ENEE 381 Problem Set #6

Due April 26, 2004

Questions (1),(4),(5),(6),(8),and (9) are very relevant for the second examination

(1) Cheng 8.26

(2) Cheng 8.28

(3) A transmission line of characteristic impedance 75ohm is terminated with an inductor of 0.1μH and 50ohm in series. The frequency of operation is 100MHz. Calculate without using the Smith Chart:

(a) $|\rho|$

(b) ϕ

(c) The standing wave ratio

(d) Where on the line closest to the load can the line be matched with the shortest possible shorted stub connected in parallel to the line?

(e) What is the length of this stub?

(4) Repeat (5) with the Smith Chart

(5) A plane wave is incident on the boundary between air and a plastic (ϵ_4=10) at an angle of incidence of 45°. The wave is incident from the air side. Use the Smith chart to find:

(a) $|\rho|$

(b) ϕ

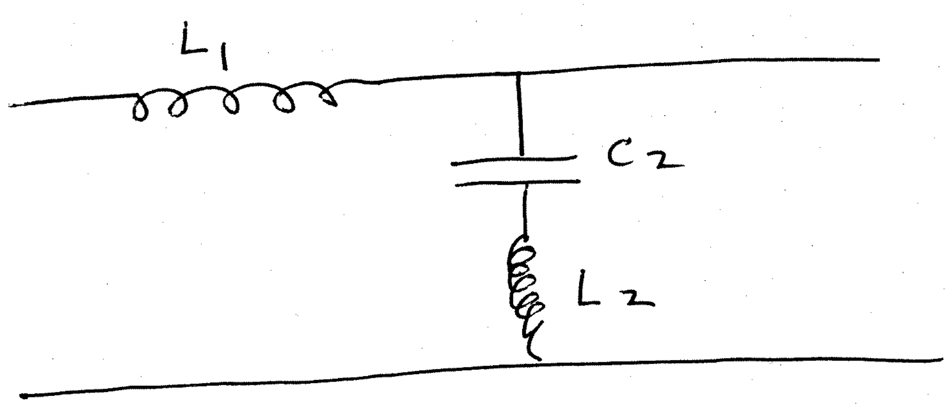
(c) The standing wave ratio

(d) The location of the nearest magnetic field maximum to the boundary

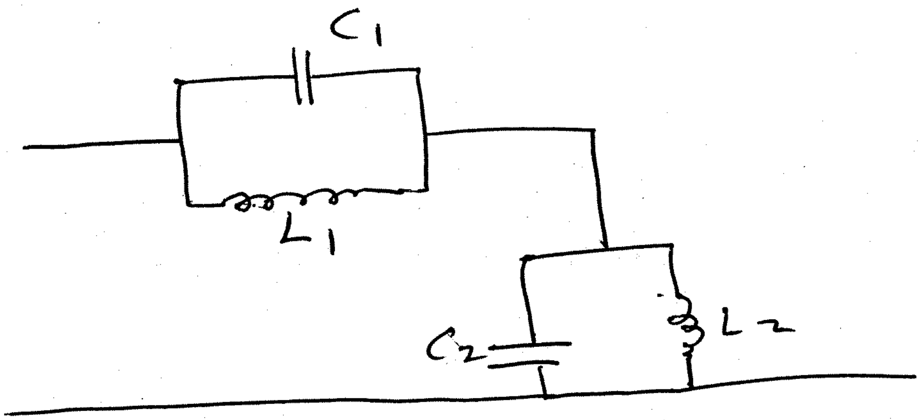
(6)

Calculate and plot the propagation constant (k or β) in the pass band(s), the attenuation constant (α) in the stop band(s), and the cutoff frequency or frequencies for the following generalized transmission lines. Use the values: L_1=10μH/m; C_1=100pF/m; L_2=15μH/m; C_2=75pF/m.

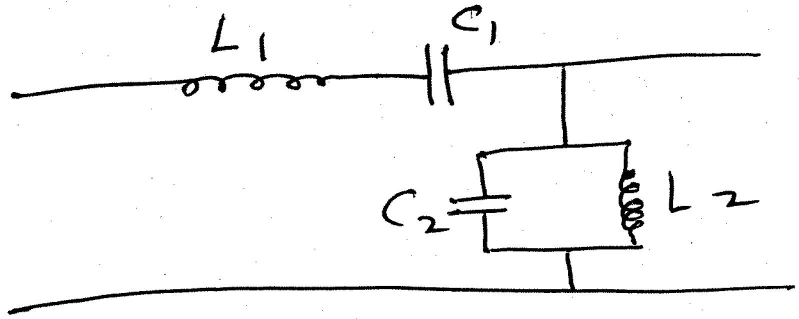
(a)



(b)



(c)



(7) For a sandwich-type radome consisting of two identical thin sheets (thickness 1.5mm, dielectric constant ϵ_r=4) on either side of a thicker foam-type dielectric (thickness 18.1mm, dielectric constant ϵ_r=1.1) calculate the reflection coefficient (amplitude and phase) for normal incidence. Solve for frequencies of 3GHz and 6GHz. Hint: Use Smith chart or the equations for the transformed impedance concept.

(8)For a certain lossless dielectric material of effectively infinite depth, reflections of a plane wave incident normally from free space produce a standing wave ratio of 2.7 in the free space. The boundary is an electric field minimum. What is the dielectric constant?

(9) A plane wave strikes the planar boundary between two lossless semi-infinite dielectrics of dielectric constants 4 and 8 at an angle of 30°. The wave is incident through the lower dielectric constant medium. Calculate

(a) the magnitude and phase of the reflection coefficient

(b) the fraction of energy transmitted into the second medium

(c) the standing wave ratio

(d) the nearest electric field maximum to the boundary

(10) Prove that when a plane wave strikes the planar boundary between two lossless dielectrics at Brewster's angle that the reflected and transmitted waves are at 90° to each other.

(11) A microwave transmitter is placed below the surface of a freshwater lake. Neglecting absorption, find the cone over which you would expect radiation to escape to the air. Use ϵ_r=70.

ENEE 381 Problem Set #5

P.8-26 For normal incidence: $1+\Gamma = \tau$, where $|\Gamma| \le 1$.

If $|\tau| = |\Gamma|$: $\Gamma < 0$ and $\eta_1 - \eta_2 = 2\eta_2 \longrightarrow \eta_1 = 3\eta_2 \longrightarrow |\Gamma| = \frac{1}{2}$.

$$\therefore\ S = \frac{1+|\Gamma|}{1-|\Gamma|} = 3 \quad —— \quad S_{dB} = 20\log_{10} 3 = 9.54\ (dB).$$

P.8-28 a) $\Gamma = \frac{E_r}{E_i} = \frac{\eta_c - \eta_0}{\eta_c + \eta_0}$; $\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi$, $\eta_c = \sqrt{j\omega\mu/\sigma}$.

$|\eta_c| << \eta_0$.

b) $$|\Gamma|^2 = \left|\frac{\eta_c - \eta_0}{\eta_c + \eta_0}\right|^2 = \left|\frac{1-\eta_c/\eta_0}{1+\eta_c/\eta_0}\right|^2 \cong \left|1 - 2\eta_c/\eta_0\right|^2$$

$$= (1 - 2\eta_c/\eta_0)(1 - 2\eta_c^*/\eta_0) \cong 1 - 4\,\mathcal{R}e(\eta_c)/\eta_0.$$

Fraction of power absorbed, $F = 1 - |\Gamma|^2 = \frac{4}{\eta_0}\mathcal{R}e\sqrt{\frac{j\omega\mu}{\sigma}}$

$$= \frac{4}{\eta_0}\sqrt{\frac{\omega\mu}{2\sigma}}.$$

c) $\omega = 2\pi \times 10^6$ (Hz). For iron: $\mu = 4{,}000 \times (4\pi 10^{-7})$ (H/m), $\sigma = 10^7$ (S/m).

$F = 4.21 \times 10^{-4}$, or 0.0421%.

ENEE 381 Problem Set #5
Solutions. Worked out using Mathcad

(3) $Z0 := 75 \quad L := 10^{-7} \quad R := 50 \quad \omega := 2\cdot\pi\cdot 10^{8}$

$ZL := R + i\cdot\omega\cdot L$

$ZL = 50 + 62.83185i$

$\frac{ZL}{Z0} = 0.66667 + 0.83776i$ normalized load

$\rho := \frac{ZL - Z0}{ZL + Z0} \qquad \rho = 0.04204 + 0.48152i$

$|\rho| = 0.48335$ Amplitude

$\frac{\arg(\rho)}{\deg} = 85.01037$ Phase angle

$S := \frac{1 + |\rho|}{1 - |\rho|}$

$S = 2.87113$ Standing wave ratio

For stub problem change to admittance

$Y0 := \frac{1}{Z0} \qquad YL := \frac{1}{ZL}$

$Yi = Y0\cdot\frac{(YL\cdot\cos(kl) + i\cdot Y0\cdot\sin(kl))}{Y0\cdot\cos(kl) + i\cdot YL\cdot\sin(kl)}$ Admittance at distance l from load

$yi = \frac{(YL\cdot\cos(kl) + i\cdot Y0\cdot\sin(kl))}{Y0\cdot\cos(kl) + i\cdot YL\cdot\sin(kl)}$

Normalized admittance at distance l from load. We want this to be 1+jS, for the shortest, shorted stub matching. Since a short, shorted stub starts on the RHS of the Smith chart and goes to -jS values first as it gets longer.

$kl := \frac{\pi}{4}$ Guess

Given

$\mathrm{Re}\left[\frac{(YL\cdot\cos(kl) + i\cdot Y0\cdot\sin(kl))}{Y0\cdot\cos(kl) + i\cdot YL\cdot\sin(kl)}\right] = 1$

$a := \mathrm{find}(kl)$

$kl := a \qquad a = 1.77949$

$l := \frac{kl}{2\cdot\pi} \qquad l = 0.28322$ 0.283λ from load is matching point

$$\frac{(YL\cdot \cos(kl) + i\cdot Y0\cdot \sin(kl))}{Y0\cdot \cos(kl) + i\cdot YL\cdot \sin(kl)} = 1 + 1.10428i$$

So shorted stub has ystub=-j1.104

For a shorted stub

$$Yi = -i\cdot Y0\cdot \cot(klstub)$$

so

$$\cot(klstub) = 1.104$$

$$klstub := \operatorname{atan}\left(\frac{1}{1.104}\right)$$

$$klstub = 0.73601$$

$$lstub := \frac{klstub}{2\cdot \pi}$$

$lstub = 0.11714$ stub length in wavelengths

(5) see Smith Chart

(6)

$C1 := 100\cdot 10^{-12}$ $L1 := 10\cdot 10^{-6}$ $C2 := 75\cdot 10^{-12}$ $L2 := 15\cdot 10^{-6}$

$\omega c1 := \sqrt{\frac{1}{L1\cdot C1}}$ $\omega c2 := \sqrt{\frac{1}{L2\cdot C2}}$ cutoff frequencies

$\omega c1 = 3.16228 \times 10^{7}$ $\omega c2 = 2.98142 \times 10^{7}$

$i := 1, 2 .. 1000$ Running integer to give different frequencies on plots

Use designations 1 for the low frequency band, 2 for the middle band, 3 for the high frequency band

$\omega_i := i\cdot \frac{5\cdot 10^{8}}{1000}$ $\omega 1_i := i\cdot \frac{\omega c2}{1001}$ $\omega 2_i := \omega c2 + i\cdot \frac{(\omega c1 - \omega c2)}{1001}$ $\omega 3_i := \omega c1 + i\cdot \frac{\omega c1}{1001}$

In a pass band γ is imaginary, in a stop band it is real

(a) SERIES L1
SHUNT L2 and C2 in series

$$Z_i := i\cdot \omega_i\cdot L1$$

$$Y_i := \frac{1}{i\cdot \omega_i\cdot L2 + \frac{1}{i\cdot \omega_i\cdot C2}}$$

$$\gamma_i := \sqrt{i\cdot\omega_i\cdot L1\cdot\frac{1}{i\cdot\omega_i\cdot L2+\frac{1}{i\cdot\omega_i\cdot C2}}}$$

$$\omega c := \frac{1}{\sqrt{L2\cdot C2}}$$

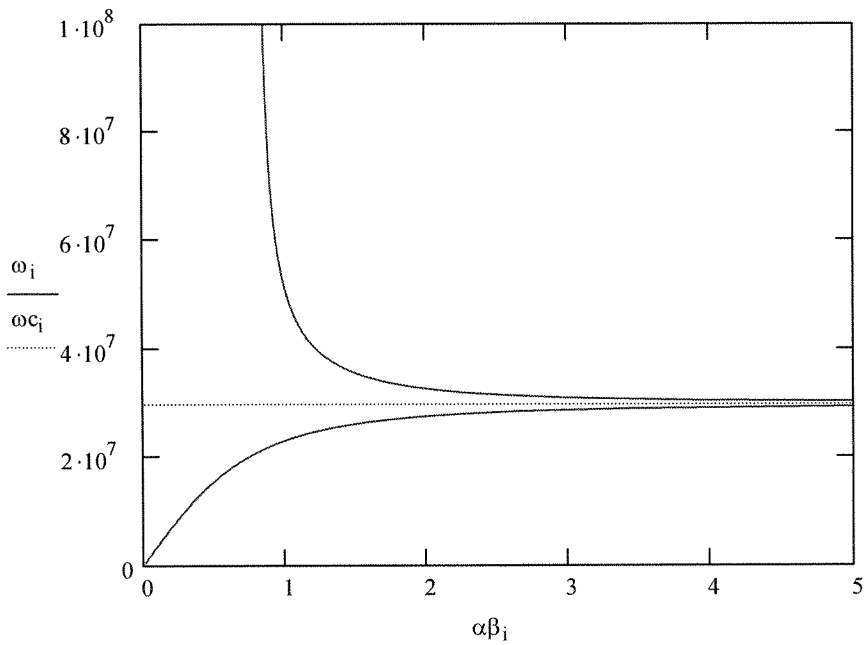
$$\gamma_i := i\cdot\omega_i\cdot\sqrt{L1\cdot C2}\cdot\sqrt{\frac{1}{1-\frac{(\omega_i)^2}{\omega c^2}}}$$

For $\omega<\omega c1$ there is a passband
for $\omega>\omega c1$ there is a stopband

$$\alpha\beta_i := \text{if}\left(\omega_i < \omega c, \text{Im}(\gamma_i), -\text{Re}(\gamma_i)\right)$$

Plots both α in the stopband and β in the passband

$\omega c_i := \omega c$ Draws the horizontal line at $\omega c1$



Note that the pass band is in the bottom, and the stop band in the top of the graph.
The α is symptotic to a value

$$\alpha_{asymptotic} := \omega c\cdot\sqrt{L1\cdot C2}$$

(b) SERIES L1 and C1 in parallel

SHUNT L2 and C2 in parallel

$$Z_i := \frac{1}{i\cdot\omega_i\cdot C1 + \frac{1}{i\cdot\omega_i\cdot L1}} \qquad Y_i := i\cdot\omega_i\cdot C2 + \frac{1}{i\cdot\omega_i L2}$$

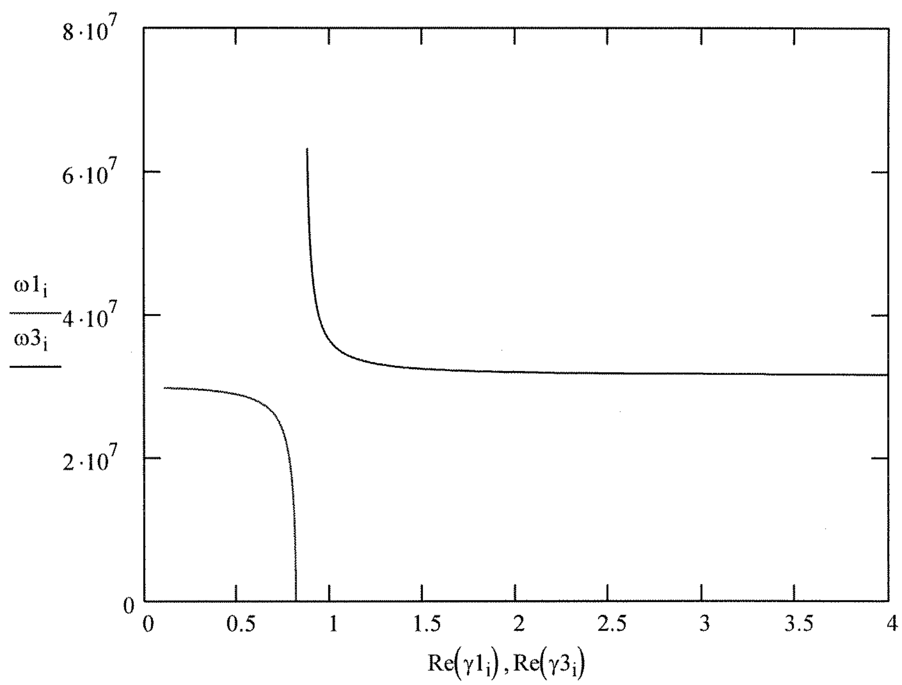
$$Z1_i := \frac{1}{i\cdot\omega1_i\cdot C1 + \frac{1}{i\cdot\omega1_i\cdot L1}} \quad Y1_i := i\cdot\omega1_i\cdot C2 + \frac{1}{i\cdot\omega1_i L2}$$

$$Z2_i := \frac{1}{i\cdot\omega2_i\cdot C1 + \frac{1}{i\cdot\omega2_i\cdot L1}} \quad Y2_i := i\cdot\omega2_i\cdot C2 + \frac{1}{i\cdot\omega2_i L2}$$

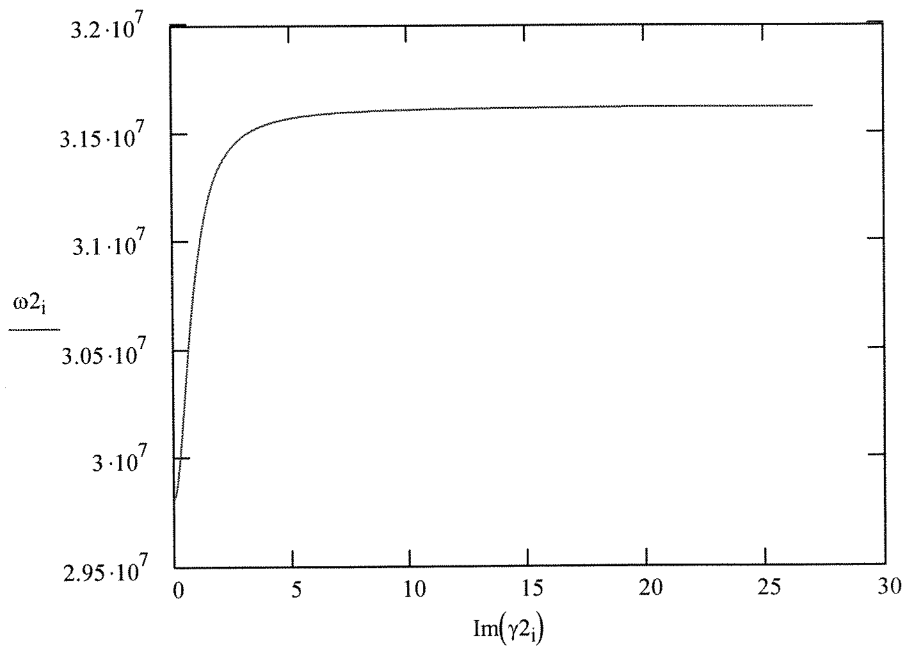
$$Z3_i := \frac{1}{i\cdot\omega3_i\cdot C1 + \frac{1}{i\cdot\omega3_i\cdot L1}} \quad Y3_i := i\cdot\omega3_i\cdot C2 + \frac{1}{i\cdot\omega3_i L2}$$

$$\gamma_i := \sqrt{Z_i\cdot Y_i} \qquad \gamma1_i := \sqrt{Z1_i\cdot Y1_i} \qquad \gamma2_i := \sqrt{Z2_i\cdot Y2_i} \qquad \gamma3_i := \sqrt{Z3_i\cdot Y3_i}$$

alpha in the stop bands



beta in the pass band



(c) SERIES L1 and C1 in series

SHUNT L2 and C2 in parallel

$$Z_i := i \cdot \omega_i L1 + \frac{1}{i \cdot \omega_i \cdot C1} \qquad Y_i := (i \cdot \omega_i \cdot C2) + \frac{1}{i \cdot \omega_i \cdot L2}$$

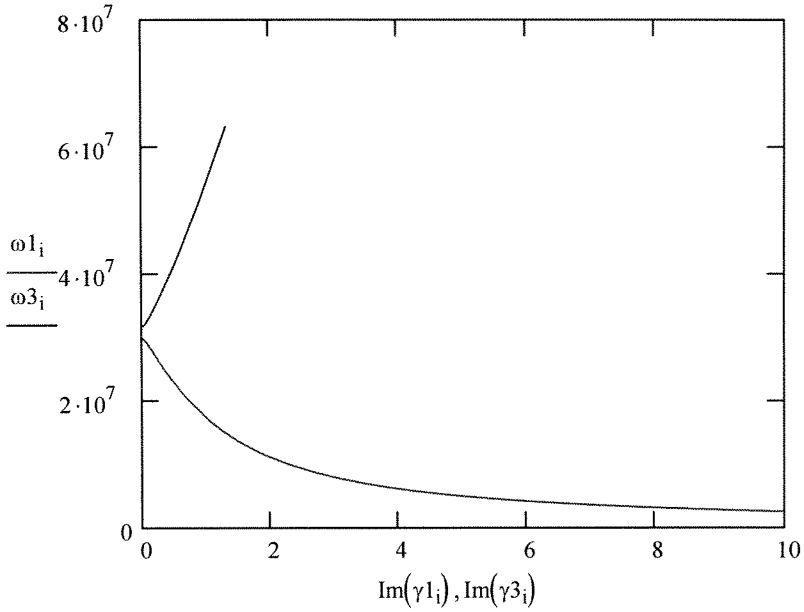
$$Z1_i := i \cdot \omega 1_i L1 + \frac{1}{i \cdot \omega 1_i \cdot C1} \qquad Y1_i := (i \cdot \omega 1_i \cdot C2) + \frac{1}{i \cdot \omega 1_i \cdot L2}$$

$$Z2_i := i \cdot \omega 2_i L1 + \frac{1}{i \cdot \omega 2_i \cdot C1} \qquad Y2_i := (i \cdot \omega 2_i \cdot C2) + \frac{1}{i \cdot \omega 2_i \cdot L2}$$

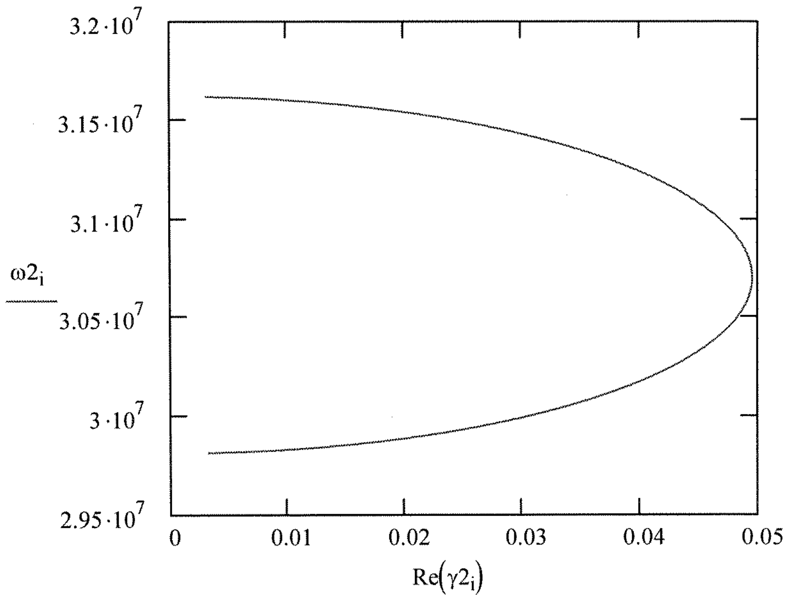
$$Z3_i := i \cdot \omega 3_i L1 + \frac{1}{i \cdot \omega 3_i \cdot C1} \qquad Y3_i := (i \cdot \omega 3_i \cdot C2) + \frac{1}{i \cdot \omega 3_i \cdot L2}$$

$$\gamma_i := \sqrt{Z_i \cdot Y_i} \qquad \gamma 1_i := \sqrt{Z1_i \cdot Y1_i} \qquad \gamma 2_i := \sqrt{Z2_i \cdot Y2_i} \qquad \gamma 3_i := \sqrt{Z3_i \cdot Y3_i}$$

Beta in the pass bands



Alpha in the stop band



(7) **Use the transformed impedance concept. Work back from last interface**

$Z1 := 376.7$ air

$Z2 := \frac{376.7}{\sqrt{4}}$ $d2 := 1.5 \cdot 10^{-3}$ $n2 := 2$

$Z3 := \frac{376.7}{\sqrt{1.1}}$ $d3 := 18.1 \cdot 10^{-3}$ $n3 := \sqrt{1.1}$ radome sandwich

$Z4 := Z2$ $d4 := d2$ $n4 := n2$

$Z5 := Z1$ air

$f := 3 \cdot 10^{9}$ **Results for 3GHz**

$c0 := 2.998 \cdot 10^{8}$

$\lambda 0 := \frac{c0}{f}$

Last interface $Z0 := Z4$ $ZL := Z5$ $kl := \frac{2 \cdot \pi \cdot n4}{\lambda 0} \cdot d4$

$$Zi := Z0 \cdot \frac{(ZL \cdot \cos(kl) + i \cdot Z0 \cdot \sin(kl))}{Z0 \cdot \cos(kl) + i \cdot ZL \cdot \sin(kl)}$$

$Zi = 340.75875 - 94.141i$

Next interface $Z0 := Z3$ $ZL := Zi$ $kl := \frac{2 \cdot \pi \cdot n3}{\lambda 0} \cdot d3$

$$Zi := Z0 \cdot \frac{(ZL \cdot \cos(kl) + i \cdot Z0 \cdot \sin(kl))}{Z0 \cdot \cos(kl) + i \cdot ZL \cdot \sin(kl)}$$

$Zi = 295.65366 + 62.84319i$

Next interface $Z0 := Z2$ $ZL := Zi$ $kl := \frac{2 \cdot \pi \cdot n2}{\lambda 0} \cdot d2$

$Zi = 295.65366 + 62.84319i$

$$\rho := \frac{Zi - Z1}{Zi + Z1}$$

$|\rho| = 0.15187$

$\frac{\arg(\rho)}{deg} = 136.87029$

$f := 6 \cdot 10^9$ **Results for 6GHz**

$c0 := 2.998 \cdot 10^8$

$\lambda 0 := \frac{c0}{f}$

Last interface $Z0 := Z4$ $ZL := Z5$ $kl := \frac{2 \cdot \pi \cdot n4}{\lambda 0} \cdot d4$

$$Zi := Z0 \cdot \frac{(ZL \cdot \cos(kl) + i \cdot Z0 \cdot \sin(kl))}{Z0 \cdot \cos(kl) + i \cdot ZL \cdot \sin(kl)}$$

$Zi = 267.72065 - 137.52415i$

Next interface $Z0 := Z3$ $ZL := Zi$ $kl := \frac{2 \cdot \pi \cdot n3}{\lambda 0} \cdot d3$

$$Zi := Z0 \cdot \frac{(ZL \cdot \cos(kl) + i \cdot Z0 \cdot \sin(kl))}{Z0 \cdot \cos(kl) + i \cdot ZL \cdot \sin(kl)}$$

$Zi = 559.91286 - 129.40928i$

Next interface $Z0 := Z2$ $ZL := Zi$ $kl := \frac{2 \cdot \pi \cdot n2}{\lambda 0} \cdot d2$

$Zi = 559.91286 - 129.40928i$

$\rho := \frac{Zi - Z1}{Zi + Z1}$

$|\rho| = 0.23723$ $\frac{\arg(\rho)}{deg} = -27.36822$

(8) $|\rho| = \frac{(S - 1)}{(S + 1)}$ $S := 2.7$

$A := \frac{(S - 1)}{(S + 1)}$

$A = 0.45946$ This is the amplitude of ρ

Since the boundary is an electric field minimum, the normalized impedance must be on the left half of the Smith chart and ϕ=180º.

$\rho := -A$

$$\rho = \frac{\frac{1}{\sqrt{\varepsilon r}} - 1}{\frac{1}{\sqrt{\varepsilon r}} + 1} \qquad \varepsilon r := \frac{(\rho - 1)^2}{(\rho + 1)^2}$$

$\varepsilon r = 7.29$ dielectric constant

(9) $\varepsilon r1 := 4$ $\varepsilon r2 := 8$ $Z0 := 376.7$

$n1 := \sqrt{\varepsilon r1}$ $n2 := \sqrt{\varepsilon r2}$

$\theta 1 := 30 \cdot deg$

$$\theta 2 := \operatorname{asin}\left(\frac{n1 \cdot \sin(\theta 1)}{n2}\right)$$

$\frac{\theta 2}{deg} = 20.70481$ angle of refraction

For TM waves
$$\rho := \frac{\frac{Z0 \cdot \cos(\theta 2)}{n2} - \frac{Z0 \cdot \cos(\theta 1)}{n1}}{\frac{Z0 \cdot \cos(\theta 2)}{n2} + \frac{Z0 \cdot \cos(\theta 1)}{n1}}$$

$|\rho| = 0.13394$ amplitude

$\frac{\arg(\rho)}{deg} = 180$ phase

$\frac{1 + |\rho|}{1 - |\rho|} = 1.30931$ standing wave ratio

The normalized load is
$$\zeta L := \frac{\frac{Z0 \cdot \cos(\theta 2)}{n2}}{\frac{Z0 \cdot \cos(\theta 1)}{n1}}$$

$\zeta L = 0.76376$ This is on the left side of the Smith chart, which is an E-field minimum. So, nearest E-field maximum appears to be 0.25λ from boundary, but because the wave is traveling at an angle the actual distance is

$$d := \frac{1}{2 \cdot \cos(\theta 1)}$$

$d = 0.57735$ **ANSWER**

For TE waves
$$\rho := \frac{\frac{Z0}{n2 \cdot \cos(\theta 2)} - \frac{Z0}{n1 \cdot \cos(\theta 1)}}{\frac{Z0}{n2 \cdot \cos(\theta 2)} + \frac{Z0}{n1 \cdot \cos(\theta 1)}}$$

$|\rho| = 0.20871$ amplitude

$\frac{\arg(\rho)}{deg} = 180$ phase

$\frac{1+|\rho|}{1-|\rho|} = 1.52753$ standing wave ratio

The normalized load is $\zeta L := \frac{\frac{Z0}{n2\cdot\cos(\theta 2)}}{\frac{Z0}{n1\cdot\cos(\theta 1)}}$

$\zeta L = 0.65465$ This is on the left side of the Smith chart, which is an E-field minimum. So, nearest E-field maximum is $0.25\lambda/\cos(\theta 1)$ from the boundary

$$d := \frac{1}{2\cdot\cos(\theta 1)}$$

$d = 0.57735$ **ANSWER**

(10) Brewster's angle satisfies

$$\tan(\theta_B) = \operatorname{atan}\left(\frac{n2}{n1}\right)$$

so
$$\sin(\theta_B) = \frac{n2}{\sqrt{n2^2 + n1^2}}$$

$$\cos(\theta_B) = \frac{n1}{\sqrt{n2^2 + n1^2}}$$

We want to prove that $\theta_B+\theta_R=90^o$, which implies that $\sin(\theta_B)=\cos(\theta_R)$

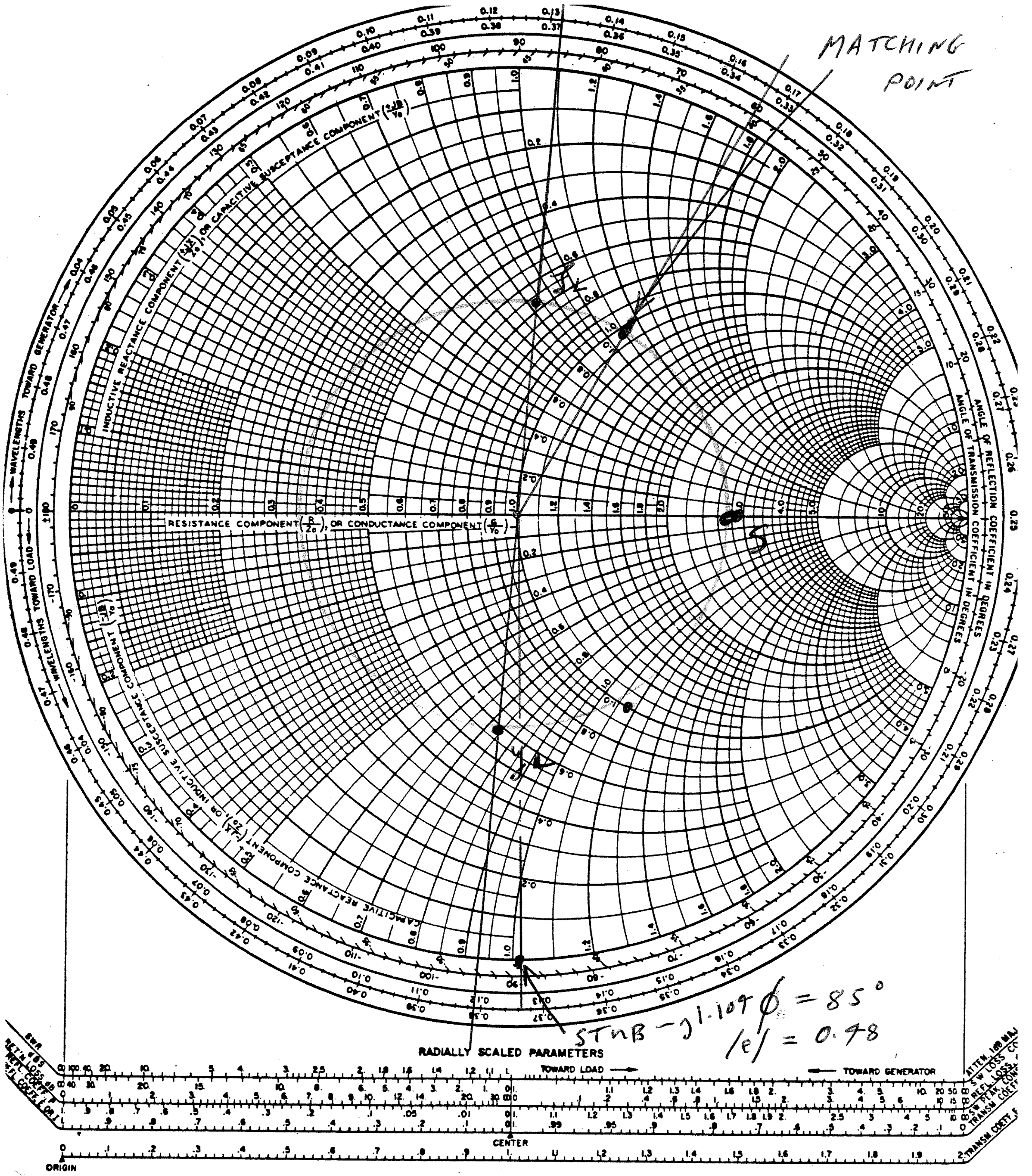
Now $n_1\sin(\theta_B)=n_2\sin(\theta_R)$, where θ_R is the angle of refraction.

Clearly $\sin(\theta_R)=n_1\sin(\theta_B)/n_2=\cos(\theta_B)$. **Q.E.D.**

(11) This is a problem about total internal reflection, which is the angle when $\theta_2=90^o$ for a wave going from high dielectric constant to low dielectric constant.

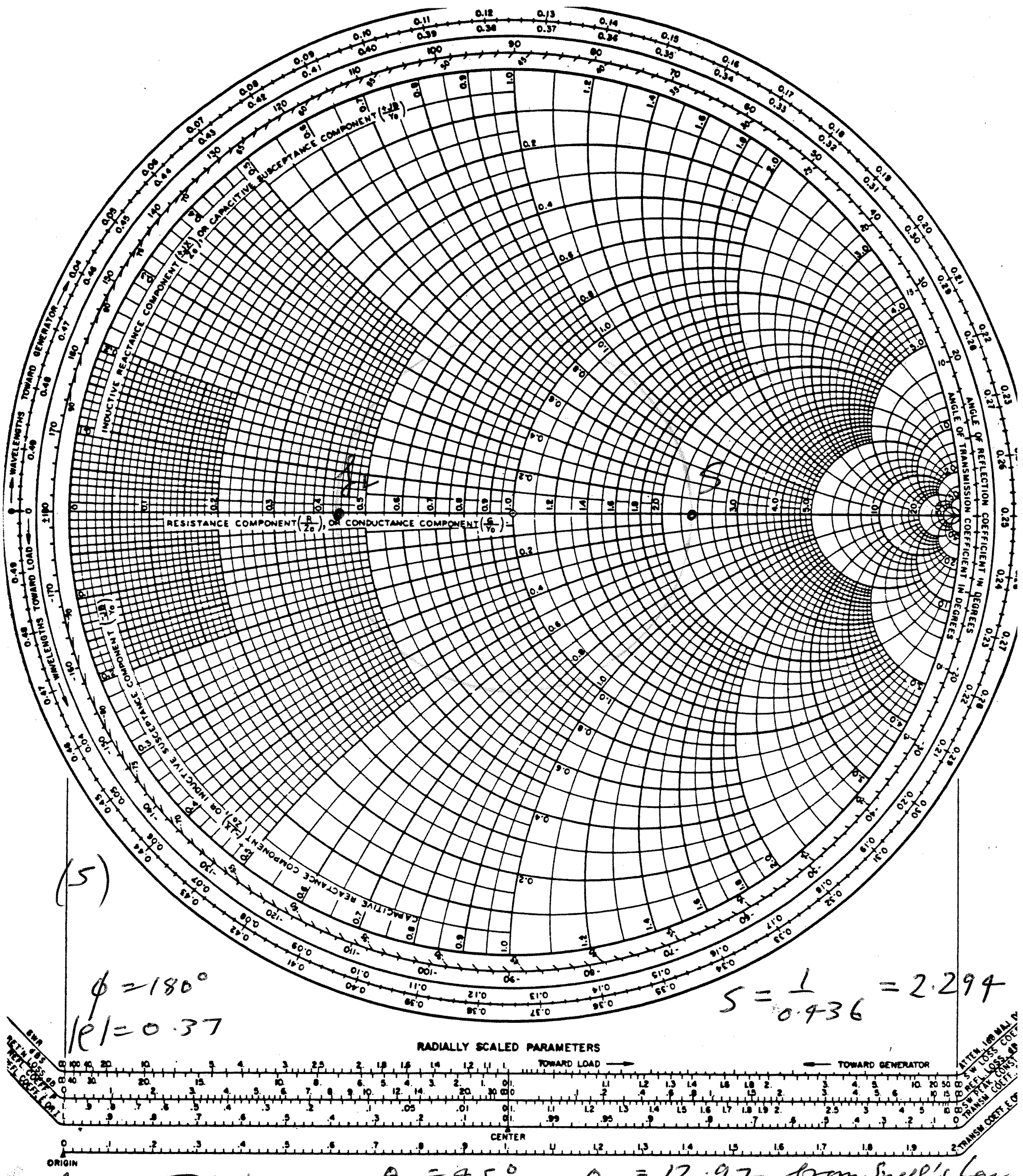
$$\theta 1 := \operatorname{asin}\left(\frac{1}{\sqrt{70}}\right)$$

$\frac{\theta 1}{deg} = 6.86457$ This is the semi-vertical angle of the cone of wave directions that can escape



$l_{stub} = 0.129\lambda$

(4)



(5)

$\phi = 180^\circ$

$|\rho| = 0.37$

$S = \frac{1}{0.436} = 2.294$

Assume TM wave $\theta_1 = 45^\circ$, $\theta_2 = 12.92$ from Snell's Law

$Z_1' = Z_0 \cos\theta_1 = 266.4$ ohm $\quad Z_2' = \frac{Z_0}{\sqrt{10}} \cos\theta_2 = 116.1$ ohm

$\rho_L = 0.436$

Magnetic field max is at boundary