ENEE 381 Problem Set #7, 2002. SOLUTIONS

(1) $a := 15 \cdot 10^{-3}$ conductor spacing

$c_0 := 2.998 \cdot 10^{8}$ velocity of light in a vacuum

The formula for the cutoff frequency is

$\nu_c = \dfrac{m \cdot c}{2 \cdot a}$ c is velocity of light in material between planes

The TE_m and TM_m have the same cutoff frequency, which is determined by the integer m

In air, for m=1

$$\nu_c := \frac{c_0}{2 \cdot a}$$

$\nu_c = 9.993 \times 10^{9}$ 10GHz

For m=2

$$\nu_c := \frac{2 \cdot c_0}{2 \cdot a}$$

$\nu_c = 1.999 \times 10^{10}$ 20GHz

For m=3

$$\nu_c := \frac{3 \cdot c_0}{2 \cdot a}$$

$\nu_c = 2.998 \times 10^{10}$ 30GHz

With ε_r=4 $c := \dfrac{c_0}{2}$

so corresponding cuttoff frequencies are 5GHz, 10GHz, and 15GHz

With an air-filled line and 8GHz excitation, all modes are cutoff. In this case the attenuation constant α is

$i := 1, 2 .. 3$ $\nu := 8 \cdot 10^{9}$ $\nu_{c_1} := 10 \cdot 10^{9}$ $\nu_{c_2} := 20 \cdot 10^{9}$ $\nu_{c_3} := 30 \cdot 10^{9}$

$$\alpha_i := i \cdot \frac{\pi}{a} \cdot \sqrt{1 - \left(\frac{\nu}{\nu_{c_i}}\right)^2}$$

$\dfrac{1}{\alpha_1} = 7.958 \times 10^{-3}$ $\dfrac{1}{\alpha_2} = 2.605 \times 10^{-3}$ $\dfrac{1}{\alpha_3} = 1.651 \times 10^{-3}$

(2) The formula for the cutoff frequency is for a mode $TE_{m,n}$ or $TM_{m,n}$ is

$$\nu_{c_{m,n}} = \frac{c}{2\cdot\pi}\cdot\sqrt{\left(\frac{m\cdot\pi}{a}\right)^2 + \left(\frac{n\cdot\pi}{b}\right)^2}$$

$a := 30\cdot 10^{-3}$ $b := 15\cdot 10^{-3}$

$\nu := 10\cdot 10^{9}$

For an air-filled guide $c := c_0$

$m := 1, 2 .. 3$ $n := 0, 1 .. 2$ different mode subscripts

$$\nu_{c_{m,n}} := \frac{c}{2\cdot\pi}\cdot\sqrt{\left(\frac{m\cdot\pi}{a}\right)^2 + \left(\frac{n\cdot\pi}{b}\right)^2}$$

$\nu_{c_{1,0}} = 4.997 \times 10^{9}$ $\nu_{c_{1,1}} = 1.117 \times 10^{10}$ $\nu_{c_{1,2}} = 2.06 \times 10^{10}$

$\nu_{c_{2,1}} = 1.413 \times 10^{10}$ $\nu_{c_{2,2}} = 2.235 \times 10^{10}$ $\nu_{c_{2,0}} = 9.993 \times 10^{9}$

The attenuation constant is

$$\alpha_{m,n} := \sqrt{\left(\frac{m\cdot\pi}{a}\right)^2 + \left(\frac{n\cdot\pi}{b}\right)^2}\cdot\sqrt{1 - \left(\frac{\nu}{\nu_{c_{m,n}}}\right)^2}$$

No attenuation for TE_{10} mode and TE_{20} mode. For other modes

$\frac{1}{\alpha_{1,1}} = 9.575 \times 10^{-3}$ $\frac{1}{\alpha_{2,2}} = 2.388 \times 10^{-3}$

$\frac{1}{\alpha_{1,2}} = 2.649 \times 10^{-3}$ $\frac{1}{\alpha_{2,1}} = 4.778 \times 10^{-3}$

With a dielectric $c := \frac{c_0}{2}$

$$\nu_{c_{m,n}} := \frac{c}{2\cdot\pi}\cdot\sqrt{\left(\frac{m\cdot\pi}{a}\right)^2 + \left(\frac{n\cdot\pi}{b}\right)^2}$$

$\nu_{c_{1,0}} = 2.498 \times 10^{9}$ $\nu_{c_{1,1}} = 5.586 \times 10^{9}$ $\nu_{c_{1,2}} = 1.03 \times 10^{10}$

$\nu_{c_{2,1}} = 7.066 \times 10^{9}$ $\nu_{c_{2,2}} = 1.117 \times 10^{10}$ $\nu_{c_{2,0}} = 4.997 \times 10^{9}$

The attenuation constant is

$$\alpha_{m,n} := \sqrt{\left(\frac{m\cdot\pi}{a}\right)^2 + \left(\frac{n\cdot\pi}{b}\right)^2}\cdot\sqrt{1 - \left(\frac{\nu}{\nu_{c_{m,n}}}\right)^2}$$

No attenuation for TE_{10}, TE_{20}, TE_{11} and TM_{11} modes. For other modes

$$\frac{1}{\alpha_{1,2}} = 9.653 \times 10^{-3} \qquad \frac{1}{\alpha_{2,2}} = 4.787 \times 10^{-3}$$

(3) $n := 1.5$ $\theta_1 := 80\cdot\text{deg}$ Take λ=1μm $\lambda := 10^{-6}$

$$\theta_2 := \text{asin}(n\cdot\sin(\theta_1))$$

$\theta_2 = 1.571 - 0.942i$ Note that because of total internal reflection θ_2 is complex

$$\cos(\theta_2) = 1.087i$$

Take the wave to be a P-wave

$$\rho := \frac{n\cdot\cos(\theta_2) - \cos(\theta_1)}{n\cdot\cos(\theta_2) + \cos(\theta_1)}$$

$(|\rho|)^2 = 1$ all energy reflects

The propagation constant of the wave in the z-direction (perpendicular to the boundary) on the air side of the boundary is with $k_{air} := 2\cdot\frac{\pi}{\lambda}$

$$k_z := k_{air}\cdot\cos(\theta_2)$$

$k_z = 6.832i \times 10^6$ Note that this is imaginary

$$\alpha := \text{Im}(k_z)$$

$$\alpha = 6.832 \times 10^6$$

$$\frac{1}{\alpha} = 1.464 \times 10^{-7}$$

$\frac{1}{\alpha\cdot\lambda} = 0.146$ penetrates about 0.146 wavelengths

(4) For the TE_{10} mode the cutoff frequency is $\nu_c = \frac{c}{2\cdot a}$

$\nu := 10\cdot 10^9$ input frequency $c := 2.998\cdot 10^8$ velocity of light

In air $\nu_c := \frac{c}{2\cdot a}$ $a := 50\cdot 10^{-3}$ $b := 10\cdot 10^{-3}$ guide dimensions

$\nu_c = 2.998\times 10^9$ Note that $\frac{c}{2\cdot b} = 1.499\times 10^{10}$ so mode only propagates in one orientation

In dielectric-filled section $c := \frac{c0}{2}$

$\lambda_0 := \frac{c0}{\nu}$ wavelength in air

$\nu d_c := \frac{c}{2\cdot a}$ new cutoff frequency

$\lambda d := \frac{c0}{2\cdot\nu}$ wavelength in dielectric

$\nu d_c = 1.499\times 10^9$

The guide wavelength in the air-filled section is

$$\lambda_g := \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{2\cdot a}\right)^2}} \qquad \lambda_g = 0.031 \qquad \lambda_0 = 0.03$$

The guide wavelength in the dielectric-filled section is

$$\lambda_g := \frac{\lambda d}{\sqrt{1-\left(\frac{\lambda d}{2\cdot a}\right)^2}} \qquad \lambda_g = 0.015 \qquad \lambda d = 0.015$$

The TE mode impedances are, using Z_0=376.7 ohm $Z_0 := 376.7$

$$ZTEair := \frac{376.7}{\sqrt{1-\left(\frac{\lambda_0}{2\cdot a}\right)^2}} \qquad ZTEair = 394.863$$

ohms

$$ZTEdiel := \frac{376.7}{2\cdot\sqrt{1-\left(\frac{\lambda d}{2\cdot a}\right)^2}} \qquad ZTEdiel = 190.502$$

alternative expressions

$$ZTEair := \frac{Z_0}{\sqrt{1-\left(\frac{\nu_c}{\nu}\right)^2}} \qquad ZTEair = 394.863$$

$$ZTEdiel := \frac{Z_0}{2\cdot\sqrt{1-\left(\frac{\nu d_c}{\nu}\right)^2}} \qquad ZTEdiel = 190.502$$

Use the transformed impedance formula, write

$Z_L := ZTEair \qquad Z_2 := ZTEdiel \qquad Z_1 := ZTEair$

$d := 10\cdot10^{-3}$ thickness of slab

$\beta l := 2\cdot\frac{\pi}{\lambda_g}\cdot d$ phase factor to use in transformed impedance equation see for example Eq. 5.7 (13) in RWD

$$Z_i := Z_2\cdot\frac{\left(Z_L\cdot\cos(\beta l) + i\cdot Z_2\cdot\sin(\beta l)\right)}{Z_2\cdot\cos(\beta l) + i\cdot Z_L\cdot\sin(\beta l)}$$

transformed impedance formula

$Z_i = 118.156 - 85.223i$

$$\rho := \frac{(Z_i - Z_1)}{Z_i + Z_1}$$

reflection coefficient

The fraction of energy transmitted is

$T := 1 - (|\rho|)^2$

$T = 0.69$