ENEE 381 Problem Set #5

4/17/03 due 4/24/03

Questions like (5), (7) and (8) could be on the second exam.

(1) Cheng 8.26

(2) Cheng 8.27

(3) Cheng 8.28

(4) Cheng 8.30

(5) Cheng 8.36

(6) A transmission line of characteristic impedance 75ohm is terminated with an inductor of 0.1μH and 50ohm in series. The frequency of operation is 100MHz. Calculate without using the Smith Chart:

(a) $|\rho|$

(b) ϕ

(c) The standing wave ratio

(d) Where on the line closest to the load can the line be matched with the shortest possible shorted stub connected in parallel to the line?

(e) What is the length of this stub?

(7) Repeat (6) with the Smith Chart

(8) A plane wave is incident on the boundary between air and a plastic (ϵ_4=10) at an angle of incidence of 45°. The wave is incident from the air side. Use the Smith chart to find:

(a) $|\rho|$

(b) ϕ

(c) The standing wave ratio

(d) The location of the nearest magnetic field maximum to the boundary

ENEE PROBLEM SET 5 SOLUTIONS

(1) P.8-26 For normal incidence: $1+\Gamma=\tau$, where $|\Gamma|\le 1$.

If $|\tau|=|\Gamma|$: $\Gamma<0$ and $\eta_1-\eta_2=2\eta_2 \longrightarrow \eta_1=3\eta_2 \longrightarrow |\Gamma|=\frac{1}{2}$.

$\therefore\ S=\dfrac{1+|\Gamma|}{1-|\Gamma|}=3$ —— $S_{dB}=20\log_{10}3=9.54$ (dB).

(2) P.8-27 a) In the lossy medium (medium 2):

$$\bar{E}_t=\bar{a}_x E_{t0}e^{-\alpha_2 z}e^{-j\beta_2 z},$$

where, from Problem P.8-9, $\alpha_2=\omega\sqrt{\frac{\mu_2\epsilon_2}{2}}\left[\sqrt{1+\left(\frac{\sigma_2}{\omega\epsilon_2}\right)^2}-1\right]^{\frac{1}{2}}$, $\beta_2=\omega\sqrt{\frac{\mu_2\epsilon_2}{2}}\left[\sqrt{1+\left(\frac{\sigma_2}{\omega\epsilon_2}\right)^2}+1\right]^{\frac{1}{2}}$.

Given: $\beta_1=6$ (rad/m) $\longrightarrow \omega=\beta_1 c=1.8\times10^9$ (rad/s).

$\tan\delta_c=\dfrac{\sigma_2}{\omega\epsilon_2}=0.5 \longrightarrow \alpha_2=2.30$ (Np/m), $\beta_2=9.76$ (rad/m).

$$\eta_2=\sqrt{\frac{\mu_2}{\epsilon_2}}=\frac{120\pi}{\sqrt{\epsilon_{r2}}(1+\tan^2\delta_c)^{1/4}}e^{j\frac{1}{2}\tan^{-1}(\sigma_2/\omega\epsilon_2)}=225\,e^{j13.3^\circ}.$$

$\bar{E}_t=\bar{a}_x E_{t0}e^{-2.30z}e^{-j9.76z}$, $\quad \bar{H}_t=\bar{a}_z\times\dfrac{\bar{E}_t}{\eta_2}=\bar{a}_y\dfrac{E_{t0}}{225}e^{-j13.3^\circ}e^{-2.30z}e^{-j9.76z}$

Let $\bar{E}_r=\bar{a}_x E_{r0}e^{j6z} \longrightarrow \bar{H}_r=-\bar{a}_y\dfrac{E_{r0}}{120\pi}e^{j6z}$. $\quad \bar{H}_i=\bar{a}_y\dfrac{10}{120\pi}e^{-j6z}$

Boundary conditions for $\bar{E}$ and $\bar{H}$ at $z=0$:
$$\begin{cases}10+E_{r0}=E_{t0},\\ 10-E_{r0}=E_{t0}\sqrt{\epsilon_{r2}}(1+\tan^2\delta_c)^{1/4}e^{-j13.3^\circ}.\end{cases}$$

$\longrightarrow E_{r0}=2.77\,e^{j157^\circ}$; $\quad E_{t0}=7.53\,e^{-j172^\circ}$

$\therefore$
$\bar{E}_r(z,t)=\bar{a}_x 2.77\cos(1.8\times10^9t+6z+157^\circ)$ (V/m),
$\bar{H}_r(z,t)=-\bar{a}_y 0.073\cos(1.8\times10^9t+6z+157^\circ)$ (A/m),
$\bar{E}_t(z,t)=\bar{a}_x 7.53\,e^{-2.30z}\cos(1.8\times10^9t-9.76z-172^\circ)$ (V/m),
$\bar{H}_t(z,t)=\bar{a}_y 0.033\,e^{-2.30z}\cos(1.8\times10^9t-9.76z+174.7^\circ)$ (A/m).

b) $(\bar{\mathcal{P}}_{av})_1=\bar{a}_z\left(\dfrac{10^2}{2\times120\pi}-\dfrac{2.77^2}{2\times120\pi}\right)=\bar{a}_z\,0.122$ (W/m²),

$(\bar{\mathcal{P}}_{av})_2=\bar{a}_z\dfrac{7.53^2}{2\times225}(\cos13.3^\circ)e^{-4.60z}=\bar{a}_z\,0.122\,e^{-4.60z}$ (W/m²).

(3) P.8-28 a) $\Gamma=\dfrac{E_r}{E_i}=\dfrac{\eta_c-\eta_0}{\eta_c+\eta_0}$; $\quad \eta_0=\sqrt{\dfrac{\mu_0}{\epsilon_0}}=120\pi$, $\eta_c=\sqrt{j\omega\mu/\sigma}$.
$|\eta_c|\ll\eta_0$.

b) $|\Gamma|^2=\left|\dfrac{\eta_c-\eta_0}{\eta_c+\eta_0}\right|^2=\left|\dfrac{1-\eta_c/\eta_0}{1+\eta_c/\eta_0}\right|^2\cong\left|1-2\eta_c/\eta_0\right|^2$
$=(1-2\eta_c/\eta_0)(1-2\eta_c^*/\eta_0)\cong 1-4\,\mathcal{R}e(\eta_c)/\eta_0$.

Fraction of power absorbed, $F=1-|\Gamma|^2=\dfrac{4}{\eta_0}\mathcal{R}e\sqrt{\dfrac{j\omega\mu}{\sigma}}$
$=\dfrac{4}{\eta_0}\sqrt{\dfrac{\omega\mu}{2\sigma}}$.

c) $\omega=2\pi\times10^6$ (Hz). For iron: $\mu=4{,}000\times(4\pi10^{-7})$ (H/m), $\sigma=10^7$ (S/m).

$F=4.21\times10^{-4}$, or 0.0421%.

(4)

P. 8-30 From Example 8-12: $\eta_2 = \sqrt{\eta_1 \eta_3} \longrightarrow \epsilon_{2r} = \sqrt{\epsilon_{r1}\epsilon_{r3}} = 2.$

a) Wavelength of red light in dielectric coating: $\lambda_2 = \frac{0.75}{\sqrt{\epsilon_{r2}}} = 0.530\ (\mu m) \longrightarrow d = \frac{\lambda_2}{4} = 0.133\ (\mu m)$

b) For violet light: $\lambda_2' = \frac{0.42}{\sqrt{2}} = 0.297\ (\mu m).$

$$\frac{d}{\lambda_2'} = 0.447 \longrightarrow \beta_2 d = 0.894\pi.$$

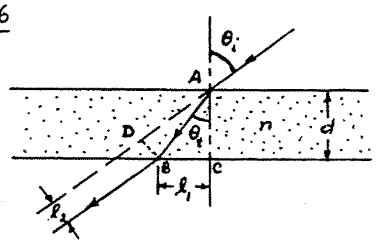
From Eq. (8-173) and using impedances normalized with respect to $\eta_1 = \eta_0$: $Z_2(0) = \eta_2 \frac{\eta_3 + j\eta_2 \tan\beta_2 d}{\eta_2 + j\eta_3 \tan\beta_2 d} = \frac{1}{\sqrt{2}} \frac{\frac{1}{2} + \frac{j}{\sqrt{2}}\tan\beta_2 d}{\frac{1}{\sqrt{2}} + \frac{j}{2}\tan\beta_2 d} = \frac{0.5 - j0.247}{1.0 - j0.247},$

$$\Gamma = \frac{Z_2(0) - 1}{Z_2(0) + 1} = 0.317\, e^{j198°}.$$

Percentage of power reflected $= |\Gamma|^2 \times 100\%$
$= (0.317)^2 \times 100\% = 10\%.$

(5)

P. 8-36



a) Snell's law:

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{1}{n},$$

$$\theta_t = \sin^{-1}\left(\frac{1}{n}\sin\theta_i\right).$$

b) $\cos\theta_t = \sqrt{1 - \left(\frac{1}{n}\sin\theta_i\right)^2}.$

$$\ell_1 = \overline{BC} = \overline{AC}\tan\theta_t = d\frac{\sin\theta_t}{\cos\theta_t} = \frac{d\sin\theta_i}{\sqrt{n^2 - \sin^2\theta_i}}.$$

c) $\ell_2 = \overline{BD} = \overline{AC}\sin(\theta_i - \theta_t) = \frac{d}{\cos\theta_t}(\sin\theta_i\cos\theta_t - \cos\theta_i\sin\theta_t)$

$$= d\sin\theta_i\left[1 - \frac{\cos\theta_i}{\sqrt{n^2 - \sin^2\theta_i}}\right].$$

(6)

$Z_0 := 75$ Characteristic impedance of line

$f := 100 \cdot 10^6$ Frequency

$\omega := 2 \cdot \pi \cdot f$

$L := 10^{-7}$ inductor

$Z_L := i \cdot \omega \cdot L + 50$ load

$Z_L = 50 + 62.832i$

$\zeta_L := \dfrac{Z_L}{Z_0}$ normalized load

$\zeta_L = 0.667 + 0.838i$

$\rho := \dfrac{Z_L - Z_0}{Z_L + Z_0}$

$\rho = 0.042 + 0.482i$ ρ is in the first quadrant of the complex plane

$|\rho| = 0.483$ magnitude

$\dfrac{\arg(\rho)}{\deg} = 85.01$ phase angle

$S := \dfrac{1 + |\rho|}{1 - |\rho|}$

$S = 2.871$ Standing wave ratio

$Y_i = Y_0 \cdot \left(\dfrac{Y_L \cdot \cos(k \cdot l) + i \cdot Y_0 \cdot \sin(k \cdot l)}{Y_0 \cdot \cos(k \cdot l) + i \cdot Y_L \cdot \sin(k \cdot l)} \right)$ input admittance distance l from load

$Y_0 := \dfrac{1}{Z_0}$ $Y_L := \dfrac{1}{Z_L}$

We need a phase velocity on the line. Since for free space Z_0=376.7 and is $Z_0/\text{sqrt}(\varepsilon_r)$ it is reasonable to use in this case

$\varepsilon_r := \left(\dfrac{376.7}{75} \right)^2$ $\varepsilon_r = 25.227$

$v := \dfrac{2.998 \cdot 10^8}{\sqrt{\varepsilon_r}}$

So the wavelength is

$\lambda := \dfrac{v}{f}$

$k := 2 \cdot \dfrac{\pi}{\lambda}$ $\lambda = 0.597$

$k = 10.526$

For matching with a stub

$$Y_L = 7.755 \times 10^{-3} - 9.745i \times 10^{-3}$$

$$1 + i\cdot S = \left(\frac{Y_L\cdot\cos(k\cdot l) + i\cdot Y_0\cdot\sin(k\cdot l)}{Y_0\cdot\cos(k\cdot l) + i\cdot Y_L\cdot\sin(k\cdot l)}\right)$$

$$Y_0 = 0.013$$

Solve for l. Make guesses for the variables

$$\lambda = 0.597$$

$$l := 0.2\cdot\lambda$$

Given

$$\mathrm{Re}\left(\left(\frac{Y_L\cdot\cos(k\cdot l) + i\cdot Y_0\cdot\sin(k\cdot l)}{Y_0\cdot\cos(k\cdot l) + i\cdot Y_L\cdot\sin(k\cdot l)}\right)\right) = 1$$

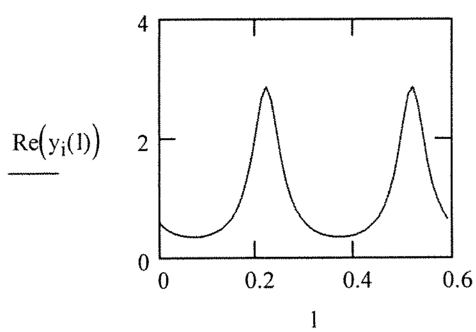
$0.0 < l < \lambda$ Constraint

$a := \mathrm{Find}(l)$

$a = 0.169$

$l := 0, 0.01 .. \lambda$

$$y_i(l) := \left(\frac{Y_L\cdot\cos(k\cdot l) + i\cdot Y_0\cdot\sin(k\cdot l)}{Y_0\cdot\cos(k\cdot l) + i\cdot Y_L\cdot\sin(k\cdot l)}\right)$$



Graph shows the various values of l for which yi=1+jS

$l := a$

$y_i(l) = 1 + 1.104i$ $\quad S := \mathrm{Im}(y_i(l))$ $\quad S = 1.104$

Minimum length from load where matching can be accomplished is at

$\frac{l}{\lambda} = 0.283$ $\quad 0.283\lambda$ from load

At this point normalized admittance is 1+j1.104

For a shorted stub

$$y_i := \left(\frac{Y_L\cdot\cos(k\cdot l) + i\cdot Y_0\cdot\sin(k\cdot l)}{Y_0\cdot\cos(k\cdot l) + i\cdot Y_L\cdot\sin(k\cdot l)}\right)$$

where Y_L is infinity

$ystub_i = -i \cdot \cot(k \cdot l_{stub})$

Find length of stub. First guess length

$l_{stub} := 0.1$ $\qquad ystub_i := i \cdot S$

Given

$ystub_i = -i \cdot \cot(k \cdot l_{stub})$

$b := \mathrm{Find}(l_{stub})$

$b = 0.229$

$l_{stub} := b$

$\frac{l_{stub}}{\lambda} = 0.383$ Need a 0.383 λ shorted stub

(8) Use P-wave (TM wave) geometry $Z_0 := 376.7$ free space impedance

For second medium

$n_2 := \sqrt{10}$

$n_1 := 1$

$\theta_1 := 45 \cdot \mathrm{deg}$

$$\theta_2 := \mathrm{asin}\left(\frac{\sin(\theta_1)}{n_2}\right)$$

$\frac{\theta_2}{\mathrm{deg}} = 12.921$ This is the angle of refraction

$Z_1 := Z_0 \cdot \cos(\theta_1)$ $\qquad Z_1 = 266.367$

$Z_2 := \frac{Z_0}{n_2} \cdot \cos(\theta_2)$ $\qquad Z_2 = 116.107$

Normalized load is

$$\zeta_L := \frac{Z_2}{Z_1}$$

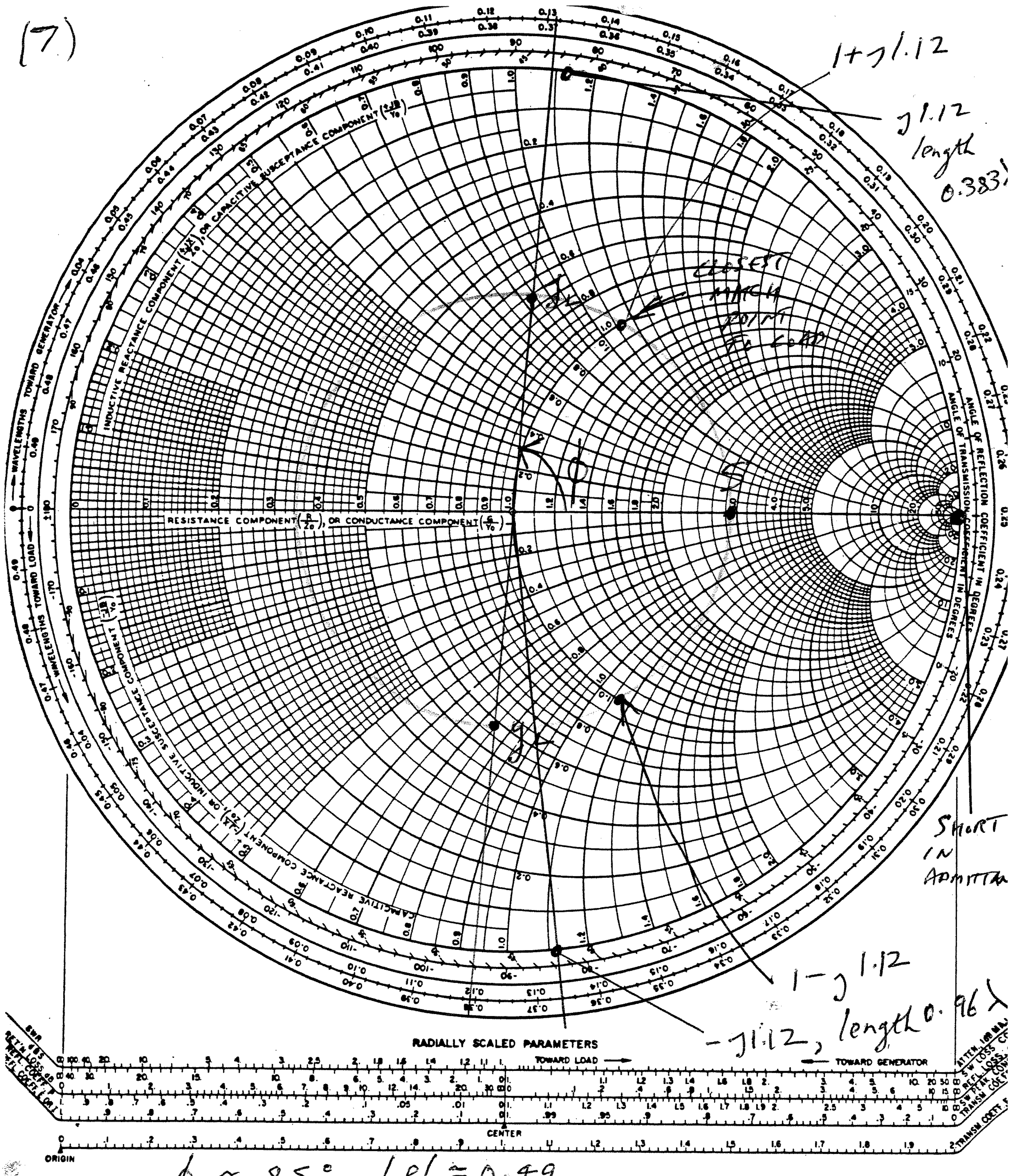
$\zeta_L = 0.436$ This is the starting point on the Smith chart

(For S-wave (TE-wave) geometry you would use $Z/\cos(\theta)$ instead)

Note: $S := \frac{1}{0.436}$ $\qquad \rho := \frac{Z_2 - Z_1}{Z_2 + Z_1}$ $\qquad \rho = -0.393$ See chart)

$S = 2.294$ $\qquad |\rho| = 0.393$ $\qquad \phi := 180 \cdot \mathrm{deg}$

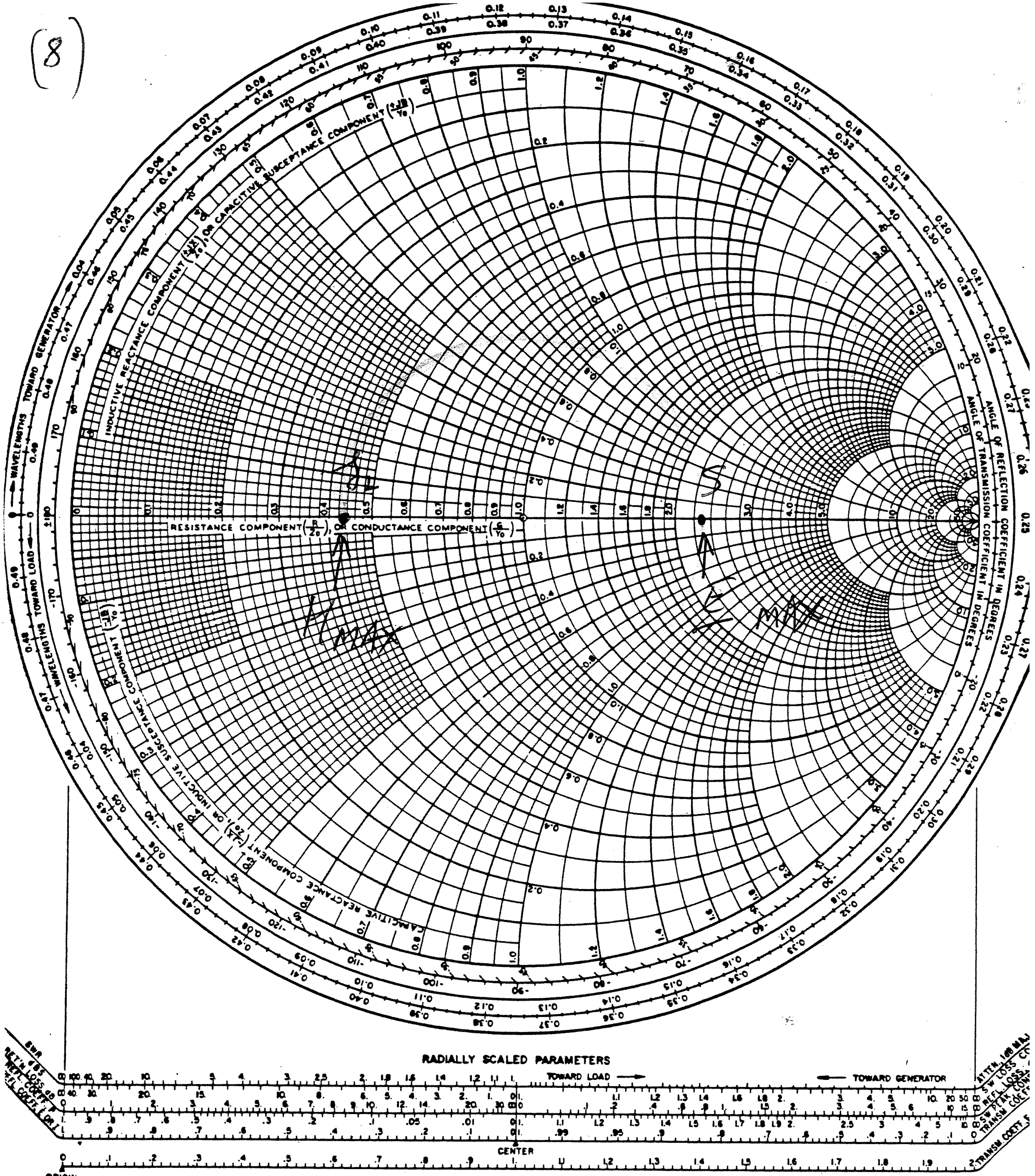
(7)



$\phi \simeq 85°$ $|\rho| \simeq 0.49$

MATCH POINT IS 0.283λ FROM LOAD

(8)



Clearly $\phi = 180°$, $S = 2.3$

$|\rho| = 0.39$ HMAX IS AT LOAD