ENEE 381 Problem Set #4

4/1/03 due 4/10/03

(1) Cheng problem 8-20

(2) Cheng problem 8-21

(3) A length of loss-less transmission line is first short-circuited at one end and then open-circuited. The impedance measured at the other end in the first case is Z_1 and Z_2 in the second. Prove that $Z_1 Z_2 = Z_0^2$. This is a convenient way for measuring the characteristic impedance of an unknown line.

(4) A 50 ohm transmission line is terminated with an impedance of 20-j30. What is the magnitude and phase of the reflection coefficient?

(5) Repeat question (4) using the Smith chart

(6) A 75 ohm transmission line is terminated with a load of $150 + j50$ ohm. Compute ρ in terms of both amplitude $|\rho|$ and ϕ. What fraction of incident power is absorbed in the load?

(7) Cheng problem 9.26

(8) Cheng problem 9.27

(9) Cheng problem 9.30

(10) Use Smith chart. A line with $Z_0 = 100\Omega$ is terminated with an unknown load. The SWR is found to be 3. A current maximum is observed 0.1λ from the load. What are:

(a) the load?
(b) the reflection coefficient ρ, magnitude and angle?
(c) how would you match the line without changing the load at the end of the line?

(11) Use Smith chart. A transmission line of characteristic impedance 75 ohm is terminated with an impedance 50+j125 ohm. 0.1λ from the load a 150ohm shorted stub 0.2λ long is connected in shunt to the main line. What are:

(a) The reflection coefficient in magnitude and phase at this point?
(b) The standing wave ratio?
(c) Where is the nearest current minimum that is greater than 0.1λ from the load?
(d) Where is the nearest point greater than 0.1λ from the load where the line can be matched with an open 75 ohm stub?

(12) Use Smith chart. Cheng problem 9.48

(13) Use Smith chart. Cheng problem 9-49.

(14) Use Smith chart. Cheng problem 9-50.

(15) Use Smith chart. Cheng problem 9-51.

ENEE 381 PROBLEM SET #4

SOLUTIONS

(1)

P. 8-20 a) $\delta = \dfrac{1}{\sqrt{\pi f \mu_0 \sigma}}$, $f = 10^4/2\pi$.

b) $\bar{H}(z,t) = \bar{a}_y H_0 e^{-z/\delta} \cos\left(10^4 t - \frac{z}{\delta}\right)$.

$$\eta_c = (1+j)\frac{\alpha}{\sigma} = (1+j)\frac{1}{\sigma\delta} = \frac{\sqrt{2}}{\sigma\delta} e^{j\pi/4}.$$

$$\bar{E}(z,t) = \bar{a}_x \frac{\sqrt{2}}{\sigma\delta} H_0 e^{-z/\delta} \cos\left(10^4 t - \frac{z}{\delta} + \frac{\pi}{4}\right).$$

c) $$\bar{\mathcal{P}}_{av} = \frac{1}{2}\mathcal{R}e\,(\bar{E} \times \bar{H}^*) = \bar{a}_z \frac{1}{2}\frac{\sqrt{2}}{\sigma\delta} H_0^2 \cos\frac{\pi}{4}$$
$$= \bar{a}_z \frac{1}{2}\left(\frac{H_0^2}{\sigma\delta}\right) \qquad (\mathrm{W/m^2}).$$

(2)

P. 8-21 Given $\bar{E}_i = E_0 (\bar{a}_x - j\bar{a}_y) e^{-j\beta z}$.

a) Assume reflected $\bar{E}_r(z) = (\bar{a}_x E_{rx} + \bar{a}_y E_{ry}) e^{j\beta z}$

Boundary condition at $z=0$: $\bar{E}_i(0) + \bar{E}_r(0) = 0$.

$\longrightarrow \bar{E}_r(z) = E_0(-\bar{a}_x + j\bar{a}_y) e^{j\beta z}$, a left-hand circularly polarized wave in $-z$ direction.

b) $\bar{a}_{n2} \times (\bar{H}_1 - \bar{H}_2) = \bar{J}_s \longrightarrow -\bar{a}_z \times [\bar{H}_i(0) + \bar{H}_r(0)] = \bar{J}_s$. ($\bar{H}_2 = 0$ in perfect conductor)

$$\bar{H}_i(0) = \frac{1}{\eta_0}\bar{a}_z \times \bar{E}_i(0) = \frac{E_0}{\eta_0}(j\bar{a}_x + \bar{a}_y), \quad \bar{H}_r(0) = \frac{1}{\eta_0}(-\bar{a}_z) \times \bar{E}_r(0) = \frac{E_0}{\eta_0}(j\bar{a}_x + \bar{a}_y).$$

$$\bar{H}_1(0) = \bar{H}_i(0) + \bar{H}_r(0) = \frac{2E_0}{\eta_0}(j\bar{a}_x + \bar{a}_y),$$

$$\bar{J}_s = -\bar{a}_z \times \bar{H}_1(0) = \frac{2E_0}{\eta_0}(\bar{a}_x - j\bar{a}_y).$$

c) $$\bar{E}_1(z,t) = \mathcal{R}e\left[\bar{E}_i(z) + \bar{E}_r(z)\right] e^{j\omega t}$$
$$= \mathcal{R}e\; E_0\left[(\bar{a}_x - j\bar{a}_y) e^{-j\beta z} + (-\bar{a}_x + j\bar{a}_y) e^{j\beta z}\right] e^{j\omega t}$$
$$= \mathcal{R}e\; E_0\left[-2j(\bar{a}_x - j\bar{a}_y)\sin\beta z\right] e^{j\omega t}$$
$$= 2E_0 \sin\beta z\,(\bar{a}_x \sin\omega t - \bar{a}_y \cos\omega t).$$

(3) The impedance observed a distance l from a load Z_L on a lossless line of impedance Z_0 is

$$Z_i = Z_0 \left[\frac{Z_L \cos kl + j Z_0 \sin kl}{Z_0 \cos kl + j Z_L \sin kl} \right]$$

If the line is shorted $Z_L = 0$ and

$Z_i = Z_1 = j Z_0 \tan kl$

If the line is open then $Z_L \to \infty$ and

$Z_i = Z_2 = -j Z_0 \cot kl$

clearly $Z_1 Z_2 = Z_0^2$.

(4) $Z_L = 20 - j30$ ohm $Z_0 = 50$ ohm

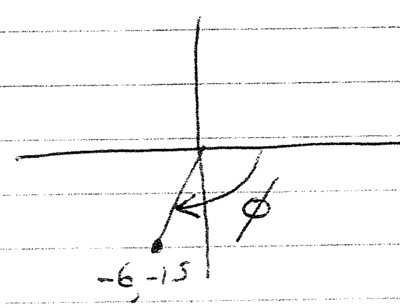
$$\rho = \frac{20 - j30 - 50}{20 - j30 + 50} = \frac{-30 - j30}{70 - j30}$$

$$\rho = \frac{-3 - j3}{7 - j3} = \frac{-(3+j3)(7+j3)}{58}$$

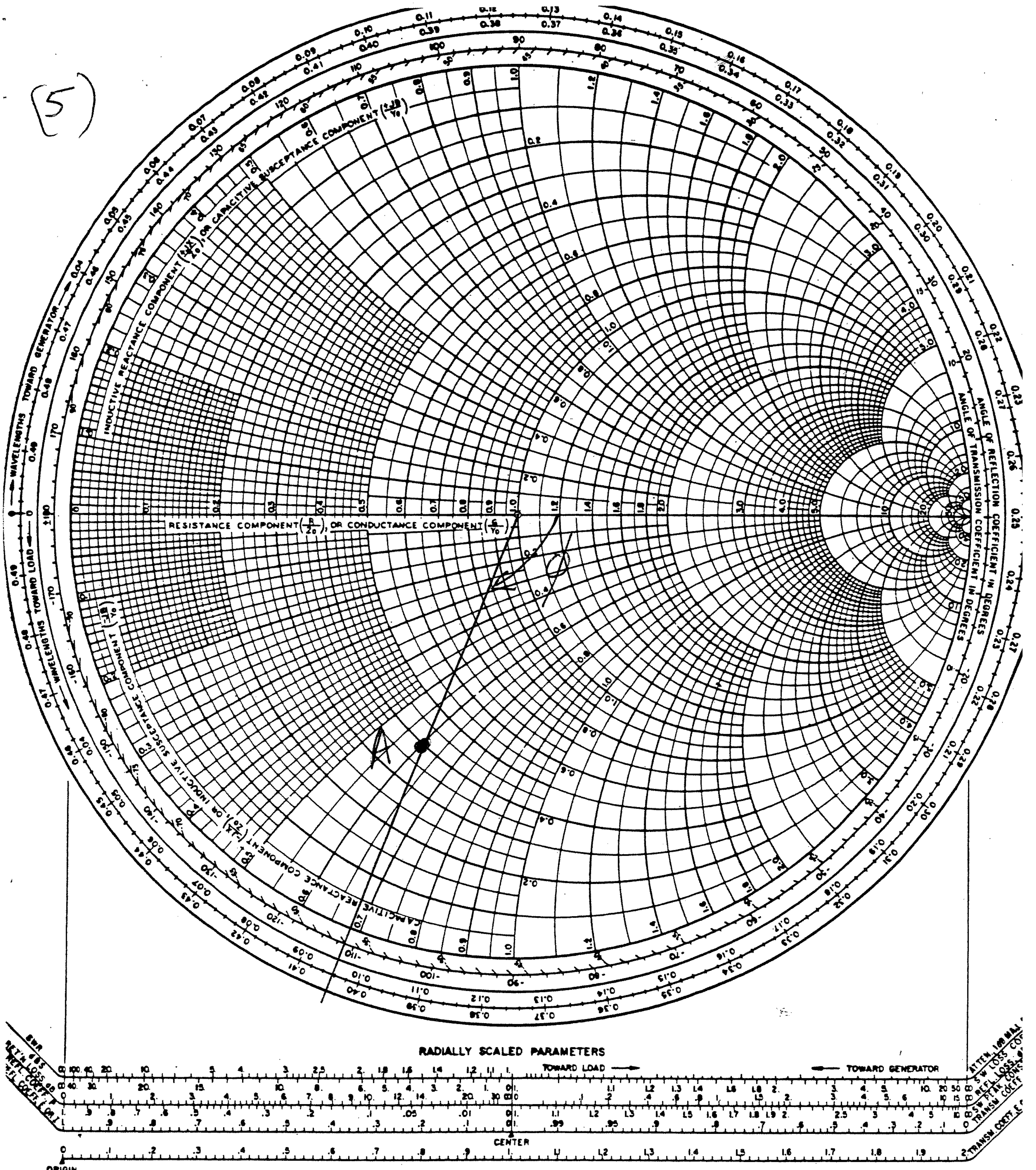
$$\rho = \frac{-12 - j30}{58} = \frac{-6 - j15}{29}$$

$|\rho| = 0.557$

$\phi = -111.8^\circ$



(5)


WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
INDUCTIVE REACTANCE COMPONENT, OR CAPACITIVE SUSCEPTANCE COMPONENT
RESISTANCE COMPONENT, OR CONDUCTANCE COMPONENT
CAPACITIVE REACTANCE COMPONENT, OR INDUCTIVE SUSCEPTANCE COMPONENT
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
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A
B
RADIALLY SCALED PARAMETERS
TOWARD LOAD
TOWARD GENERATOR
CENTER
ORIGIN

(5) see chart Normalized load is

$$z_L = 0.4 - j0.6$$

$$|\rho| = 0.56 \quad \phi = -111.8^\circ$$

(6) $Z_L = 150 + j50$

$Z_0 = 75$

Normalized load $z_L = \dfrac{150 + j50}{75} = 2 + j0.667$

$$\rho = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{75 + j50}{225 + j50} = \frac{3 + j2}{9 + j2} = \frac{(3 + j2)(9 - j2)}{81 + 4}$$

$$\rho = \frac{31 + 12j}{85} = |\rho| e^{j\phi}$$

$$|\rho| = \frac{1}{85}\sqrt{31^2 + 12^2} = 0.391 \qquad \phi = \tan^{-1}\frac{12}{31} = 21.16^\circ$$

Fraction of energy reflected is $|\rho|^2$, Fraction into load $1 - |\rho|^2 = 0.847$

(7) P.9-26 From Eq.(9-147): $Z_i = R_0' \dfrac{Z_L + jR_0't}{R_0' + jZ_Lt} \longrightarrow Z_L = R_0' \dfrac{Z_i - jR_0't}{R_0' - jZ_it}$, $t = \tan\beta\ell'$

$$Z_i = R_0' \frac{Z_L + jR_0't}{R_0' + jZ_Lt} \longrightarrow Z_L = R_0' \frac{Z_i - jR_0't}{R_0' - jZ_it}.$$

With $Z_i = 50$ (Ω) and $Z_L = 40 + j10$ (Ω), we have

$$40 + j10 = R_0' \frac{50 - jR_0't}{R_0' - j50t} \longrightarrow \begin{cases} 40R_0' + 500t = 50R_0', \\ 10R_0' - 2000t = -R_0'^2 t. \end{cases}$$

$\therefore R_0' = 38.7$ (Ω); $t = \tan\beta\ell' = 0.775 \longrightarrow \ell' = 0.105\lambda$.

(8) P.9-27 a) $|\Gamma| = \dfrac{S-1}{S+1} = \dfrac{2-1}{2+1} = \dfrac{1}{3}$.

Eq. (9-133a): $V(z') = \frac{I_L}{2}(Z_L + Z_0) e^{j\beta z'}\left[1 + |\Gamma| e^{j\phi}\right]$.

Eq. (9-134): $\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0} = |\Gamma| e^{j\theta_r}$, $\quad \phi = \theta_r - 2\beta z'$

Voltage is a minimum when $\phi = \pm\pi \longrightarrow \theta_r = 2\left(\frac{2\pi}{\lambda}\right) \times 0.3\lambda - \pi = 0.2\pi$.

$\therefore \Gamma = \frac{1}{3} e^{j0.2\pi}$.

b) $Z_L = Z_0\left(\dfrac{1+\Gamma}{1-\Gamma}\right) = 466 + j206$ (Ω).

c) Terminating resistance $R_m = \dfrac{R_0}{S} = \dfrac{300}{2} = 150$ (Ω),

$\ell_m = \frac{\lambda}{2} - z_m' = (0.5 - 0.3)\lambda = 0.2\lambda$.

Another set of solution is: $R_m' = SR_0 = 600$ (Ω) & $\ell_m' = 0.45\lambda$.

(9)

P.9-30 a) Given: $V_g = 0.1\angle 0^\circ$ (V), $Z_g = Z_0 = 50$ (Ω), $R_L = 25$ (Ω) $= 0.5 Z_0$.

$$V_i = \frac{Z_i}{Z_0 + Z_i} V_g, \qquad I_i = \frac{V_g}{Z_0 + Z_i},$$

where $Z_i = Z_0 \dfrac{0.5 Z_0 + jZ_0 \tan\beta\ell}{Z_0 + j0.5 Z_0 \tan\beta\ell} = Z_0 \dfrac{1 + j2\tan\beta\ell}{2 + j\tan\beta\ell}$.

$$\therefore \quad V_i = \frac{1 + j2\tan\beta\ell}{3(1 + j\tan\beta\ell)} V_g = \frac{1}{30}\left(\frac{1 + j2\tan\beta\ell}{1 + j\tan\beta\ell}\right) \text{ (V)},$$

$$I_i = \frac{2 + j\tan\beta\ell}{3Z_0(1 + j\tan\beta\ell)} V_g = \frac{2}{3}\left(\frac{2 + j\tan\beta\ell}{1 + j\tan\beta\ell}\right) \text{ (mA)}.$$

Setting $Z_g = Z_0$ and $\Gamma_g = 0$ in Eqs. (9-120a) and (9-120b), we have

$$V_L = V(z'=0) = \frac{V_g Z_0}{Z_0 + Z_g} e^{-j\beta\ell}(1+\Gamma) \qquad \left\{\Gamma = \frac{R_L - Z_0}{R_L + Z_0} = -\frac{1}{3}\right\}$$

$$= \frac{1}{30} e^{-j\beta\ell} \text{ (V)},$$

$$I_L = I(z'=0) = \frac{V_g}{Z_0 + Z_g} e^{-j\beta\ell}(1-\Gamma) = \frac{4}{3} e^{-j\beta\ell} \text{ (mA)}.$$

b) $$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = 2.$$

c) $$(P_{av})_L = \frac{1}{2}\mathcal{R}e(V_L I_L^*) = \frac{1}{2}\left(\frac{1}{30}\right)\left(\frac{4}{3}\times 10^{-3}\right) = 2.22\times 10^{-5} \text{ (W)} = 0.0222 \text{ (mW)}.$$

If $R_L = 50$ (Ω) $= Z_0$, $V_L = \dfrac{V_g}{2} e^{-j\beta\ell}$, $I_L = \dfrac{V_g}{2Z_0} e^{-j\beta\ell}$

$$\longrightarrow \text{Max.}(P_{av})_L = \frac{V_g^2}{8Z_0} = 2.50\times 10^{-5} \text{ (W)}.$$

(10) $Z_0 = 100\,\Omega$

VSWR = 3 POINT A on Chart

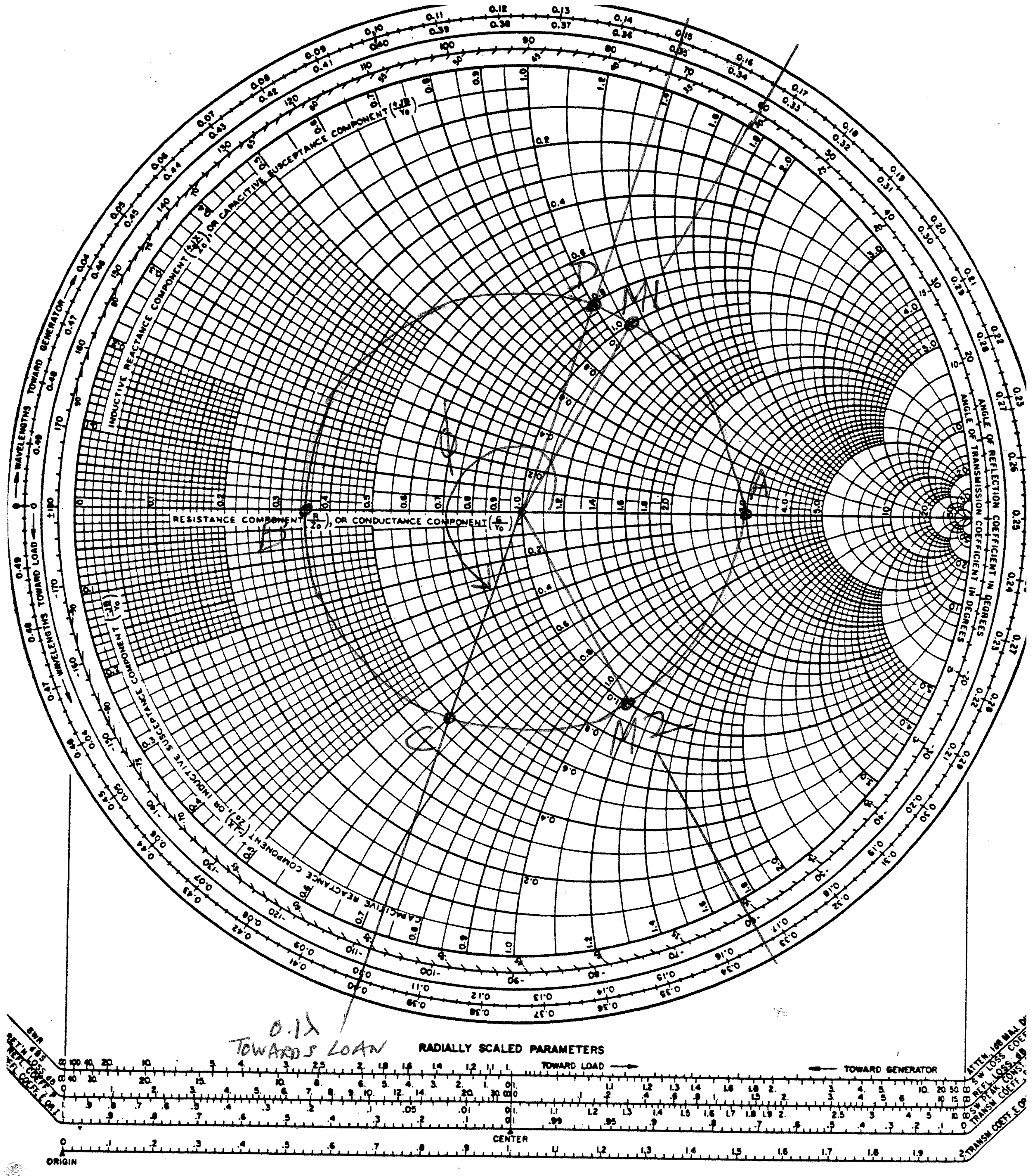
The current maximum is at point B
This is 0.1λ from load. GO 0.1λ Towards load from ~~point~~ B

point C is normalized load $0.5 - j0.6$
Actual load is $Z_L = 50 - j60$, Y_L IS AT POINT D

$|\rho| = 0.49$ $\phi = 252^\circ$

Could match at M1 with an inductor in parallel. M1 is 0.16λ towards generator

Could match at M2 with a capacitor in parallel. M2 is 0.204λ towards generator


WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
INDUCTIVE REACTANCE COMPONENT $\left(\frac{+jX}{Z_0}\right)$, OR CAPACITIVE SUSCEPTANCE COMPONENT $\left(\frac{+jB}{Y_0}\right)$
RESISTANCE COMPONENT $\left(\frac{R}{Z_0}\right)$, OR CONDUCTANCE COMPONENT $\left(\frac{G}{Y_0}\right)$
CAPACITIVE REACTANCE COMPONENT $\left(\frac{-jX}{Z_0}\right)$, OR INDUCTIVE SUSCEPTANCE COMPONENT $\left(\frac{-jB}{Y_0}\right)$
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES
D
M1
A
B
C
M2
0.1λ
TOWARDS LOAN
RADIALLY SCALED PARAMETERS
TOWARD LOAD
TOWARD GENERATOR
CENTER
ORIGIN

(11) $Z_L = 50 + j125 \qquad Z_0 = 75$

$$y_L = \frac{50 + j125}{75} = 0.667 + j1.667 \quad \text{POINT A}$$

y_L is normalized admittance, point B

$y_L = 0.22 - j0.54$

0.1λ from this point, towards generator is point C $y_i' = 0.18 + j0.13$

the shorted stub 0.2λ has normalized admittance $y_{stub} = -j0.322$

ACTUAL STUB ADMITTANCE IS $-j0.332 \times \left(\frac{1}{150}\right)$

$$\text{Renormalize} \quad y' = \frac{-j0.332 \times \left(\frac{1}{150}\right)}{(1/75)} = -j0.166$$

ADD THIS TO POINT VALUE

$$y'' = 0.18 + j0.13 - j0.166 = 0.18 - j0.036$$

POINT D

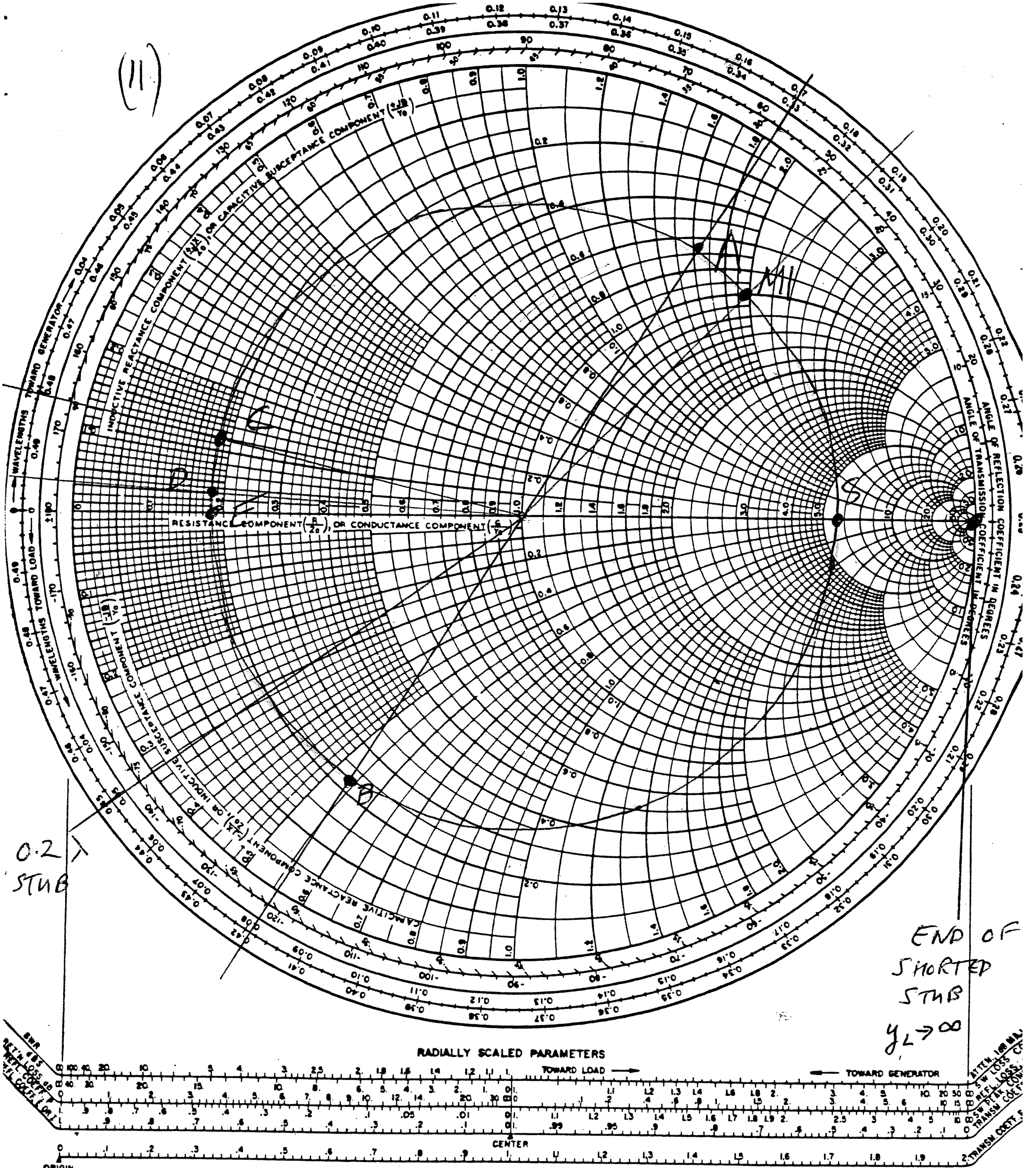
(a) $|\rho| = 0.7 \qquad \phi = 176^\circ$

(b) $S = 5.6$

(c) Nearest current minimum to load is point E, which is 0.081λ from load. Next current minimum is 0.5λ further. So, ANSWER 0.581λ

(d) Nearest matching point is point M, $0.187 + 0.081\ \lambda$ FROM LOAD, 0.268λ FROM LOAD

(11)



A
M1
B
C
D
E
S
0.2 λ
STUB
END OF
SHORTED
STUB
$y_L \to \infty$
WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
INDUCTIVE REACTANCE COMPONENT $\left(\frac{+jX}{Z_o}\right)$, OR CAPACITIVE SUSCEPTANCE COMPONENT $\left(\frac{+jB}{Y_o}\right)$
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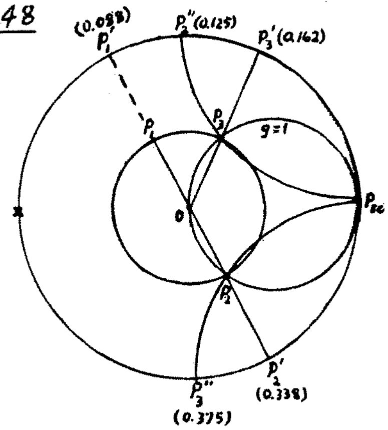
RADIALLY SCALED PARAMETERS
TOWARD LOAD
TOWARD GENERATOR
SWR
RET'N LOSS, dB
REFL. COEFF, P
REFL. COEFF, E OR I
ATTEN. 1dB MAJ.
S.W. LOSS COEFF.
S.W. PEAK (CONST. P)
TRANSM. COEFF, P
TRANSM. COEFF, E OR I
CENTER
ORIGIN

(12)

P.9-48



$z_L = 0.5 + j0.5$,

$y_L = 1 - j$.

a) See construction.

P_1 : $z_L = 0.5 + j0.5$.

P_2 : $y_L = 1 - j1 = y_2 \longrightarrow d_2 = 0$.

P_3 : $y_3 = 1 + j1$.

$\longrightarrow d_3 = 0.162\lambda + (0.5 - 0.338)\lambda$
$= 0.324\lambda$.

P_2'' : $b_2 = j1 \longrightarrow \ell_2 = (0.5 + 0.125)\lambda$
$= 0.375\lambda$.

P_3'' : $b_3 = -j1 \longrightarrow \ell_3 = (0.375 - 0.25)\lambda$
$= 0.125\lambda$.

b) For $Z_0' = 75 = 1.5 Z_0$, $Y_0' = 0.667 Y_0$.

The required normalized stub admittances are $b_2' = -b_3' = \frac{j}{0.667} = j1.5$.

	$(Z_0)_{stub} = (Z_0)_{Line}$	$(Z_0)_{stub} = 1.5 (Z_0)_{Line}$
$z_L = 0.5 + j0.5$	$d_2 = 0$, $\ell_2 = 0.375\lambda$	$d_2' = 0$, $\ell_2' = 0.406\lambda$
$y_L = 1 - j1$	$d_3 = 0.324\lambda$, $\ell_3 = 0.125\lambda$	$d_3' = 0.324\lambda$, $\ell_3' = 0.0936\lambda$

(13)

P.9-49 $z_L = 0.5 + j0.5$

Use Smith chart as an impedance chart. Same construction as that in problem P. 9-34 except P_{sc} would be on the extreme left (marked by a x), and g=1 circle becomes r=1 circle.

P_1: $z_L = 0.5 + j0.5$; P_2: $z_{i2} = 1 + j1$ with $d_2 = (0.162 - 0.088)\lambda$
$= 0.074\lambda$.

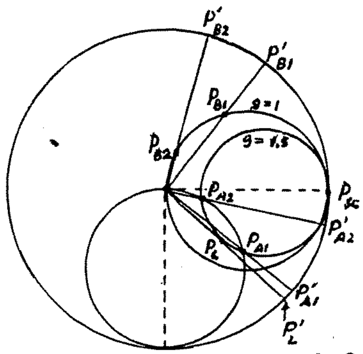
P_3: $z_{i3} = 1 - j1$ with $d_3 = (0.338 - 0.088)\lambda = 0.250\lambda$.

To achieve a match with a series stub having $R_0' = \frac{35}{50} R_0$, we need a normalized stub susceptance $-j\frac{50}{35} = -j1.43$ for solution corresponding to P_2. From Smith chart we obtain the required stub length $\ell_2 = 0.347\lambda$.

Similarly for solution corresponding to P_3, a stub with a normalized susceptance $+j1.43$ is needed, which requires a stub length $\ell_3 = 0.153\lambda$.

(14)

P.9-50



$z_L = 0.33 + j0.33$.

P_L : $y_L = 1.50 - j1.50$ (0.306λ at P_L'),

P_{A1} : $y_{A1} = 1.50 - j1.80$ (0.304λ at P_{A1}'),

P_{A2} : $y_{A2} = 1.50 - j0.14$ (0.269λ at P_{A2}'),

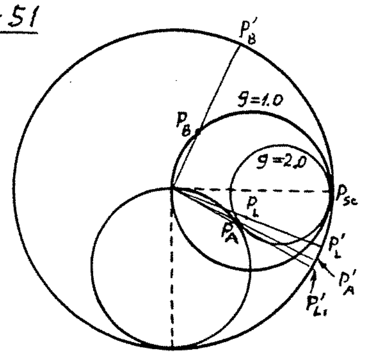
P_{B1} : $y_{B1} = 1.00 + j1.60$ (0.179λ at P_{B1}'),

P_{B2} : $y_{B2} = 1.00 + j0.40$ (0.144λ at P_{B2}').

	a) Short-circuited stubs	b) Open-circuited stubs
$(y_{SA})_1 = y_{A1} - y_L = -j0.30$	$\ell_{A1} = 0.203\lambda$	$\ell_{A1} = 0.453\lambda$
$(y_{SA})_2 = y_{A2} - y_L = j1.36$	$\ell_{A2} = 0.399\lambda$	$\ell_{A2} = 0.149\lambda$
$(y_{SB})_1 = -j1.60$	$\ell_{B1} = 0.089\lambda$	$\ell_{B1} = 0.339\lambda$
$(y_{SB})_2 = -j0.40$	$\ell_{B2} = 0.189\lambda$	$\ell_{B2} = 0.439\lambda$

(15)

P.9-51



$y_L = \dfrac{300}{100+j50} = 2.4 - j1.2$.

Point P_L on Smith chart. (0.280λ at P_L')

Since the rotated $g=1.0$ circle is tangent to the $g=2.0$ circle, an added line length d_L is needed to convert g_L (2.4) to 2.0, moving from P_L along the $|\Gamma|$-circle to P_{L1} (not shown) on the $g=2.0$ circle (0.291λ at P_{L1}'). Note that P_{L1} is different from P_A, the point of tangency between the $g=2.0$ and rotated $g=1.0$ circles.

a) Min. $d_L = 0.291\lambda - 0.280\lambda = 0.011\lambda$.

b) P_A : $y_A = 2 - j1$ (0.287λ at P_A').

P_B : $y_B = 1 + j1$ (0.162λ at P_B').

$y_{SA} = y_A - y_{L1} = (2-j1) - (2-j1.35) = j0.35 \longrightarrow \ell_A = 0.304\lambda$.

$y_{SB} = -j1 \longrightarrow \ell_B = 0.125\lambda$.