**ENEE 381 Problem Set #4**

**4/1/04 due 4/13/04**

(1) A length of loss-less transmission line is first short-circuited at one end and then open-circuited. The impedance measured at the other end in the first case is $Z_1$ and $Z_2$ in the second. Prove that $Z_1 Z_2 = Z_0^2$. This is a convenient way for measuring the characteristic impedance of an unknown line.

(2) A 50 ohm transmission line is terminated with an impedance of 20-j30. What is the magnitude and phase of the reflection coefficient?

(3) Repeat question (4) using the Smith chart

(4) A 75 ohm transmission line is terminated with a load of $150 + j50$ ohm. Compute $\rho$ in terms of both amplitude $|\rho|$ and $\phi$. What fraction of incident power is absorbed in the load?

(5) Cheng problem 9.27

(6) Cheng problem 9.30

(7) Use Smith chart. A line with $Z_0 = 100\Omega$ is terminated with an unknown load. The SWR is found to be 3. A current maximum is observed $0.1\lambda$ from the load. What are:
(a) the load?
(b) the reflection coefficient $\rho$, magnitude and angle?
(c) how would you match the line without changing the load at the end of the line?

(8) Use Smith chart. A transmission line of characteristic impedance 75 ohm is terminated with an impedance 50+j125 ohm. $0.1\lambda$ from the load a 150ohm shorted stub $0.2\lambda$ long is connected in shunt to the main line. What are:
(a) The reflection coefficient in magnitude and phase at this point?
(b) The standing wave ratio?
(c) Where is the nearest current minimum that is greater than $0.1\lambda$ from the load?
(d) Where is the nearest point greater than $0.1\lambda$ from the load where the line can be matched with an open 75 ohm stub?

(9) Use Smith chart. Cheng problem 9.48

(10) Use Smith chart. Cheng problem 9-49.

(11) Use Smith chart. Cheng problem 9-50.

(12) Use Smith chart. Cheng problem 9-51.

## ENEE 381 Problem Set 4. SOLUTIONS

(1) The transformed impedance equation is

$$Z_i = Z_0 \cdot \frac{\left(Z_L \cdot \cos(kl) + j \cdot Z_0 \cdot \sin(kl)\right)}{Z_0 \cdot \cos(kl) + j \cdot Z_L \cdot \sin(kl)}$$

For a shorted line $Z_L$=0, so

$$Z_{short} = j \cdot Z_0 \cdot \tan(kl)$$

For an open line $Z_L$ is infinite, so

$$Z_{open} = -j \cdot Z_0 \cdot \cot(kl)$$

$$Z_{short} \cdot Z_{open} = Z_0^2$$ Q.E.D.

(2) $Z_0 := 50$ $\quad j := i$

$Z_L := 20 - j \cdot 30$

$$\rho := \frac{Z_L - Z_0}{Z_L + Z_0}$$

$\rho = -0.207 - 0.517i$

$|\rho| = 0.557$ magnitude

$\frac{\arg(\rho)}{\deg} = -111.801$ phase angle in degrees

(3) On chart the normalized impedance is

$$\zeta_L := \frac{Z_L}{Z_0}$$

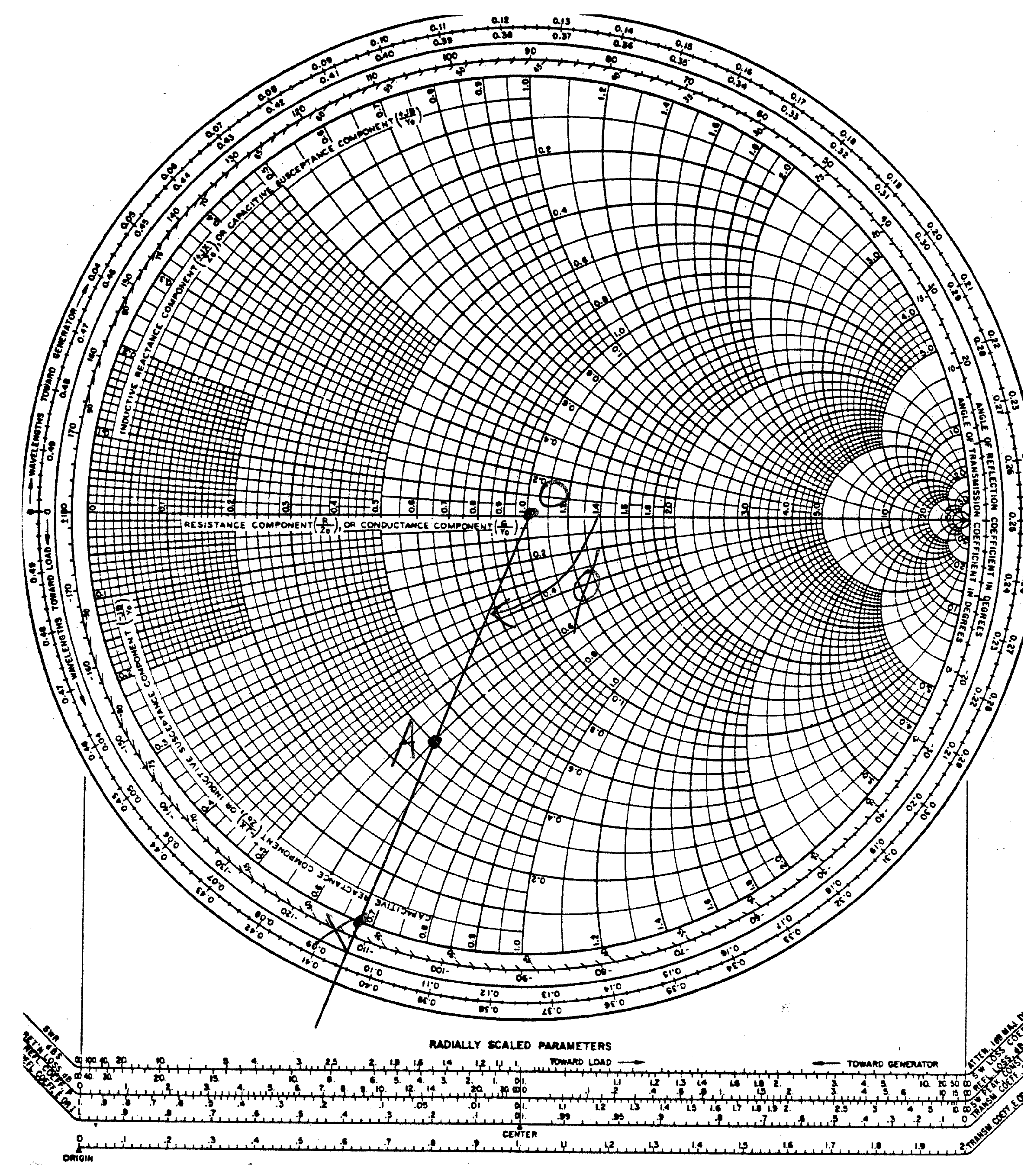
$\zeta_L = 0.4 - 0.6i$ Point A on chart

(4) $Z_0 := 75$ $\quad Z_L := 150 + j \cdot 50$

$$\rho := \frac{Z_L - Z_0}{Z_L + Z_0}$$

$(|\rho|)^2 = 0.153$ Fraction of power reflected

$1 - (|\rho|)^2 = 0.847$ Fraction of power absorbed in load




(3) $z_L = 0.4 - j0.6$

$|\rho| = \dfrac{OA}{OX} = 0.56 \qquad \phi = -111^\circ$

(5) P.9-27 a) $|\Gamma| = \frac{S-1}{S+1} = \frac{2-1}{2+1} = \frac{1}{3}$.

Eq. (9-133a): $V(z') = \frac{I_L}{2}(Z_L + Z_0)e^{j\beta z'}\left[1 + |\Gamma| e^{j\phi}\right]$.

Eq. (9-134): $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = |\Gamma| e^{j\theta_r}, \qquad \phi = \theta_r - 2\beta z'$

Voltage is a minimum when $\phi = \pm\pi \longrightarrow \theta_r = 2\left(\frac{2\pi}{\lambda}\right)\times 0.3\lambda - \pi = 0.2\pi$.

$\therefore\ \Gamma = \frac{1}{3} e^{j0.2\pi}$

b) $Z_L = Z_0\left(\frac{1+\Gamma}{1-\Gamma}\right) = 466 + j206 \ \ (\Omega)$.

c) Terminating resistance $R_m = \frac{R_0}{S} = \frac{300}{2} = 150\ (\Omega)$,

$\ell_m = \frac{\lambda}{2} - z'_m = (0.5 - 0.3)\lambda = 0.2\lambda$.

Another set of solution is: $R'_m = SR_0 = 600\,(\Omega)$ & $\ell'_m = 0.45\lambda$.

(6) P.9-30 a) Given: $V_g = 0.1\angle 0°$ (V), $Z_g = Z_0 = 50\ (\Omega)$, $R_L = 25\,(\Omega) = 0.5 Z_0$.

$$V_i = \frac{Z_i}{Z_0 + Z_i} V_g, \qquad I_i = \frac{V_g}{Z_0 + Z_i},$$

where $Z_i = Z_0 \frac{0.5Z_0 + jZ_0\tan\beta\ell}{Z_0 + j0.5Z_0\tan\beta\ell} = Z_0 \frac{1 + j2\tan\beta\ell}{2 + j\tan\beta\ell}$.

$$\therefore\ V_i = \frac{1 + j2\tan\beta\ell}{3(1 + j\tan\beta\ell)} V_g = \frac{1}{30}\left(\frac{1 + j2\tan\beta\ell}{1 + j\tan\beta\ell}\right) \quad \text{(V)},$$

$$I_i = \frac{2 + j\tan\beta\ell}{3Z_0(1 + j\tan\beta\ell)} V_g = \frac{2}{3}\left(\frac{2 + j\tan\beta\ell}{1 + j\tan\beta\ell}\right) \quad \text{(mA)}.$$

Setting $Z_g = Z_0$ and $\Gamma_g = 0$ in Eqs. (9-120a) and (9-120b), we have

$$V_L = V(z'=0) = \frac{V_g Z_0}{Z_0 + Z_g} e^{-j\beta\ell}(1+\Gamma) \qquad \left(\Gamma = \frac{R_L - Z_0}{R_L + Z_0} = -\frac{1}{3}\right)$$

$$= \frac{1}{30} e^{-j\beta\ell} \quad \text{(V)},$$

$$I_L = I(z'=0) = \frac{V_g}{Z_0 + Z_g} e^{-j\beta\ell}(1-\Gamma) = \frac{4}{3} e^{-j\beta\ell} \quad \text{(mA)}.$$

b) $S = \frac{1+|\Gamma|}{1-|\Gamma|} = 2$.

c) $(P_{av})_L = \frac{1}{2}\mathcal{R}e(V_L I_L^*) = \frac{1}{2}\left(\frac{1}{30}\right)\left(\frac{4}{3}\times 10^{-3}\right) = 2.22\times 10^{-5}$ (W) $= 0.0222$ (mW).

If $R_L = 50\,(\Omega) = Z_0$, $V_L = \frac{V_g}{2} e^{-j\beta\ell}$, $I_L = \frac{V_g}{2Z_0} e^{-j\beta\ell}$

$$\longrightarrow \text{Max.}(P_{av})_L = \frac{V_g^2}{8Z_0} = 2.50\times 10^{-5}\ \text{(W)}.$$

(7) $Z_0 := 100$ $S := 3$

On chart mark point on real axis for S=3, draw circel abour center of chart $I_{max}$ is on left hand side. Load is 0.1λ away at point 0.5-j0.6

$|\rho| = 0.49$ $\phi = 108$ degrees

Sanity check

$$\zeta_L := 0.5 - j\cdot 0.6$$

$$Z_L := Z_0 \cdot \zeta_L$$

$$\rho := \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$|\rho| = 0.483 \qquad \frac{\arg(\rho)}{\deg} = -108.004$$

Matching points are at B or C. You can match with series or parallel components but the distances from the load vary depending on whether you use the chart in impedance or admittance

(8) $Z_0 := 75$ $Z_L := 50 + j\cdot 125$

$\zeta_L := \frac{Z_L}{Z_0}$ Normalized load

$\zeta_L = 0.667 + 1.667i$ point A

$$y_L := \frac{1}{\zeta_L}$$

$y_L = 0.207 - 0.517i$ Normalized admittance of load, point B

Shorted stub 0.2λ long starts at point C and has its normalized admittance at point D

$$y_{stub150} := -j\cdot 0.32$$

Renormalized $Y_{stub} := \frac{1}{150}\cdot y_{stub150}$ multiplying by admittance of 150 ohm line

Normalize again but this time to 75 ohm line

$$y_{stub75} := \frac{Y_{stub}}{\frac{1}{75}}$$

$$y_{stub75} = -0.16i$$

$0.1\lambda$ from the load is point E, where

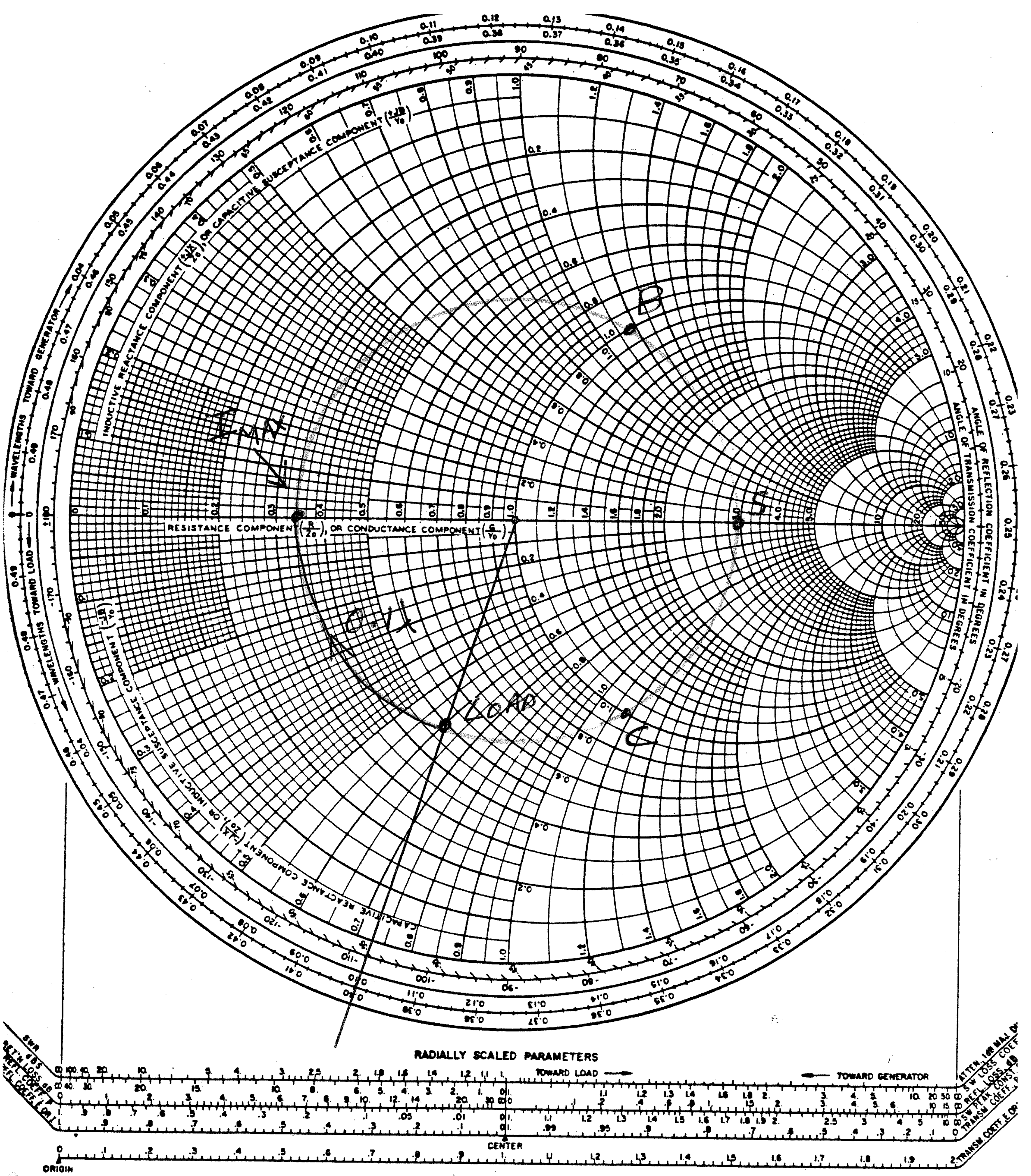
$y_i := 0.17 + j \cdot 0.14$

After adding stub

$y_{total} := y_i + y_{stub75}$

$y_{total} = 0.17 - 0.02i$ Point F

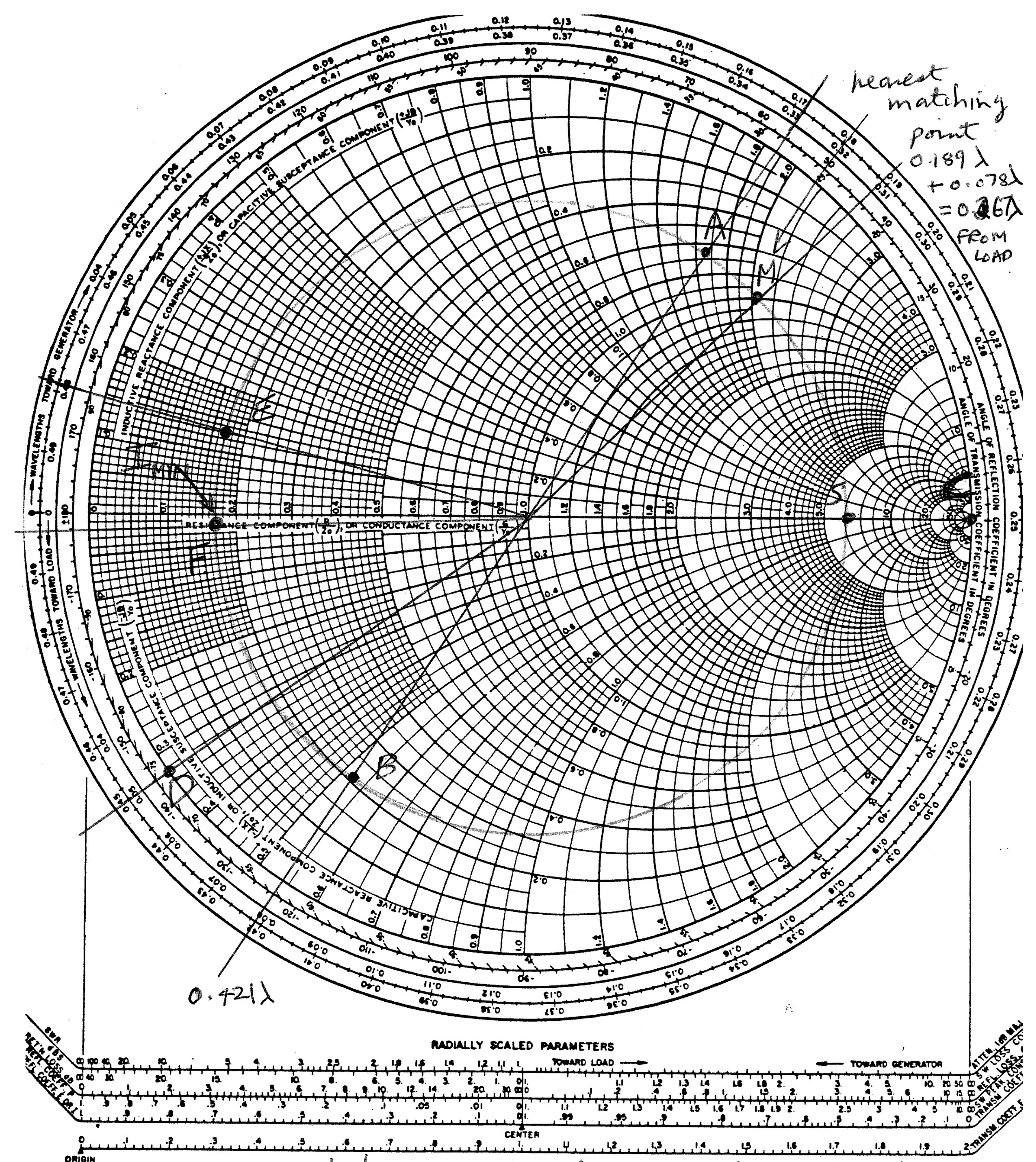
See chart for rest of problem




(7) $z_L = 0.5 - j0.6$ $\quad Z_L = 50 - j60$

$|\rho| = 0.49 \quad \phi = -108^\circ$ MATCH AT B or C

EXAMPLE $B = 1 + j1.16$ ADD normalized $-j1.16$ IN SERIES

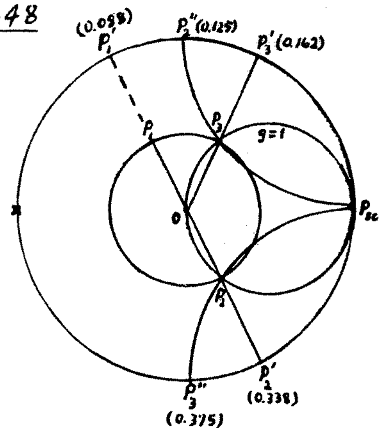



(8) At F $|\rho| = 0.71$ $\phi = -178°$ $S = 6$

S is also the location of $I_{MAX}$, $I_{MIN}$ is just past point F. The nearest $I_{MIN}$ > 0.1λ from load is 0.078λ + 0.5λ = 0.578λ from load

(9)

P.9-48




$z_L = 0.5 + j0.5$.

$y_L = 1 - j$.

a) See construction.

$P_1$ : $z_L = 0.5 + j0.5$.

$P_2$ : $y_L = 1 - j1 = y_2 \longrightarrow d_2 = 0$.

$P_3$ : $y_3 = 1 + j1$.

$\longrightarrow d_3 = 0.162\lambda + (0.5 - 0.338)\lambda$
$= 0.324\lambda$.

$P_2''$ : $b_2 = j1 \longrightarrow \ell_2 = (0.5 + 0.125)\lambda$
$= 0.375\lambda$.

$P_3''$ : $b_3 = -j1 \longrightarrow \ell_3 = (0.375 - 0.25)\lambda$
$= 0.125\lambda$.

b) For $Z_0' = 75 = 1.5 Z_0$, $Y_0' = 0.667 Y_0$.

The required normalized stub admittances are $b_2' = -b_3' = \frac{j}{0.667} = j1.5$.

| | $(Z_0)_{stub} = (Z_0)_{line}$ | $(Z_0)_{stub} = 1.5(Z_0)_{line}$ |
|---|---|---|
| $z_L = 0.5 + j0.5$ | $d_2 = 0$, $\ell_2 = 0.375\lambda$ | $d_2' = 0$, $\ell_2' = 0.406\lambda$ |
| $y_L = 1 - j1$ | $d_3 = 0.324\lambda$, $\ell_3 = 0.125\lambda$ | $d_3' = 0.324\lambda$, $\ell_3' = 0.0936\lambda$ |

(10)

P.9-49 $\quad z_L = 0.5 + j0.5$

Use Smith chart as an impedance chart. Same construction as that in problem P.9-34 except $P_{sc}$ would be on the extreme left (marked by a x), and $g=1$ circle becomes $r=1$ circle.

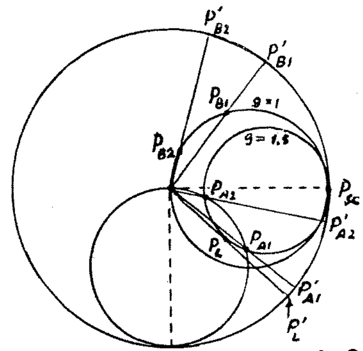
$P_1$: $z_L = 0.5 + j0.5$; $P_2$: $z_{i2} = 1 + j1$ with $d_2 = (0.162 - 0.088)\lambda$
$= 0.074\lambda$.

$P_3$: $z_{i3} = 1 - j1$ with $d_3 = (0.338 - 0.088)\lambda = 0.250\lambda$.

To achieve a match with a series stub having $R_0' = \frac{35}{50} R_0$, we need a normalized stub susceptance $-j\frac{50}{35} = -j1.43$ for solution corresponding to $P_2$. From Smith chart we obtain the required stub length $\ell_2 = 0.347\lambda$.

Similarly for solution corresponding to $P_3$, a stub with a normalized susceptance $+j1.43$ is needed, which requires a stub length $\ell_3 = 0.153\lambda$.

(11) P.9-50




$z_L = 0.33 + j0.33$.

$P_L$: $y_L = 1.50 - j1.50$ ($0.306\lambda$ at $P'_L$),

$P_{A1}$: $y_{A1} = 1.50 - j1.80$ ($0.304\lambda$ at $P'_{A1}$),

$P_{A2}$: $y_{A2} = 1.50 - j0.14$ ($0.269\lambda$ at $P'_{A2}$),

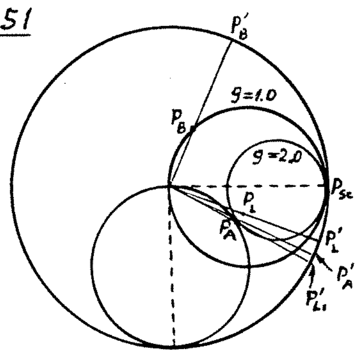
$P_{B1}$: $y_{B1} = 1.00 + j1.60$ ($0.179\lambda$ at $P'_{B1}$),

$P_{B2}$: $y_{B2} = 1.00 + j0.40$ ($0.144\lambda$ at $P'_{B2}$).

| | a) Short-circuited stubs | b) Open-circuited stubs |
|---|---|---|
| $(y_{sA})_1 = y_{A1} - y_L = -j0.30$ | $\ell_{A1} = 0.203\lambda$ | $\ell_{A1} = 0.453\lambda$ |
| $(y_{sA})_2 = y_{A2} - y_L = j1.36$ | $\ell_{A2} = 0.399\lambda$ | $\ell_{A2} = 0.149\lambda$ |
| $(y_{sB})_1 = -j1.60$ | $\ell_{B1} = 0.089\lambda$ | $\ell_{B1} = 0.339\lambda$ |
| $(y_{sB})_2 = -j0.40$ | $\ell_{B2} = 0.189\lambda$ | $\ell_{B2} = 0.439\lambda$ |

(12)

P.9-51




$y_L = \frac{300}{100+j50} = 2.4 - j1.2$.

Point $P_L$ on Smith chart. ($0.280\lambda$ at $P'_L$)

Since the rotated $g=1.0$ circle is tangent to the $g=2.0$ circle, an added line length $d_L$ is needed to convert $g_L$ (2.4) to 2.0, moving from $P_L$ along the $|\Gamma|$-circle to $P_{L1}$ (not shown) on the $g=2.0$ circle ($0.291\lambda$ at $P'_{L1}$). Note that $P_{L1}$ is different from $P_A$, the point of tangency between the $g=2.0$ and rotated $g=1.0$ circles.

a) Min. $d_L = 0.291\lambda - 0.280\lambda = 0.011\lambda$.

b) $P_A$ : $y_A = 2 - j1$ ($0.287\lambda$ at $P'_A$).

$P_B$ : $y_B = 1 + j1$ ($0.162\lambda$ at $P'_B$).

$y_{sA} = y_A - y_{L1} = (2-j1) - (2-j1.35) = j0.35 \longrightarrow \ell_A = 0.304\lambda$.

$y_{sB} = -j1 \longrightarrow \ell_B = 0.125\lambda$.