ENEE 381, Spring 2004. Homework Set 2

Due March 4, 2004

(1) Cheng 7-3

(2) Cheng 7-19

(3) Cheng Problem 7-25

(4) Cheng Problem 7-26

(5) Cheng Problem 7-27

(6) The electric field of a plane wave is

$$E_x = 100\cos(\omega t - \pi/6).$$

Its magnetic field in a medium with μ_r=1, ϵ_r=12 is

$$H_y = H_0\cos(\omega t + \pi/4).$$

What is the magnitude H_0?
What is the time averaged Poynting vector?

(7) The electric field of a plane wave is

$$E_x = E_0 e^{(\omega t - \pi/12)}.$$

Its magnetic field in a medium with μ_r=1, ϵ_r=12 is

$$H_y = 100 e^{(\omega t + \pi/7)}.$$

What is the magnitude E_0?
What is the time averaged Poynting vector?

(8) Cheng 8-6

(9) Cheng 8-16

(10) Cheng 8-18

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Homework # 2 SOLUTIONS

(1) P.7-3 In the rectangur loop with the assigned direction for i_2

$$L_{12}\frac{di_1}{dt} = L\frac{di_2}{dt} + Ri_2, \qquad ①$$

where $L_{12} = \frac{\Phi_{12}}{i_1} = \frac{h}{i_2}\int_d^{d+w} B_{12}\,dr = \frac{h}{i_2}\int_d^{d+w}\frac{\mu_0 i_1}{2\pi r}\,dr$

$$= \frac{\mu_0 h}{2\pi}\ln\left(1+\frac{w}{d}\right). \qquad ②$$

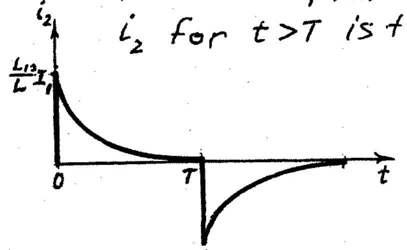
a) At $t=0$, $i_1(t) = I_1 U(t)$ is applied and ① becomes

$$L\frac{di_2}{dt} + Ri_2 = L_{12} I_1 \delta(t). \qquad ③$$

Solution of ③: $i_2 = \frac{L_{12}}{L} I_1 e^{-(R/L)t}$, $0<t<T$ ④

At $t=T$, $i_2 = \frac{L_{12}}{L} I_1 e^{-RT/L}$, when a negative step function $-I_1 U(t)$ is applied. If $T \gg L/R$, then i_2 for $t>T$ is the reverse of i_2 for $0<t<T$;

i.e., $i_2 = -\frac{L_{12}}{L} I_1 e^{-(R/L)(t-T)}$, $t>T$.



b). Energy dissipated in R,

$$W \cong 2\left(\frac{L_{12}}{L}I_1\right)^2 R\int_0^{\infty} e^{-(2R/L)t}\,dt$$
$$= \frac{1}{L}(L_{12}I_1)^2.$$

(2) P.7-19 Medium 1: Free space.
Medium 2: $\mu_2 \to \infty$. H_2 must be zero so that B_2 is not infinite.

Boundary Conditions: $\bar{a}_{n2}\times \bar{H}_1 = \bar{J}_s$, $B_{1n} = B_{2n}$.
$E_{1t} = E_{2t}$, $\bar{a}_{n2}\cdot(\bar{D}_1 - \bar{D}_2) = \rho_s$.

(3) P.7-25 $\bar{E} = \bar{a}_y 0.1 \sin(10\pi x)\cos(6\pi 10^9 t - \beta z)$ (V/m).

Use phasors: $\bar{H} = -\frac{1}{j\omega\mu_0}\bar{\nabla}\times\bar{E} = \frac{j}{\omega\mu_0}\left[\bar{a}_x j0.1\beta\sin(10\pi x) + \bar{a}_z 0.1(10\pi)\cos(10\pi x)\right]\cdot e^{-j\beta z}$, ①

$$\bar{E} = \frac{1}{j\omega\epsilon_0}\bar{\nabla}\times\bar{H} = \bar{a}_y\frac{0.1}{\omega^2\mu_0\epsilon_0}\left[(10\pi)^2+\beta^2\right]\sin(10\pi x)e^{-j\beta z}. \qquad ②$$

Phase form for given $\bar{E}$: $\bar{E} = \bar{a}_y\, 0.1\sin(10\pi x)e^{-j\beta z}$. ③

Equating ② and ③: $(10\pi)^2 + \beta^2 = \omega^2\mu_0\epsilon_0 = 400\pi^2$.
$\longrightarrow \beta = \sqrt{300}\,\pi = 54.4$ (rad/m).

From ①: $\bar{H}(x,z;t) = \mathcal{R}e(\bar{H}e^{j\omega t})$
$= -\bar{a}_x 2.30\times10^{-4}\sin(10\pi x)\cos(6\pi 10^9 t - 54.4z)$
$-\bar{a}_z 1.33\times10^{-4}\cos(10\pi x)\sin(6\pi 10^9 t - 54.4z)$ (A/m).

(4)

P.7-26 $\bar{H}(x,z;t)=\bar{a}_y 2\cos(15\pi x)\sin(6\pi 10^9 t-\beta z)$ (A/m).

Phasor: $\bar{H}=\bar{a}_y 2\cos(15\pi x)e^{-j\beta z}$

Similar to Problem P.7-25: $(15\pi)^2+\beta^2=\omega^2\mu_0\epsilon_0=\frac{(6\pi 10^9)^2}{(3\times 10^8)^2}=400\pi^2$

$\longrightarrow \beta=13.2\pi=41.6$ (rad/m).

$$\bar{E}=\frac{1}{j\omega\epsilon_0}\bar{\nabla}\times\bar{H}=\left[\bar{a}_x 158\pi\cos(15\pi x)+\bar{a}_z j180\pi\sin(15\pi x)\right]e^{-j\beta z}$$

$$\bar{E}(x,z;t)=\mathcal{I}m(\bar{E}e^{j\omega t})=\bar{a}_x 496\cos(15\pi x)\sin(6\pi 10^9 t-41.6z)$$
$$+\bar{a}_z 565\sin(15\pi x)\cos(6\pi 10^9 t-41.6z)\ \text{(V/m)}.$$

(5)

P.7-27 Use phasors: $\bar{E}=\bar{a}_\theta\frac{E_0}{R}\sin\theta\cdot e^{-jkR}$

$$\bar{\nabla}\times\bar{E}=\bar{a}_\phi\frac{1}{R}\frac{\partial}{\partial R}(RE_\theta)=\bar{a}_\phi(-jk)\frac{E_0}{R}\sin\theta\cdot e^{-jkR}$$
$$=-j\omega\mu_0\bar{H}\longrightarrow\bar{H}=\bar{a}_\phi\frac{kE_0}{\omega\mu_0 R}\sin\theta\cdot e^{-jkR}$$

In free space, $k=\omega\sqrt{\mu_0\epsilon_0}\longrightarrow\bar{H}=\bar{a}_\phi\frac{E_0}{R}\sqrt{\frac{\epsilon_0}{\mu_0}}\sin\theta\cdot e^{-j\omega\sqrt{\mu_0\epsilon_0}R}$,

$$\bar{H}(R,\theta;t)=\bar{a}_\phi\frac{E_0}{R}\sqrt{\frac{\epsilon_0}{\mu_0}}\sin\theta\cos\omega(t-\sqrt{\mu_0\epsilon_0}R).$$

(6)

$$E_x=100\cos(wt-\pi/6)=100e^{j(wt-\pi/6)}$$
$$H_y=H_0\cos(wt+\pi/4)=H_0e^{j(wt+\pi/4)}$$
$$\left|\frac{E_x}{H_y}\right|=Z=\frac{376.7}{\sqrt{12}}=108.74\ \text{ohm}$$
$$H_0=\frac{100}{108.74}=\underline{91.96\ A/m}$$

The time averaged Poynting vector is

$$\underline{P}=\frac{1}{2}\mathcal{R}e(E_x H_y^*)$$
$$\underline{P}=\frac{1}{2}\mathcal{R}e\left(100\cdot 91.96\ e^{-j(\pi/6+\pi/4)}\right)$$
$$\underline{P}=50\cdot 91.96\cos\left(\frac{5\pi}{12}\right)=\underline{1190\ W/m^2}$$

(7) $E_o = Z H_o \qquad Z = 108.74$ ohm

$\underline{E_o = 10{,}874 \ V/m}$

$\underline{P} = \frac{1}{2} E_o H_o \,\mathcal{R}e\left(e^{-j(\pi/12 + \pi/7)}\right)$

$\underline{P} = \underline{4.121 \times 10^{5} \ W/m^2}$

(8) P.8-6 Phasor: $\bar{E} = \bar{a}_x 2 e^{-jz/\sqrt{3}} + \bar{a}_y j e^{-jz/\sqrt{3}}$ (V/m).

a) $\omega = 10^8$ (rad/s) $\longrightarrow f = 10^8/2\pi = 1.59 \times 10^7$ (Hz),

$\beta = 1/\sqrt{3}$ (rad/m) $\longrightarrow \lambda = 2\pi/\beta = 2\sqrt{3}\,\pi$ (m).

b) $u = \frac{c}{\sqrt{\epsilon_r}} = \frac{\omega}{\beta} \longrightarrow \epsilon_r = \left(\frac{\beta c}{\omega}\right)^2 = 3.$

c) Left-hand elliptically polarized.

d) $\eta = \sqrt{\frac{\mu}{\epsilon}} = \frac{120\pi}{\sqrt{\epsilon_r}} = \frac{120\pi}{\sqrt{3}}$ (Ω),

$\bar{H} = \frac{1}{\eta} \bar{a}_z \times \bar{E} = \frac{\sqrt{3}}{120\pi}\left(\bar{a}_y 2 e^{-jz/\sqrt{3}} - \bar{a}_x j e^{-jz/\sqrt{3}}\right),$

$\bar{H}(z,t) = \frac{\sqrt{3}}{120\pi}\left[\bar{a}_x \sin(10^8 t - z/\sqrt{3}) + \bar{a}_y \cos(10^8 t - z/\sqrt{3})\right]$ (A/m).

(9) P.8-16 $\mathcal{P}_{av} = |E|^2/2\eta_0 = 10^{-2}$ (W/cm²).

a) $|E| = \sqrt{0.02\eta_0} = 2.75$ (V/cm) $= 275$ (V/m),

$|H| = |E|/\eta_0 = 7.28 \times 10^{-3}$ (A/cm) $= 0.728$ (A/m).

b) $\mathcal{P}_{av} = |E|^2/2\eta_0 = 1300$ (W/m²).

$|E| = 990$ (V/m), $\qquad |H| = 2.63$ (A/m).

(10) P.8-18 $\bar{E} = \bar{a}_\theta E_\theta + \bar{a}_\phi E_\phi,$

$\bar{H} = \frac{1}{\eta} \bar{a}_R \times \bar{E} = \frac{1}{\eta}(\bar{a}_\phi E_\theta - \bar{a}_\theta E_\phi).$

$\bar{\mathcal{P}}_{av} = \frac{1}{2}\mathcal{R}e(\bar{E} \times \bar{H}^*) = \bar{a}_z \frac{1}{2\eta}\left(|E_\theta|^2 + |E_\phi|^2\right).$