ENEE 381 Problem Set # 2

SOLUTIONS

(1) The waves from the radio transmitter are distributed in all directions over the surface of a sphere of radius R.

The power flux (Watts / m^2) at radius R is

$$I = \frac{P}{4\pi R^2}$$

The power flux is often called the Intensity, I. It is the average value of the magnitude of the Poynting vector

$$I = \overline{|\underline{S}|} = \overline{\underline{E} \times \underline{H}} = \frac{E^2}{2Z_0}$$

Therefore
$$E = \sqrt{2Z_0 I} = \sqrt{\frac{2Z_0 P}{4\pi R^2}} = \sqrt{\frac{Z_0 P}{2\pi R^2}}$$

Plug in $P = 4W$, $R = 10^3$ m, $Z_0 = 376.7$ ohm

$$\frac{E}{H} = Z_0 \quad \text{Therefore} \longrightarrow \quad \begin{aligned} E &= 1.55 \times 10^{-2}\ V/m \\ H &= 4.11 \times 10^{-5}\ A/m \end{aligned}$$

For a laser beam with $I = 10\,GW/cm^2 = 10^{14}\ W/m^2$

$$E = \sqrt{2Z_0 I} = \underline{2.74 \times 10^8\ V/m}$$

$$H = \frac{E}{Z_0} = \underline{7.29 \times 10^5\ A/m}$$

(2) $E = \sqrt{\dfrac{Z_0 P}{2\pi R^2}}$

Plug-in $Z_0 = 376.7$

$P = 1\,W$

$R = 50 \times 10^{-3}\,m$

$\underline{E = 155\,V/m}$

$\underline{H = 0.41 A/m}$

If we assume that 1W is absorbed in a sphere of radius 100 mm then the average energy absorption is

$$u = \frac{P}{\frac{4}{3}\pi R^3} = 23.9\,J\,s^{-1}\,m^{-3}$$

If we take a reasonable value for the density equal to the density of water $= 1kg/m^3$ then the mass absorbs $0.024\,J/kg \equiv 2.39 \times 10^{-5}\,J/g$

Now 1 calorie $= 4.2J$, therefore the absorbed power is 5.69×10^{-6} calories/g.

Therefore the rate of heating is 5.69×10^{-6} degrees/sec

(3) P.7-7 $\Phi(t) = \bar{B}(t)\cdot\bar{S}(t) = -(5\cos\omega t)\times 0.2\,(0.7-x)$

$$= -0.35\cos\omega t\,(1+\cos\omega t) \quad (mT),$$

$$i = -\frac{1}{R}\frac{d\Phi}{dt} = -\frac{1}{R}\,0.35\,\omega(\sin\omega t + \sin 2\omega t)$$

$$= -1.75\,\omega\sin\omega t\,(1+2\cos\omega t) \quad (mA).$$

(4) P.7-10 a) $\mu_r = 1 + \chi_m,\quad \chi_m = 5000-1 = 4999,$

$$\bar{H} = \frac{\bar{M}}{\chi_m} = \bar{a}_z\frac{M_0}{4999};\qquad \bar{B} = \mu_0\mu_r\bar{H} = \bar{a}_z\frac{5000}{4999}\mu_0 M_0.$$

b) $V_0 = \oint \bar{u}\times\bar{B}\cdot d\bar{\ell} = \int_b^a (\bar{a}_\phi\,\omega r)\times(\bar{a}_z B)\cdot\bar{a}_r\,dr$

$$= -\frac{\omega B}{2}(b^2-a^2) = -\frac{2500}{4999}\mu_0 M_0\,\omega(b^2-a^2).$$

c) $\bar{E}_r = \bar{E}_r' - \bar{u}\times\bar{B} = \frac{\bar{J}_r}{\sigma} - (\bar{a}_\phi u)\times(\bar{a}_z B) = \bar{a}_r\left(\frac{i}{2\pi r h\sigma} - \omega r B\right)$

at r

Induced voltage $V = \int_a^b E_r\,dr = \frac{i}{2\pi h\sigma}\ln\frac{b}{a} - \frac{\omega B}{2}(b^2-a^2).$

$$= iR + V_0.$$

Short circuit: $V_0 = 0,\quad i_{sc} = \frac{\omega B}{2R}(b^2-a^2)$, where $R = \frac{\ln(b/a)}{2\pi h\sigma}.$

(5) P.7-23 $\bar{E}_1(z,t) = \bar{a}_x\,0.03\sin 10^8\pi\left(t-\frac{z}{c}\right) = \bar{a}_x\,\mathcal{R}e\left[0.03\,e^{-j\pi/2}e^{j10^8\pi(t-z/c)}\right],$

$$\bar{E}_2(z,t) = \bar{a}_x\,0.04\cos\left[10^8\pi\left(t-\frac{z}{c}\right)-\frac{\pi}{3}\right] = \bar{a}_x\,\mathcal{R}e\left[0.04\,e^{-j\pi/3}e^{j10^8\pi(t-z/c)}\right].$$

Phasors: $\bar{E} = \bar{E}_1 + \bar{E}_2 = \bar{a}_x\left[0.03\,e^{-j\pi/2} + 0.04\,e^{-j\pi/3}\right]$

$$= \bar{a}_x\left[-j0.03 + (0.02 - j\,0.02\sqrt{3})\right] = \bar{a}_x(0.068\,e^{-j1.27}) = \bar{a}_x E_0 e^{j\theta}.$$

$\therefore\ E_0 = 0.068,\qquad \theta = -1.27$ (rad), or -72.8°

(b) An intensity I (Watts/m^2) corresponds to a photon flux $\frac{I}{h\nu}$ photons/m^{2}s

each photon carries momentum $\frac{h}{\lambda}$

For a reflective surface the momentum change per square meter per second is

$$p = \left(\frac{I}{h\nu}\right) 2 \left(\frac{h}{\lambda}\right) = \frac{2I}{c}$$

reflection reverses momentum

The force on area A is

$$F = \frac{2IA}{c}$$

The acceleration is $a = \frac{2IA}{mc}$

where m is the mass.

To reach velocity v takes a time t where $v = at$. Therefore for $v = \frac{c}{10}$

$$t = \frac{c}{10a} = \frac{mc^2}{20IA}$$

Plugging $c = 3\times10^8$ m/s, $M = 10^4$ kg, $A = 10^8$ m^2, $I = 1$

$$t = \underline{4.5\times10^{11} \text{ secs}} = \underline{14,269 \text{ years}}$$