ENEE 381, Spring 2004. Homework Set 3

Due Tuesday March 30, 2004

(1) Cheng Problem 8-20

(2) Cheng Problem 8-22

(3) Cheng Problem 8-25

(4) Cheng Problem 9-23

(5) Cheng Problem 9-24

(6) Cheng Problem 9-26

(7) A transmission line of Z_0=75ohm is terminated in a load of 60+j50 ohm. What are
(a) ρ
(b)$|\rho|$
(c) ϕ
(d) The VSWR

(8) A transmission line of Z_0=50ohm is terminated with a capacitor of 10nF in parallel with an inductor of 10nH. The line is being operated at 100MHz. Calcuate:
(a) ρ
(b)$|\rho|$
(c) ϕ
(d) The VSWR

(9) A transmission line of characteristic impedance 50 ohm is terminated with a load Z_L. The line is being driven at a freqency of 100 MHz. T he line is found to be matched by adding an inductance in parallel to the line at 0.1 λ from the load. The inductance needed is 79.58 nH. What is the value of the load? What is the reflection magnitude and phase at the load when the line is unmatched? What is the standing wave ratio when the line is unmatched?

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SOLUTIONS TO PROBLEM SET # 3

(1) Cheng 8-20

(a) Skin depth is $\delta = \frac{1}{\sqrt{\mu_0 \mu_r \pi \nu \sigma}}$

In this case $\mu_r = 1$ $\nu = \frac{10^4}{2\pi}$

(b) Just inside the ocean surface $H_{t_2} = H_{t_1}$ (outside)
The magnetic field decays according to the skin effect

$$H(z,t) = \hat{j} H_0 e^{-z/\delta} e^{-jz/\delta} e^{j\omega t} \qquad \omega = 10^4$$

$$= \hat{j} H_0 e^{-z/\delta} \cos\left(\omega t - \frac{z}{\delta}\right)$$

The impedance of sea water is $Z = \sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_r}}$

$\varepsilon_r = \varepsilon' - j\varepsilon''$, where for a very good conductor

$$\varepsilon'' = \frac{\sigma}{\omega \varepsilon_0} \quad \text{and} \quad \varepsilon'' >> \varepsilon'$$

in this case $Z = \sqrt{\frac{j\mu_0 \omega \varepsilon_0}{\varepsilon_0 \sigma}} = \sqrt{\frac{\mu_0 \omega}{\sigma}} \sqrt{j} = \frac{\sqrt{2}}{\sigma \delta} e^{j\pi/4}$

Therefore $E(z,t) = Z\, H(z,t) = \hat{i} \frac{\sqrt{2}}{\sigma\delta} H_0 e^{-z/\delta} \cos\left(\omega t - \frac{z}{\delta} + \frac{\pi}{4}\right)$

(c) The total current in the "skin" $= J_s$, which by Amperes Law = the magnetic field just inside the surface
The ohmic dissipation is $\frac{1}{2} R_s J_s^2$, where R_s is the surface resistance $R_s = \frac{1}{\sigma \delta}$

There ohmic dissipation is $\frac{1}{2} \frac{H_0^2}{\sigma \delta}$ W/m^2

(2) Cheng P.8-22 Given $\bar{E}_i(x,z) = \bar{a}_y 10 e^{-j(6x+8z)}$ (V/m).

a) $k_x = 6,\ k_z = 8 \longrightarrow k = \beta = \sqrt{k_x^2 + k_z^2} = 10$ (rad/m).

$\lambda = 2\pi/k = 2\pi/10 = 0.628$ (m); $f = c/\lambda = 4.78\times10^8$ (Hz); $\omega = kc = 3\times10^9$ (rad/s).

b) $\bar{E}_i(x,z;t) = \bar{a}_y 10\cos(3\times10^9 t - 6x - 8z)$ (V/m).

$$\bar{H}_i(x,z) = \frac{1}{\eta_0}\bar{a}_{ni}\times\bar{E}_i \qquad \left(\bar{a}_{ni} = \frac{\bar{k}}{k} = \bar{a}_x 0.6 + \bar{a}_z 0.8.\right)$$

$$= \frac{1}{120\pi}(\bar{a}_x 0.6 + \bar{a}_z 0.8)\times\bar{a}_y 10 e^{-j(6x+8z)} = \left(-\bar{a}_x\frac{1}{15\pi} + \bar{a}_z\frac{1}{20\pi}\right)e^{-j(6x+8z)}$$

$$\bar{H}_i(x,z;t) = \left(-\bar{a}_x\frac{1}{15\pi} + \bar{a}_z\frac{1}{20\pi}\right)\cos(3\times10^9 t - 6x - 8z) \quad \text{(A/m)}.$$

c) $\cos\theta_i = \bar{a}_{ni}\cdot\bar{a}_z = \left(\frac{\bar{k}}{k}\right)\cdot\bar{a}_z = 0.8 \longrightarrow \theta_i = \cos^{-1}0.8 = 36.9°$.

d) $\bar{E}_i(x,0) + \bar{E}_r(x,0) = 0 \longrightarrow \bar{E}_r(x,z) = -\bar{a}_y 10 e^{-j(6x-8z)}$

$$\bar{H}_r(x,z) = \frac{1}{\eta_0}\bar{a}_{nr}\times\bar{E}_r(x,z) \qquad (\bar{a}_{nr} = \bar{a}_x 0.6 - \bar{a}_z 0.8)$$

$$= -\left(\bar{a}_x\frac{1}{15\pi} + \bar{a}_z\frac{1}{20\pi}\right)e^{-j(6x-8z)}$$

e) $\bar{E}_1(x,z) = \bar{E}_i(x,z) + \bar{E}_r(x,z) = \bar{a}_y 10(e^{-j8z} - e^{j8z})e^{-j6x}$

$$= -\bar{a}_y j20 e^{-j6x}\sin 8z \quad \text{(V/m)}.$$

$$\bar{H}_1(x,z) = \bar{H}_i(x,z) + \bar{H}_r(x,z) = -\left(\bar{a}_x\frac{2}{15\pi}\cos 8z + \bar{a}_z\frac{j}{10\pi}\sin 8z\right)e^{-j6x} \quad \text{(A/m)}.$$

(3) Cheng P.8-25 a) From Eqs. (8-128) and (8-129):

$$\bar{E}_1(x,z;t) = -2E_{i0}\big[\bar{a}_x\cos\theta_i\sin(\beta_1 z\cos\theta_i)\cos(\omega t - \beta_1 x\sin\theta_i) + \bar{a}_z\sin\theta_i\cos(\beta_1 z\cos\theta_i)\sin(\omega t - \beta_1 x\sin\theta_i)\big];$$

$$\bar{H}_1(x,z;t) = \bar{a}_y\frac{2E_{i0}}{\eta_1}\cos(\beta_1 z\cos\theta_i)\sin(\omega t - \beta_1 x\sin\theta_i).$$

b) $$\bar{\mathcal{P}}_{av} = \frac{1}{2}\mathcal{R}e(\bar{E}\times\bar{H}^*) = \bar{a}_x\frac{2E_{i0}^2}{\eta_1}\sin\theta_i\cos^2(\beta_1 z\cos\theta_i).$$

(4) Cheng P.9-23 a) $$|\Gamma| = \frac{S-1}{S+1} = \frac{\left|\frac{Z_L}{Z_0} - 1\right|}{\left|\frac{Z_L}{Z_0} + 1\right|} = \frac{\sqrt{(r_L-1)^2 + x_L^2}}{\sqrt{(r_L+1)^2 + x_L^2}},$$

where $r_L = R_L/Z_0$ and $x_L = X_L/Z_0$.

$$\longrightarrow x_L = \pm\left[\frac{\left(\frac{S-1}{S+1}\right)^2(r_L+1)^2 - (r_L-1)^2}{1 - \left(\frac{S-1}{S+1}\right)^2}\right]^{1/2}.$$

When $S = 3$, $x_L = \pm\sqrt{(10r_L - 3r_L^2 - 3)/3}$.

b) $S = 3$ and $r_L = 150/75 = 2 \longrightarrow x_L = \pm\sqrt{5/3}$.

$X_L = x_L Z_0 = \pm 96.8$ (Ω).

c) From Eq. (9-147): $r_L + jx_L = \dfrac{r_m + jt}{1 + jr_m t}$,

where $r_m = R_m/Z_0$, and $t = \tan\beta\ell_m$.

$$\longrightarrow r_m = \frac{(1 + r_L^2 + x_L^2) \pm \sqrt{(1+r_L^2+x_L^2)^2 - 4r_L^2}}{2r_L}$$

$$= 3 \text{ or } \frac{1}{3}, \quad \text{for } r_L = 2 \text{ and } x_L^2 = 5/3.$$

Also, $x_L = \dfrac{(1-r_m^2)t}{1 + r_m^2 t^2} \longrightarrow t = \dfrac{1}{2x_L r_m^2}\left[(1-r_m^2) \pm \sqrt{(1-r_m^2)^2 - 4x_L^2 r_m^2}\right]$.

$r_m = 3$ yields negative t (discard).

For $r_m = \frac{1}{3}$, $t = \begin{cases} 3\sqrt{3/5} \longrightarrow \ell_m = 0.1865\lambda \\ \text{or } \sqrt{15} \longrightarrow \ell_m = 0.2098\lambda \end{cases}$

Use $\ell_m = 0.2098\lambda$ to obtain V_{min} nearest to the load at $(0.5 - 0.2098)\lambda = 0.2902\lambda$.

(5) Cheng P.9-24 a) $|\Gamma|^2 = \left|\dfrac{(R_L - Z_0) + jX_L}{(R_L + Z_0) + jX_L}\right| = \dfrac{(R_L - Z_0)^2 + X_L^2}{(R_L + Z_0)^2 + X_L^2}$

$$\frac{\partial|\Gamma|^2}{\partial Z_0} = 0 \longrightarrow Z_0 = \sqrt{R_L^2 + X_L^2}.$$

If $Z_L = 40 + j30$ (Ω), $Z_0 = 50$ (Ω).

b) $\text{Min. } |\Gamma| = \sqrt{\dfrac{Z_0 - R_L}{Z_0 + R_L}} = \sqrt{\dfrac{50-40}{50+40}} = \dfrac{1}{3}$.

$\text{Min. } S = \dfrac{1 + \frac{1}{3}}{1 - \frac{1}{3}} = 2$.

c) From Eq. (9-147): $r_i + jx_i = \dfrac{r_m + jt}{1 + jr_m t} = 0.8 + j0.6$.

$\longrightarrow t = \dfrac{1}{2x_i r_m^2}\left[(1 - r_m^2) \pm \sqrt{(1-r_m^2)^2 - 4x_i^2 r_m^2}\right]$ (See problem P.9-23.)

At voltage minimum, $r_m = \dfrac{1}{S} = \dfrac{1}{2}$.

$t = 1$ (Use negative sign.)

$\tan\beta\ell_m = \tan(2\pi\ell_m/\lambda) = 1 \longrightarrow \ell_m = \dfrac{\lambda}{8}$.

∴ Voltage minimum nearest to the load is $(\frac{\lambda}{2} - \frac{\lambda}{8})$ or $3\lambda/8$ from the load.

(6) Cheng P.9-26 From Eq. (9-147): $Z_i = R_0' \dfrac{Z_L + jR_0' t}{R_0' + jZ_L t} \longrightarrow Z_L = R_0' \dfrac{Z_i - jR_0' t}{R_0' - jZ_i t}$, $t = \tan\beta\ell'$

$$Z_i = R_0' \frac{Z_L + jR_0' t}{R_0' + jZ_L t} \longrightarrow Z_L = R_0' \frac{Z_i - jR_0' t}{R_0' - jZ_i t}.$$

With $Z_i = 50$ (Ω) and $Z_L = 40 + j10$ (Ω), we have

$$40 + j10 = R_0' \frac{50 - jR_0' t}{R_0' - j50t} \longrightarrow \begin{cases} 40R_0' + 500t = 50R_0', \\ 10R_0' - 2000t = -R_0'^2 t. \end{cases}$$

∴ $R_0' = 38.7$ (Ω); $t = \tan\beta\ell' = 0.775 \longrightarrow \ell' = 0.105\lambda$.

(7) $Z_0 := 75$

$Z_L := 60 + i \cdot 50$

$$\rho := \frac{Z_L - Z_0}{Z_L + Z_0}$$

(a) $\rho = 0.023 + 0.362i$

(b) $|\rho| = 0.363$

(c) $\phi := \text{atan}\left(\frac{\text{Im}(\rho)}{\text{Re}(\rho)}\right)$

$\phi = 1.508$ radians

$\frac{\phi}{\text{deg}} = 86.376$ degrees

(d) $S := \frac{1 + |\rho|}{1 - |\rho|}$

$S = 2.138$

(8) $\omega := 2 \cdot \pi \cdot 10^8$ $L := 10 \cdot 10^{-9}$ $C := 10 \cdot 10^{-9}$

$Y_{Cap} := i \cdot \omega \cdot C$ $Y_{Ind} := \frac{1}{i \cdot \omega \cdot L}$

$Y_{Cap} = 6.283i$ $Y_{Ind} = -0.159i$

$Y_L := Y_{Cap} + Y_{Ind}$ Load Admittance

$Y_0 := \frac{1}{50}$ Characteristic admitttance of line

$$\rho := \frac{Y_L - Y_0}{Y_L + Y_0}$$

(a) $\rho = 1 + 6.532i \times 10^{-3}$

(b) $|\rho| = 1$

(c) $\phi := \text{atan}\left(\frac{\text{Im}(\rho)}{\text{Re}(\rho)}\right)$

$\phi = 6.532 \times 10^{-3}$ radians

$\frac{\phi}{\text{deg}} = 0.374$ degrees

(d) Since $|\rho| = 1$ The VSWR is infiinite

(9) $Y_0 := \frac{1}{50}$ Characteristic admittance of line $Z_0 := 50$

$\omega := 2\cdot\pi\cdot 10^{8}$

$L := 79.58\cdot 10^{-9}$

$Y_{Ind} := \frac{1}{i\cdot\omega\cdot L}$

$Y_{Ind} = -0.02i$

$y_{Ind} := \frac{Y_{Ind}}{Y_0}$ normalized matching admittance

$y_{Ind} = -i$

The normalized line admittance at the matching point is

$y_i := 1 + i$

The actual line admittance is

$Y_i := y_i\cdot Y_0$

$Y_i = 0.02 + 0.02i$

The admittance measured at a distance from the load is

$$Y_i = \frac{Y_0\cdot\left(Y_L\cdot\cos(kl) + i\cdot Y_0\cdot\sin(kl)\right)}{Y_0\cdot\cos(kl) + i\cdot Y_L\cdot\sin(kl)}$$

In this case $kl := 2\cdot\pi\cdot 0.1$

$Y_L := 0.03$ Guess

given

$$Y_i = \frac{Y_0\cdot\left(Y_L\cdot\cos(kl) + i\cdot Y_0\cdot\sin(kl)\right)}{Y_0\cdot\cos(kl) + i\cdot Y_L\cdot\sin(kl)}$$

$ans := find(Y_L)$

$Y_L := ans$

$Y_L = 8.709\times 10^{-3} + 6.832i\times 10^{-3}$

$Z_L := \frac{1}{Y_L}$ $Z_L = 71.078 - 55.764i$ Load impedance

$$\rho := \frac{Z_L - Z_0}{Z_L + Z_0}$$

(a) $\rho = 0.319 - 0.314i$

(b) $|\rho| = 0.447$

(c) $\phi := \operatorname{atan}\left(\frac{\operatorname{Im}(\rho)}{\operatorname{Re}(\rho)}\right)$

$\phi = -0.778$ radians

$\frac{\phi}{deg} = -44.565$ degrees

(d) $S := \frac{1 + |\rho|}{1 - |\rho|}$

$S = 2.618$