ENEE 381 Problem Set #6 Solutions

(1)

$$c_0 := 2.998 \cdot 10^8$$

$$\lambda := \frac{c_0}{9 \cdot 10^9}$$ Wavelength at 9GHz

$$d := \frac{\lambda}{4}$$ Distance of film from plane conductor

$$i := 1, 2 .. 1001$$

$$\nu_i := 6 \cdot 10^9 + \frac{(i-1) \cdot 12 \cdot 10^9}{1000}$$ Different frequencies

$$Z_0 := 376.6$$

$$Z_c := 0$$

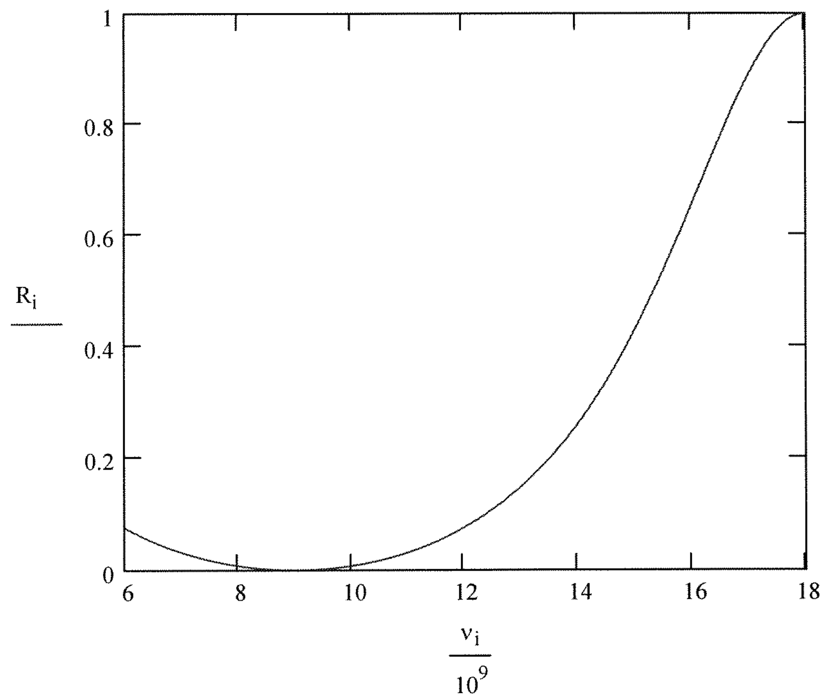
$$k_i := \frac{2 \cdot \pi \cdot \nu_i}{c_0}$$

$$Z_i := Z_0 \cdot \left(\frac{Z_c \cdot \cos(k_i \cdot d) + i \cdot Z_0 \cdot \sin(k_i \cdot d)}{Z_0 \cdot \cos(k_i \cdot d) + i \cdot Z_c \cdot \sin(k_i \cdot d)} \right)$$ Input impedance at distance d from the film at the different frequencies

$$ZL_i := \frac{1}{\frac{1}{Z_0} + \frac{1}{Z_i}}$$ Input impedance is in parallel with sheet

$$\rho_i := \frac{ZL_i - Z_0}{ZL_i + Z_0}$$

$$R_i := \left(\left| \rho_i \right| \right)^2$$



$R_1 = 0.077$ $R_{251} = 0$ $R_{501} = 0.077$ $R_{751} = 0.429$ $R_{1000} = 1$

$\nu_1 = 6 \times 10^9$ $\nu_{251} = 9 \times 10^9$ $\nu_{501} = 1.2 \times 10^{10}$ $\nu_{751} = 1.5 \times 10^{10}$

(2) This is a slab problem with four interfaces, three layers between air and air

$\varepsilon_4 := 4$ $\varepsilon_3 := 1.1$ $\varepsilon_2 := 4$

$Z_5 := Z_0$ $Z_4 := \frac{Z_0}{\sqrt{\varepsilon_4}}$ $Z_3 := \frac{Z_0}{\sqrt{\varepsilon_3}}$ $Z_2 := \frac{Z_0}{\sqrt{\varepsilon_2}}$

$Z_1 := Z_0$

$d_4 := 1.5 \cdot 10^{-3}$ $d_3 := 18.1 \cdot 10^{-3}$ $d_2 := 1.5 \cdot 10^{-3}$ $i := 1, 2 .. 2$

Use transformed impedance idea three times

$$\lambda 0_1 := \frac{c_0}{3\cdot 10^9} \qquad \lambda 0_2 := \frac{c_0}{6\cdot 10^9}$$

The two wavelengths for calculation

$$k4_i := \frac{2\cdot\pi\cdot\sqrt{\varepsilon_4}}{\lambda 0_i} \qquad k3_i := \frac{2\cdot\pi\cdot\sqrt{\varepsilon_3}}{\lambda 0_i} \qquad k2_i := k4_i$$

$$Z4_i := Z_4\cdot\left(\frac{Z_5\cdot\cos\left(k4_i\cdot d_4\right) + i\cdot Z_4\cdot\sin\left(k4_i\cdot d_4\right)}{Z_4\cdot\cos\left(k4_i\cdot d_4\right) + i\cdot Z_5\cdot\sin\left(k4_i\cdot d_4\right)}\right)$$

transformed impedance formula to go from last interface to third interface

$$Z3_i := Z_3\cdot\left(\frac{Z4_i\cdot\cos\left(k3_i\cdot d_3\right) + i\cdot Z_3\cdot\sin\left(k3_i\cdot d_3\right)}{Z_3\cdot\cos\left(k3_i\cdot d_3\right) + i\cdot Z4_i\cdot\sin\left(k3_i\cdot d_3\right)}\right)$$

transformed impedance formula to go from

$$Z2_i := Z_2\cdot\left(\frac{Z3_i\cdot\cos\left(k2_i\cdot d_2\right) + i\cdot Z_2\cdot\sin\left(k2_i\cdot d_2\right)}{Z_2\cdot\cos\left(k2_i\cdot d_2\right) + i\cdot Z3_i\cdot\sin\left(k2_i\cdot d_2\right)}\right)$$

$$\rho_i := \frac{Z2_i - Z_0}{Z2_i + Z_0}$$

$$\left|\rho_1\right| = 0.086 \qquad \frac{\arg\left(\rho_1\right)}{\deg} = 175.78$$

$$\left|\rho_2\right| = 0.455 \qquad \frac{\arg\left(\rho_2\right)}{\deg} = -101.327$$

$$\left(\left|\rho_1\right|\right)^2 = 7.424\times 10^{-3}$$

$$\left(\left|\rho_2\right|\right)^2 = 0.207$$

(3)

$S = \frac{(1 + |\rho|)}{1 - |\rho|}$ $S := 2.7$ $n1 := 1$ refractive index for air

$\rho := \frac{-(S-1)}{S+1}$ negative sign because interface is an electric field minimum

$\rho = -0.459$ reflection coefficient

$\frac{(1 + |\rho|)}{1 - |\rho|} = 2.7$ check on VSWR

$\rho = \frac{n1 - n2}{n2 + n1}$ reflection coefficient in normal incidence

$n2 := -n1 \cdot \frac{(\rho - 1)}{(\rho + 1)}$

$n2 = 2.7$

$\varepsilon 2 := n2^2$

$\varepsilon 2 = 7.29$ dielectric constant

θ

(4) $n1 := \sqrt{4}$ $n2 := \sqrt{8}$ refractive indices involved

$\theta 1 := 30 \cdot deg$

$\theta 2 := asin\left(\frac{n1 \cdot sin(\theta 1)}{n2}\right)$

$\frac{\theta 2}{deg} = 20.705$ refraction angle

effective impedances

$Z21 := \frac{Z_0}{n2} \cdot cos(\theta 2)$

$Z11 := \frac{Z_0}{n1} \cdot cos(\theta 1)$

$$\rho := \frac{\frac{Z_0}{n2} \cdot cos(\theta 2) - \frac{Z_0}{n1} \cdot cos(\theta 1)}{\frac{Z_0}{n2} \cdot cos(\theta 2) + \frac{Z_0}{n1} \cdot cos(\theta 1)}$$

The normalized load is $\zeta_L := \frac{Z21}{Z11}$

$\zeta_L = 0.764$

$\rho = -0.134$

$|\rho| = 0.134$

$\frac{\arg(\rho)}{deg} = 180$

$T := 1 - (|\rho|)^2$

$T = 0.982$

$S := \frac{1 + |\rho|}{1 - |\rho|}$

$S = 1.309$

The normalized load is to the left of the center of the Smith Chart so the load is an E field minimum. The nearest E field maximum is half a wavelength in front of the boundary.

(5) θ1=arc tan (n2/n1) at Brewster's angle

θ2=arctan (n1/n2) at Brewster's angle

Clearly tan(θ1)=cot(θ2), so θ1+θ2=90 degrees

(6) At the critical angle coming up from below the water surface

$\theta 1 := \operatorname{asin}\left(\frac{1}{\sqrt{70}}\right)$

$\theta 2 := \operatorname{asin}\left(\sqrt{70} \cdot \sin(\theta 1)\right)$

$\frac{\theta 2}{deg} = 90$ At critical angel refracted ray is at 90 degrees

$\frac{\theta 1}{deg} = 6.865$ this is the cone semi-vertical angle below the water that allows rays to escape

(7) To make an AR layer we need $Z21 = \sqrt{Z11 \cdot Z31}$

where Z11, Z21, Z31 are effective impedances,so

$$\frac{Z_0}{n2} \cdot \cos(\theta 2) = \sqrt{\frac{Z_0}{n1} \cdot \cos(\theta 1) \cdot \frac{Z_0}{n_3} \cdot \cos(\theta 3)}$$ for P-waves

$$\frac{Z_0}{n2} \cdot \frac{1}{\cos(\theta 2)} = \sqrt{\frac{Z_0}{n1} \cdot \frac{1}{\cos(\theta 1)} \cdot \frac{Z_0}{n_3} \cdot \frac{1}{\cos(\theta 3)}}$$ for S-waves

Clearly we need $n2 = \sqrt{n1 \cdot n3}$ and a $\lambda/4$ layer

$$\theta 1 := 45 \cdot deg$$

$$\theta 2 := \operatorname{asin}\left(\frac{1 \cdot \sin(\theta 1)}{\sqrt{\sqrt{10}}}\right)$$

$$\frac{\theta 2}{deg} = 23.43$$

$$\theta 3 := \operatorname{asin}\left(\frac{\sqrt{\sqrt{10}} \cdot \sin(\theta 2)}{\sqrt{70}}\right)$$

$$\frac{\theta 3}{deg} = 4.848$$

$$\theta 3 := \operatorname{asin}\left(\frac{1. \sin(\theta 1)}{\sqrt{70}}\right)$$

alternative way of finding $\theta 3$

$$\frac{\theta 3}{deg} = 4.848$$

Layer thickness must be $d\cos(\theta 2)=\lambda 2/4$

Write $d/\lambda 2=F$

$$F := \frac{1}{4(\cos(\theta 2))}$$

$F = 0.272$

Actual thickness if 0.272 λ