

ENEE 380 ELECTROMAGNETIC THEORY HOMEWORK #3

Solution to Problem #1

The capacitance of the coaxial capacitor with inner radius a , outer radius b , length x is

$$C = \frac{2\pi\epsilon_0\epsilon_r x}{\ln(b/a)}.$$

The stored energy in the capacitor is

$$W_e = \frac{1}{2}CV^2,$$

which gives

$$W_e = \frac{\pi\epsilon_0\epsilon_r x V^2}{\ln(b/a)}.$$

$$\frac{\partial W_e}{\partial x} = \frac{\pi\epsilon_0\epsilon_r V^2}{\ln(b/a)}.$$

Therefore, the force pulling the inner conductor into the outer *at constant potential* is

$$F = Mg = \frac{\pi\epsilon_0\epsilon_r V^2}{\ln(b/a)}.$$

where g is the acceleration due to gravity. The mass M required is

$$M = \frac{\pi\epsilon_0\epsilon_r V^2}{g \ln(b/a)}.$$

Note that the force is not calculated from

$$F = -\frac{\partial W_e}{\partial x}$$

because the power supply that keeps the potential constant provides energy to keep the potential constant while the capacitance is changing. In other words

$$\left(\frac{\partial W_e}{\partial x}\right)_V = -\left(\frac{\partial W_e}{\partial x}\right)_Q$$