\frac{1}{x}+\frac{1}{D-x}\right)dx$$

$$=\frac{\rho_\ell}{2\pi\epsilon_0}\left(\ln\frac{D-b}{b}-\ln\frac{b}{D-b}\right)=\frac{\rho_\ell}{\pi\epsilon_0}\ln\frac{D-b}{b}\cong\frac{\rho_\ell}{\pi\epsilon_0}\ln\frac{D}{b}$$

$$C'=\frac{\rho_\ell}{V_0}=\frac{\pi\epsilon_0}{\ln(D/b)} \quad (F/m).$$

$$\bar{F}'=\bar{\nabla}W_e=\bar{a}_x\frac{V_0}{2}\frac{\partial C'}{\partial D}=-\bar{a}_x\frac{\pi\epsilon_0 V_0^2}{2D[\ln(D/b)]^2},$$

—— in the direction of decreasing D.

(8) P.3-48 a) Switch closed $\longrightarrow$ $V=V_0=$ constant.

$$W_e=\frac{1}{2}CV_0^2, \qquad C=\frac{w}{d}[\epsilon x+\epsilon_0(L-x)].$$

$$\bar{F}_V=\bar{\nabla}W_e=\bar{a}_x\frac{V_0^2}{2}\frac{\partial C}{\partial x}=\bar{a}_x\frac{V_0^2 w}{2d}(\epsilon-\epsilon_0).$$

b) Switch open $\longrightarrow$ $Q=$ constant $=CV_0$.

$$W_e=\frac{Q^2}{2C},$$

$$\bar{F}_Q=-\bar{\nabla}W_e=-\bar{a}_x\frac{Q^2}{2}\frac{\partial}{\partial x}\left(\frac{1}{C}\right)$$

$$=\bar{a}_x\frac{Q^2 d}{2w}\frac{\epsilon-\epsilon_0}{[\epsilon x+\epsilon_0(L-x)]^2}=\bar{a}_x\frac{V_0^2 w}{2d}(\epsilon-\epsilon_0).$$