

**ENEE 381 First Examination**  
**October 8, 2002, 12:30 - 1:45pm**  
**ANSWER 3 QUESTIONS**

*If more than 3 are answered, best 3 will count*

(1) A linearly polarized plane electromagnetic wave propagating in the  $z$ -direction has an electric field that can be represented as

$$E_x = E_0 e^{j(\omega t - kz)}.$$

Use  $\text{curl} \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$  to find the magnetic field of this wave. If the wave has amplitude  $E_0 = 1 \text{ V/m}$  and is propagating through a material with  $\epsilon_r = 10$ ,  $\mu_r = 1$ , what is the magnetic field amplitude? (5pts.) What is the average Poynting vector of the wave? (2pts.)

If the electric field and magnetic field happened to be of the form

$$E_x = 10 e^{j(\omega t - kz + \pi/5)}$$

and

$$H_y = \frac{10}{Z} e^{j(\omega t - kz - \pi/3)}$$

What would be the value of the average Poynting vector? (3pts.)

(2) Calculate the total energy flux  $W$  (Watts) into a closed volume  $V$  by using

$$W = - \oint (\mathbf{E} \times \mathbf{H}) \cdot d\mathbf{S}.$$

Use the vector identity

$$\text{div}(\mathbf{E} \times \mathbf{H}) = \mathbf{H} \cdot \text{curl} \mathbf{E} - \mathbf{E} \cdot \text{curl} \mathbf{H}$$

to show where the energy flowing into the volume goes in terms of stored energy and ohmic dissipation. (5pts.)

A circular wire of radius 1mm and conductivity  $3 \times 10^7 \text{ S/m}$  has a low frequency voltage applied to it that gives a field in the wire of 1V/m. Show that the ohmic dissipation per unit length in the wire is equal to the Poynting vector flux into unit length of the cylindrical surface of the wire. (5pts.)

(3) Use Maxwell's equations to show that when a plane electromagnetic traveling in the  $+z$ -direction enters a conductor filling the space  $z > 0$  that the field amplitudes decay as (4pts.)

$$E_x \sim e^{-z/\delta}.$$

Derive an expression for  $\delta$  in terms of the frequency and conductivity of the metal. (2pts.)

If the magnetic field right at the surface of the conductor ( $z = 0^+$ ) is 3A/m, what is the total surface current (A/m) in the conductor? (2pts.) Approximately, what is the electric field just at the surface if the metal is a very good conductor? (2pts.)

(4) Do two of the following:

- (a) A transmission line of characteristic impedance 75 ohm is terminated in a load of  $30+j50$  ohm. What is the reflection coefficient in magnitude and phase.
- (b) Explain how the scalar potential definition  $\mathbf{E} = -\text{grad}\phi$  has to be modified when electromagnetic fields change with time.
- (c) What is the pressure on a very good conductor when a plane wave of intensity (average value of the Poynting vector) of  $10\text{W}/\text{m}^2$  strikes the conductor normally and then reflects off.

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ F/m}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ Henry/m.}$$

$$c_0 = 2.998 \times 10^8 \text{ m/s}$$