ENEE 381 Problem Set # 2

SOLUTIONS

(1) The waves from the radio transmitter are distributed in all directions over the surface of a sphere of radius R.

The power flux (Watts / m^2) at radius R is

$$I = \frac{P}{4\pi R^2}$$

The power flux is often called the Intensity, I. It is the average value of the magnitude of the Poynting vector

$$I = \overline{|\underline{S}|} = \overline{\underline{E} \times \underline{H}} = \frac{E^2}{2Z_0}$$

Therefore

$$E = \sqrt{2Z_0 I} = \sqrt{\frac{2Z_0 P}{4\pi R^2}} = \sqrt{\frac{Z_0 P}{2\pi R^2}}$$

Plug in $P = 4W$, $R = 10^3$ m, $Z_0 = 376.7$ ohm

$\frac{E}{H} = Z_0$ Therefore $\longrightarrow$ $E = 1.55 \times 10^{-2}$ V/m, $H = 4.11 \times 10^{-5}$ A/m

For a laser beam with $I = 10\,GW/cm^2 = 10^{14}\,W/m^2$

$E = \sqrt{2Z_0 I} = 2.74 \times 10^8$ V/m

$H = \frac{E}{Z_0} = 7.29 \times 10^5$ A/m

(2) $$E = \sqrt{\frac{Z_0 P}{2\pi R^2}}$$

Plug-in $Z_0 = 376.7$

$P = 1\,W$

$R = 50 \times 10^{-3}\,m$

$\underline{E = 155\,V/m}$

$\underline{H = 0.41\,A/m}$

If we assume that 1W is absorbed in a sphere of radius 100 mm then the average energy absorption is

$$U = \frac{P}{\frac{4}{3}\pi R^3} = 23.9\,J\,s^{-1}\,m^{-3}$$

If we take a reasonable value for the density equal to the density of water $= 1\,kg/m^3$ then the mass absorbs $0.024\,J/kg \equiv 2.39 \times 10^{-5}\,J/g$

Now 1 calorie $= 4.2\,J$, therefore the absorbed power is 5.69×10^{-6} calories/g.

Therefore the rate of heating is 5.69×10^{-6} degrees/sec

(3)

3.8c (i) (rect.)

$\vec{E} = \hat{x}\, C e^{-jz\omega\sqrt{\mu\epsilon}}$, $\quad \nabla\cdot\vec{E} = \frac{\partial E_x}{\partial x} = 0 \quad$ so $\rho = 0$

$\nabla\times\vec{E} = -j\omega\mu\vec{H} = \hat{y}\frac{\partial E_x}{\partial z} = -j\omega\sqrt{\mu\epsilon}\, C e^{-jz\omega\sqrt{\mu\epsilon}}$

so $H_y = \sqrt{\frac{\epsilon}{\mu}}\, C e^{-jz\omega\sqrt{\mu\epsilon}}$, $\quad \nabla\cdot\vec{B} = \mu\nabla\cdot\vec{H} = \mu\frac{\partial H_y}{\partial y} = 0$

$\nabla\times\vec{H} = -\hat{x}\frac{\partial H_y}{\partial z} = +j\omega\sqrt{\mu\epsilon}\,\hat{x}\sqrt{\frac{\epsilon}{\mu}}\, C e^{-j\omega z\sqrt{\mu\epsilon}} = \vec{J} + j\omega\epsilon\vec{E}$

satisfied with $\vec{J} = 0$ (plane wave in homogen. dielectric)

(ii) (circ. cyl.)

$\vec{H} = \hat{\phi}\frac{C}{r}e^{-j\omega z\sqrt{\mu\epsilon}}$, $\quad \nabla\cdot\vec{H} = \frac{1}{r}\frac{\partial}{\partial r}(rH_r) + \frac{1}{r}\frac{\partial H_\phi}{\partial\phi} + \frac{\partial H_z}{\partial z} = 0$

$\nabla\times\vec{H} = \frac{\hat{z}}{r}\frac{\partial}{\partial r}(rH_\phi) - \hat{r}\frac{\partial H_\phi}{\partial z} = j\omega\sqrt{\mu\epsilon}\frac{\hat{r}C}{r}e^{-j\omega z\sqrt{\mu\epsilon}} = \vec{J} + j\omega\epsilon\vec{E}$

try $\vec{J} = 0$ and $\vec{E} = \hat{r}\sqrt{\frac{\mu}{\epsilon}}\frac{C}{r}e^{-j\omega z\sqrt{\mu\epsilon}}$,

$\nabla\cdot\vec{E} = \frac{1}{r}\frac{\partial}{\partial r}(rE_r) = 0$, $\quad \nabla\times\vec{E} = \hat{\phi}\frac{\partial E_r}{\partial z} = \hat{\phi}\left[-j\omega\mu\frac{C}{r}\right]e^{-j\omega z\sqrt{\mu\epsilon}} = -j\omega\mu\vec{H}$. Checks.

(iii) (spherical)

$\vec{E} = \hat{\theta}\frac{C}{r}e^{-j\omega r\cos\theta\sqrt{\mu\epsilon}}$

$\nabla\cdot\vec{E} = \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, D_\theta) = \epsilon C\left[\frac{\cot\theta}{r^2} + \frac{jk}{r}\sin\theta\right]e^{-jkr\cos\theta}$, $\quad k = \omega\sqrt{\mu\epsilon}$

$\nabla\times\vec{E} = \frac{\hat{\phi}}{r}\frac{\partial}{\partial r}(rE_\theta) = \hat{\phi}\frac{C}{r}\left[-j\omega\cos\theta\sqrt{\mu\epsilon}\right]e^{-jkr\cos\theta} = -j\omega\mu\vec{H}$

so $\vec{H} = \hat{\phi}\sqrt{\frac{\epsilon}{\mu}}\frac{C\cos\theta}{r}e^{-jkr\cos\theta}$, $\quad \nabla\cdot\vec{H} = \frac{1}{r\sin\theta}\frac{\partial H_\phi}{\partial\phi} = 0$

$\nabla\times\vec{H} = \frac{\hat{r}}{r\sin\theta}\frac{\partial}{\partial\theta}[H_\phi\sin\theta] - \frac{\hat{\theta}}{r}\frac{\partial}{\partial r}(rH_\phi) = \vec{J} + j\omega\epsilon\vec{E}$

so $\vec{J} = \frac{\hat{r}}{r^2\sin\theta}\sqrt{\frac{\epsilon}{\mu}}\, C[\cos\theta\sin^2\theta(jkr) + \cos 2\theta]e^{-jkr\cos\theta} + \frac{\hat{\theta}}{r}(-j\omega\epsilon C\cos^2\theta)e^{-jkr\cos\theta}$

Now this must satisfy continuity $\nabla\cdot\vec{J} = -j\omega\rho$. Using $\nabla\cdot\vec{J}$ formula from front cover of text: $\nabla\cdot\vec{J} = \left[-\frac{j\omega C\cos\theta}{r^2\sin\theta} + \frac{C\omega^2\epsilon\sqrt{\mu\epsilon}\sin\theta}{r}\right]e^{-jkr\cos\theta}$

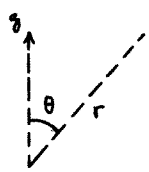
$-j\omega\rho = -j\omega\nabla\cdot\vec{D} = -j\omega\epsilon\nabla\cdot\vec{E} = -\frac{j\omega C\epsilon}{r^2\sin\theta}[\cos\theta - j\omega\sqrt{\mu\epsilon}\, r\sin^2\theta]e^{-jkr\cos\theta}$

So $\nabla\cdot\vec{J} = -j\omega\rho$ and the given function is seen to satisfy Maxwell's equations.

Notes to instructor: (1) These may be simplified if desired by asking only for charge-free, current-free solutions. (2) $r = 0$ in parts (ii) and (iii) excluded.

(4)

3.13b



$\vec{P}_{av} = \frac{1}{2}\mathrm{Re}[\vec{E}\times\vec{H}^*] = \frac{1}{2}\mathrm{Re}[\hat{\theta}E_\theta\times\hat{\phi}H_\phi^*] = \frac{1}{2}\mathrm{Re}[\hat{r}E_\theta H_\phi^*]$

$= \hat{r}\frac{E_\theta E_\theta^*}{2\eta} = \hat{r}\frac{A^2}{2\eta r^2}\sin^2\theta$

$W = \int_0^{2\pi}\int_0^{\pi}(P_r)_{av}\, r^2\sin\theta\, d\theta\, d\phi = \frac{2\pi A^2}{2\eta}\int_0^{\pi}\sin^3\theta\, d\theta$

$= \frac{\pi A^2}{\eta}\left[\int_0^{\pi}-(1-\cos^2\theta)\, d(\cos\theta)\right] = -\frac{\pi A^2}{\eta}\left[\cos\theta - \frac{\cos^3\theta}{3}\right]_0^{\pi}$

$= \frac{\pi A^2}{\eta}\left[1 - \frac{1}{3} + 1 - \frac{1}{3}\right] = \frac{4}{3}\frac{\pi A^2}{\eta}$

(5)

3.17b $E_z = \frac{J_0}{\sigma} e^{-(1+j)x/\delta}$

$\nabla \times \vec{E} = -j\omega\mu\vec{H} \Rightarrow -\frac{\partial E_z}{\partial x} = -j\omega\mu H_y = \frac{(1+j)J_0}{\sigma\delta} e^{-(1+j)x/\delta}$

so $H_y = \frac{j(1+j)J_0}{\sigma\delta\omega\mu} e^{-(1+j)x/\delta} = \frac{(1-j)J_0\delta}{2} e^{-(1+j)x/\delta}$

but $\frac{(1-j)}{2} = \frac{1}{1+j}$ so this is the same as Eq. 3.17 (1) at $x=0$.

(6)

3.18b If incident wave $H_y = H_0 e^{-jkz}$

$(P_z)_{av} = \frac{1}{2}\eta H_0^2$, $H_0 = \left[\frac{2(P_z)_{av}}{\eta}\right]^{\frac{1}{2}} = \left[\frac{2\times10^6}{377}\right]^{\frac{1}{2}} = 72.8\ \frac{A}{m}$

so if magnetic field at surface is doubled,

$W_L = \frac{R_s}{2}(2H_0)^2$; for Al @ 1GHz , $R_s = 3.26\times10^{-7}\sqrt{10^9} = 1.03\times10^{-2}\,\Omega$

so $W_L = \frac{1.03\times10^{-2}}{2}(2\times72.8)^2 = 109.2\ \frac{W}{m^2}$

thus $\frac{W_L}{(P_z)_{av}} = \frac{109.2}{10^6} = 1.09\times10^{-4}$

(7) An intensity I (Watts $/m^2$) corresponds to a photon flux $\frac{I}{h\nu}$ photons$/m^2 s$

each photon carries momentum $\frac{h}{\lambda}$

For a reflective surface the momentum change per square meter per second is

$$p = \left(\frac{I}{h\nu}\right) 2 \left(\frac{h}{\lambda}\right) = \frac{2I}{c}$$

reflection reverses momentum

The force on area A is

$$F = \frac{2IA}{c}$$

The acceleration is $a = \frac{2IA}{mc}$

where m is the mass.

To reach velocity v takes a time t where $v = at$. Therefore for $v = \frac{c}{10}$

$$t = \frac{c}{10a} = \frac{mc^2}{20IA}$$

Plug in $c = 3 \times 10^8$ m/s. $M = 10^4$ kg, $A = 10^8$ m^2, $I = 1$

$$t = \underline{4.5 \times 10^{11} \text{ secs}} = \underline{14,269 \text{ years}}$$