

ESEE 381 Problem Set #1

9/10/02 - due 9/17/02

(1)(380 Review) The current density in a certain region is

$$\mathbf{J} = 0.1e^{-10^6 t} \hat{\mathbf{r}}/r$$

in spherical coordinates. At $t=1\mu\text{s}$ how much current is crossing the surface $r=5$?

(2) (380 review) A current density $\mathbf{J} = 5\hat{\mathbf{j}}\text{A/m}^2$ exists wherever $|z| < 2\text{m}$. (a) Find \mathbf{H} for $|z| < 2$ and $|z| > 2$. Find the magnetic vector potential \mathbf{A} for $|z| < 2$ if $\mathbf{A} = 0$ at the origin.

(3) A circular coil of 100 turns of radius 50mm, total resistance 1 ohm, and no self inductance is rotated about a vertical diameter with uniform angular velocity 100 rad/s in a horizontal magnetic flux of 0.2 Tesla. Calculate the average power needed to keep the coil in motion.

The mean power required to keep the coil in motion is

$$W = n^2 \pi^2 a^4 b^2 \omega^2 / (2R)$$

What is the ohmic power dissipated in the coil?

(4) A small magnetic needle, which is free to turn slowly in a horizontal plane, is placed at the center of the coil in question (3). Calculate the angle with respect to \mathbf{B} at which it reaches equilibrium.

Show that it will set at an angle ϕ to B where

$$\cot \phi = 4R / (\pi n^2 \mu_0 \omega a)$$

(5) A charged particle starts from rest at the origin of coordinates in a region where there is a uniform electric field \mathbf{E} parallel to the x -axis, and a uniform magnetic flux density \mathbf{B} parallel to the z -axis. Show that the coordinates of the particle at a time t later will be

$$\begin{aligned} x &= \left(\frac{E}{\omega B}\right)(1 - \cos(\omega t)), \\ y &= \left(\frac{E}{\omega B}\right)(\omega t - \sin \omega t), \\ z &= 0, \end{aligned}$$

where $\omega = eB/m$. ($E = |\mathbf{E}|$, $B = |\mathbf{B}|$)(This path is called a cycloid.)

(6) Electrons are liberated with zero velocity from the negative plate of a parallel plate capacitor, to which is applied a magnetic flux density \mathbf{B} parallel to the plates. Prove that these electrons will not reach the positive plate if the plate separation d is greater than $2mE/eB^2$, where $E = |\mathbf{E}|$ is the field between the plates.

(7) A plane circular disk of radius a rotates at a speed of $2\pi f$ rad/s about an axis through the center of the disk perpendicular to the plane of the disk. There is a uniform magnetic flux \mathbf{B} parallel to the axis of rotation. Prove that the emf between the center of the disk and its rim is of magnitude $V = fB\pi a^2$. ($B = |\mathbf{B}|$.)

