ENEE 380H ELECTROMAGNETIC THEORY

Solutions to First Examination, Fall 2000

(1) Gauss's theorem is

$$\oint D \cdot dS = \int \mathrm{div} D \, dV$$

but $\mathrm{div} D = \rho$, therefore

$$\oint D \cdot dS = \int \rho \, dV = Q:$$

This is Gauss's Law

Erect a small Gaussian box with face area ΔS in the boundary of the conductor, the normal component of E (E_n) is zero on the conductor side. Therefore, application of Gauss's law gives

$$\oint D \cdot dS = \epsilon E_n \Delta S = \rho_s \Delta S;$$

which gives

$$E = \frac{\rho_s}{\epsilon}\hat{n}:$$

At 0.5m $D_n = \epsilon_0 \epsilon_r E_n$, so $E_n = \rho_s/(\epsilon_o \epsilon_r)$; which gives E_n=11.29 V/m

At 2m, $D_n = \epsilon_0 E_n$, so $E_n = \rho_s/\epsilon_o$; which gives E_n=112.9V/m.

(2)Distances from each charge to P are: $r_{AP} = \sqrt{17}$; $r_{BP} = \sqrt{5}$; $r_{CP} = \sqrt{8}$.

The potential at P relative to V=0 at ∞ is

$$V_P = \sum_{i=A,B,C} \frac{Q_i}{4\pi\epsilon_0 r_{iP}} = 23.94V:$$

The potential at P if V at ∞ is 3V is 3V higher than this 26.94V.

The distances of the three charges from (0,0,0) are $r_{A0} = \sqrt{14}$; $r_{B0} = \sqrt{14}$; and $r_{C0} = \sqrt{3}$.

Therefore relative to V=0 at ∞, V(0,0,0)is

$$V(0;0;0) = \sum_{i=A,B,C} \frac{Q_i}{4\pi\epsilon_0 r_{i0}} = 48.33V:$$

Therefore, relative to V=0 at (0,0,0) V_P=23.94-48.33=-24.39V.

(3)

$$C = \frac{Q}{V} = \frac{\int \rho_s \, dS}{-\int_1^2 E \cdot dl}:$$

Between the spheres draw an imaginary Gaussian spherical surface and integrate to find radial field.

$$E_r = \frac{Q}{4\pi\epsilon_0\epsilon_r r^2} dr:$$

Between inner sphere and surface of dielectric coating

$$V_{12} = -\int_1^2 \frac{Q}{4\pi\epsilon_0\epsilon_r r^2}dr = \frac{10^3 Q}{80\pi\epsilon_0}.$$

Between outer surface of dielectric layer and outer sphere

$$V_{23} = -\int_4^2 \frac{Q}{4\pi\epsilon_0 r^2}dr = \frac{10^3 Q}{16\pi\epsilon_0}.$$

The arrangement is equivalent to two capacitors in series, C_1 and C_2. $C_1 = 80\pi\epsilon_0/10^3$, $C_2 = 16\pi\epsilon_0/10^3$. Overall $C = 40\pi\epsilon/(3 \cdot 10^3) = 0.371$pF.

The force is the same as for two charges $+Q$ and $-Q$ spaced 4mm apart:

$$F = \frac{10^6 Q^2}{64\pi\epsilon_0}.$$

If you want to you can calculate stored energy U and evaluate $-dU/dr$.

(4)Use method of images. Put extra charge of -1C at (1,1,-1) and 1C at (-1,-1,-1). At point (1,1,0) the distances to the four charges are 1,1, 3, and 3. The fields from the charges at (1,1,1) and (1,1,-1) add up and act normal to the surface plane of the conductor. The charges at (-1,-1,1) and (-1,-1,-1) are equal but they act at an angle to the surface with $\cos(\theta) = 1/3$.

At (1,1,0) the normal E field is

$$E_n = -\left(\frac{2}{4\pi\epsilon_0} - \frac{2\cos(\theta)}{18\pi\epsilon_0}\right),$$

which gives

$$E_n = \frac{-13}{27\pi\epsilon_0}.$$

this field acts in the $-z$ direction. The surface charge density at (1,1,0) is $\rho_s = \epsilon_0 E_n = -13/27\pi\ C/m^2$ At (-1,1,0) the surface charge density is $13/27\pi\ C/m^2$

At (0,0,0) there is no E_n, so $\rho_s = 0$.