ENEE 381 Problem Set #3

3/4/03- due 3/11/03

Questions like (1) - (5) could be on the first examination.

(1) The electric vector of a wave propagating in the z-direction varies according to

$$E_y = E_0 \cos(\pi x/2a) e^{j\omega t},$$

where E_0=1V/m, a=1m. The frequency of the wave is ν=100MHz. How much energy flow per second passes through the region $-1 \leq x \leq 1$ (m), $-1 \leq y \leq 1$ (m).

(2) A point source transmitter at (0,0,0) emits a total power of 5W. What is the value of the Poynting vector at the point (10,10,10)? What is the total power flux into the surface bounded by the two concentric spheres R=5, and R=7?

(3) A point source transmitter at (0,0,0) emits a total power of 5W. What is the total power flux through the surface of a cube centered at (0,0,0) with sides of length 1m?

(4) A plane wave with magnetic field $H_y = 1$A/m and electric field E_x traveling in the z-direction through a vacuum strikes an infinite planar copper medium. What is the value of the electric field and magnetic field at the surface of the sheet? What is the value of the surface resistance R_s? How much energy is dissipated per unit area of the copper? For copper σ=5.8$\times$10^7 S/m.

(5) How are the answers to question (4) modified if the wave is traveling through a dielectric with ϵ_r=30 when it strikes the copper

(6) Cheng Problem 8-11

(7) Cheng Problem 8-12

(8) Cheng Problem 8-18

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SOLUTIONS

$$E_y = E_0 \cos\left(\frac{\pi x}{2a}\right) e^{j(\omega t - kz)}$$

From the curl equation $\frac{\partial E_y}{\partial z} = \mu \frac{\partial H_x}{\partial t}$

so

$$-jk\, E_0 \cos\left(\frac{\pi x}{2a}\right) e^{j(\omega t - kz)} = \mu j \omega H_x$$

$$H_x = -\frac{k E_0}{\mu \omega} \cos\left(\frac{\pi x}{2a}\right) e^{j(\omega t - kx)}$$

$$= -\frac{E_0}{Z} \cos\left(\frac{\pi x}{2a}\right) e^{j(\omega t - kx)}$$

$Z = \sqrt{\frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}}$. Assuming $\varepsilon_r = \mu_r = 1$ $Z = 376.7\,\text{ohm}$

The Poynting vector is

$$S_z = -E_y H_x = \frac{E_0^2}{Z} \cos^2\left(\frac{\pi x}{2a}\right) e^{2j(\omega t - kx)}$$

Time averaging $\overline{S_z} = \frac{E_0^2}{2Z} \cos^2\left(\frac{\pi x}{2a}\right)$

Power flow required is $\frac{E_0^2}{2Z} \int_{-1}^{1} \int_{-1}^{1} \cos^2\left(\frac{\pi x}{2a}\right) dx\, dy = \overline{S_z}$

$$\overline{S_z} = \frac{E_0^2}{2Z} \int_{-1}^{1} \left(1 + \cos\left(\frac{\pi x}{a}\right)\right) dx$$

$$\overline{S_z} = \frac{E_0^2}{2Z} \frac{a}{\pi} \left(x + \sin \frac{\pi x}{a}\right)_{-1}^{1}$$, with $E_0 = 1\,V/m$, $a = 1$

$\overline{S_z} = 845\,\mu W$.

(2) The distance from $(0,0,0)$ to $(10,10,10)$

is $R = \sqrt{100+100+100} = \sqrt{300}$

The Poynting vector at this point is

$$S_r = \frac{P}{4\pi R^2}\,\hat{\underline{r}} = \frac{5}{4\pi \times 300} = 1.33\ mW/m^2$$

$\uparrow$
area of surface
of sphere

Flux of energy into $R=5$ flows out of $R=7$ so no net flux into the volume between the 2 spheres

(3) all the power of 5 W flows outwards through the surface of the cube surrounding the origin, So, ANSWER IS 5 W

(4) Copper is a very good conductor so E_x (at the surface) $\simeq 0$

H_y at the surface is 2 A/m (incident wave + reflected wave)

Therefore the surface current $J_s = 2A/m$

The surface resistance is $R_s = \frac{1}{\sigma \delta}$

$\delta = \frac{1}{\sqrt{\pi \nu \mu \sigma}}$. The electric field at the surface is $(E_x)_0 = (1+j)\frac{J_s}{\sigma \delta}$

The energy dissipated per unit area is $\frac{1}{2} R_s J_s^2$

For copper $\delta = \dfrac{0.066}{\sqrt{\nu}}$

For $\nu = 100\,MHz$ $\quad \delta = 6.6 \times 10^{-6}\ m$

$\dfrac{1}{\sigma \delta} = 2.6 \times 10^{-3}$ ohm $= R_s$

Energy dissipated is $5.225\ mW/m^2$

$(E_x)_0 = (1+j)\, 5.25 \times 10^{-3}\ V/m$

(see page 154 in text)

(5) If the wave travels through a dielectric with $\varepsilon_r = 30$, the magnetic field is still $H_y = 1A/m$ (incident wave) $H_y = 2A/m$ at the surface

so J_s is the same, R_s is the same and $(E_x)_0$ is the same. In other words answer is same as question (4).

(6) P.8-11 $f = 3\times10^9$ (Hz), $\epsilon_r = 2.5$, $\tan\delta_c = \frac{\epsilon''}{\epsilon'} = 10^{-2}$.

a) Eq. (8-48): $\alpha = \frac{\omega\epsilon''}{2}\sqrt{\frac{\mu}{\epsilon'}} = \frac{\omega}{2}\left(\frac{\epsilon''}{\epsilon'}\right)\frac{\sqrt{\epsilon_r}}{c} = 0.497$ (Np/m),

$$e^{-\alpha x} = \frac{1}{2} \longrightarrow x = \frac{1}{\alpha}\ln 2 = 1.395 \text{ (m)}.$$

b) Eq. (8-50): $\eta_c = \frac{1}{\sqrt{\epsilon_r}}\sqrt{\frac{\mu_0}{\epsilon_0}}\left(1 + j\frac{\epsilon''}{2\epsilon'}\right) = 238(1+j0.005) = 238\underline{/0.29^\circ}$ (Ω),

Eq. (8-49): $\beta = \omega\sqrt{\mu\epsilon}\left[1 + \frac{1}{8}\left(\frac{\epsilon''}{\epsilon}\right)^2\right] = 31.6\pi$ (rad/m),

$$\lambda = \frac{2\pi}{\beta} = 0.063 \text{ (m)},$$

$$u_p = \frac{\omega}{\beta} = 1.8973\times10^8 \text{ (m/s)},$$

$$u_g = \frac{1}{\frac{d\beta}{d\omega}} = \frac{c}{\sqrt{\epsilon_r}}\left[1 + \frac{1}{8}\left(\frac{\epsilon''}{\epsilon'}\right)^2\right] = 1.8975\times10^8 \text{ (m/s)}.$$

c) At $x = 0$, $\bar{E} = \bar{a}_y e^{j\pi/3}$,

$$\bar{H} = \frac{1}{\eta_c}\bar{a}_x\times\bar{E} = \bar{a}_z\, 0.210\, e^{j(\frac{\pi}{3} - 0.0016\pi)}.$$

$$\bar{H}(x,t) = \bar{a}_z\, 0.210\, e^{-0.497x}\sin(6\pi10^9 t - 31.6\pi x + 0.332\pi) \text{ (A/m)}.$$

(7) P.8-12 $\sigma/\omega\epsilon = 4/10^{10}\pi\times80\times(\frac{1}{36}\times10^{-9}) = 0.18$.

a) $\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]^{1/2} = 84$ (Np/m),

$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]^{1/2} = 300\pi$ (rad/m),

$\eta_c = \sqrt{\frac{\mu}{\epsilon_c}} = \frac{120\pi}{\sqrt{\epsilon_r}\left[1+(\sigma/\omega\epsilon)^2\right]^{1/4}} e^{\frac{j}{2}\tan^{-1}(\sigma/\omega\epsilon)} = 41.8\, e^{j0.0283\pi}$ (Ω),

$u_p = \omega/\beta = 33.3\times10^6$ (m/s), $\lambda = 2\pi/\beta = 0.67$ (cm), $\delta = \frac{1}{\alpha} = 1.19$ (cm).

b) $e^{-\alpha y} = \frac{1}{10}$, $y = \frac{1}{\alpha}\ln 10 = 2.74$ (cm).

c) $\bar{H}(y,t) = \bar{a}_x\, 0.1\, e^{-84\times0.5}\sin(10^{10}\pi t - 300\pi\times0.5 - \pi/3)$

$= \bar{a}_x\, 5.75\times10^{-20}\sin(10^{10}\pi t - \pi/3)$ (A/m),

$\bar{E}(y,t) = \mathcal{I}m\left[\eta_c \bar{H}(y)\times\bar{a}_y\right]e^{j\omega t} = \bar{a}_z 2.41\times10^{-18}\sin(10^{10}\pi t - \frac{\pi}{3} + 0.0283\pi)$ (V/m).

(8) P.8-18 $\bar{E} = \bar{a}_\theta E_\theta + \bar{a}_\phi E_\phi$,

$$\bar{H} = \frac{1}{\eta}\bar{a}_R\times\bar{E} = \frac{1}{\eta}(\bar{a}_\phi E_\theta - \bar{a}_\theta E_\phi).$$

$$\bar{\mathcal{P}}_{av} = \frac{1}{2}\mathcal{R}e(\bar{E}\times\bar{H}^*) = \bar{a}_z\frac{1}{2\eta}(|E_\theta|^2 + |E_\phi|^2).$$