ENEE381 Problem Set #6 Solutions

(1) $a := 10 \cdot 10^{-3}$ $\quad c := 3 \cdot 10^{8}$

$$K_c = \frac{m \cdot \pi}{a}$$

$$\gamma = \sqrt{K_c^{2} - \frac{\omega^{2}}{c^{2}}}$$

Cutoff frequency obeys

$$\omega_c = \frac{m \cdot \pi \cdot c}{a} \quad \text{or} \quad \nu_c = \frac{m \cdot c}{2 \cdot a}$$

Between parallel planes the cutoff frequencies are the same for TE and TM modes with the same m.

For TM and TE

m=1 $\quad \nu_c := \frac{c}{2 \cdot a}$ $\quad \nu_c = 15 \cdot 10^{9}$ $\quad$ 15GHz

m=2 $\quad \nu_c := \frac{2 \cdot c}{2 \cdot a}$ $\quad \nu_c = 30 \cdot 10^{9}$ $\quad$ 30GHz

m=3 $\quad \nu_c := \frac{3 \cdot c}{2 \cdot a}$ $\quad \nu_c = 45 \cdot 10^{9}$ $\quad$ 45GHz

(2) $a := 50 \cdot 10^{-3}$ $\quad \varepsilon_r := 2.25$ $\quad c_0 := 3 \cdot 10^{8}$ $\quad \nu := 10 \cdot 10^{9}$

$c := \frac{c_0}{\sqrt{\varepsilon_r}}$ $\quad \omega := 2 \cdot \pi \cdot \nu$ $\quad Z := \frac{376.7}{\sqrt{\varepsilon_r}}$

(a) For TEM wave β=k $\quad k := \frac{\omega}{c}$ $\quad k = 314.159$

phase velocity and group velocity =c

$Z := \frac{376.7}{\sqrt{\varepsilon_r}}$ $\quad Z = 251.133$ $\quad$ characteristic impedance

guide wavelength is the same as free space wavelength

$\lambda := \frac{c}{\nu}$ $\quad \lambda = 0.02$

$$\beta := \frac{\omega}{c}\cdot\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}$$

$\beta = 287.932$

$v_p = \frac{\omega}{\beta}$ $\quad v_p := \frac{\omega}{\beta}$ $\quad v_p = 2.182\cdot 10^8$

$$\beta := \frac{\omega}{c}\cdot\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}$$

$$v_p := \frac{c}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}} \qquad v_g := c\cdot\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}$$

$v_g = 1.833\cdot 10^8$

$$Z_z := Z\cdot\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2} \qquad Z_z = 230.168$$

$$\lambda_g := \frac{2\cdot\pi}{\beta} \qquad \lambda_g = 0.022$$

(3) For a rectangular waveguide

$$K_c = \sqrt{\left(\frac{m\cdot\pi}{a}\right)^2 + \left(\frac{n\cdot\pi}{b}\right)^2}$$

$a := 50\cdot 10^{-3}$

$b := 20\cdot 10^{-3}$ $\quad c := 3\cdot 10^8$

(a) $m := 1$ $\quad n := 0$

$$\nu_c := \frac{c}{2}\cdot\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \qquad \nu_c = 3\cdot 10^9$$

(b) $m := 2$ $\quad n := 0$

$$\nu_c := \frac{c}{2}\cdot\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \qquad \nu_c = 6\cdot 10^9$$

(c) $m := 2$ $\quad n := 2$

$$\nu_c := \frac{c}{2}\cdot\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \qquad \nu_c = 1.616\cdot 10^{10}$$

(d) $m := 1 \qquad n := 1$

$$\nu_c := \frac{c}{2}\cdot\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \qquad \nu_c = 8.078\cdot 10^{9}$$

(e) $m := 2 \qquad n := 3$

$$\nu_c := \frac{c}{2}\cdot\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \qquad \nu_c = 2.329\cdot 10^{10}$$

If the waveguide is filled with a material with dielectric constant 4, all cutoff frequencies are reduced by a factor of 2, since the velocity of light is reduced by a factor of 2

(4) $m := 1 \qquad n := 0$

$$\nu_c := \frac{c}{2}\cdot\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \qquad \nu_c = 3\cdot 10^{9}$$

$$\nu := 1.1\cdot\nu_c$$

$$\omega := 2\cdot\pi\cdot\nu \qquad \omega_c := 2\cdot\pi\cdot\nu_c$$

$$\beta := \frac{\omega}{c}\cdot\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2} \qquad \beta = 28.793$$

$$Z_{TE} := \frac{376.7}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}} \qquad Z_{TE} = 904.229$$

$$v_p := \frac{c}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}} \qquad v_p = 7.021\cdot 10^{8}$$

$$v_g := c\cdot\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2} \qquad v_g = 1.25\cdot 10^{8}$$

$\lambda_g := 2 \cdot \frac{\pi}{\beta}$ $\lambda_g = 0.218$

(5) $m := 1$ $n := 1$ $a := 20 \cdot 10^{-3}$ $b := 10 \cdot 10^{-3}$

$\nu_c := \frac{c}{2} \cdot \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$ $\nu_c = 1.677 \cdot 10^{10}$

$\nu := 1.2 \cdot \nu_c$

$\omega := 2 \cdot \pi \cdot \nu$ $\omega_c := 2 \cdot \pi \cdot \nu_c$

$\beta := \frac{\omega}{c} \cdot \sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}$ $\beta = 232.987$

$Z_{TM} := 376.7 \cdot \sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}$ $Z_{TM} = 208.229$

$v_p := \frac{c}{\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}}$ $v_p = 5.427 \cdot 10^8$

$v_g := c \cdot \sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}$ $v_g = 1.658 \cdot 10^8$

$\lambda_g := 2 \cdot \frac{\pi}{\beta}$ $\lambda_g = 0.027$

(6) $m := 1 \quad n := 0 \quad c := 3\cdot10^{8} \quad a := 50\cdot10^{-3} \quad b := 20\cdot10^{-3}$

$$\nu_c := \frac{c}{2}\cdot\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \qquad \nu_c = 3\cdot10^{9}$$

$$\nu := 1.1\cdot\nu_c$$

$$\omega := 2\cdot\pi\cdot\nu \qquad \omega_c := 2\cdot\pi\cdot\nu_c$$

$$Z_{TE1} := \frac{376.7}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}} \qquad Z_{TE1} = 904.229$$ First section of waveguide

Second section of waveguide $c := \frac{3\cdot10^{8}}{\sqrt{3}}$

$$\nu_c := \frac{c}{2}\cdot\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \qquad \nu_c = 1.732\cdot10^{9}$$

$$\omega_c := 2\cdot\pi\cdot\nu_c$$

$$Z_{TE2} := \frac{\frac{376.7}{\sqrt{3}}}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}} \qquad Z_{TE2} = 255.511$$ Second section of waveguide

$$\rho := \frac{Z_{TE2} - Z_{TE1}}{Z_{TE2} + Z_{TE1}}$$

$|\rho| = 0.559$ magnitude of reflection coefficient

$\frac{\arg(\rho)}{deg} = 180$ phase of reflection coefficient

$S := \frac{1-|\rho|}{1+|\rho|} \qquad s = 0.283$ VSWR

$1-(|\rho|)^2 = 0.687$ Fraction of power into second guide region

(7)

$m := 1 \quad n := 0 \quad c := 3\cdot 10^{8} \quad a := 20\cdot 10^{-3} \quad b := 10\cdot 10^{-3}$

$$\nu_c := \frac{c}{2}\cdot\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \qquad \nu_c = 7.5\cdot 10^{9}$$

$\nu := 10\cdot 10^{9}$

$\omega := 2\cdot\pi\cdot\nu \qquad \omega_c := 2\cdot\pi\cdot\nu_c$

$$Z_{TE} := \frac{376.7}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}} \qquad Z_{TE} = 569.517$$

The spatial variation of the field is of the form

$$E_{.y} = E_0\cdot\sin\left(\pi\cdot\frac{x}{a}\right)$$

The time averaged S is

$$S_{max} = \frac{E_0^{\,2}}{2\cdot Z_{TE}}\cdot\sin^2\left(\frac{\pi\cdot x}{a}\right)$$

Integrate over waveguide cross-section

$$\text{Power} = \frac{a\cdot b}{4\cdot Z_{TE}}\cdot E_0^{\,2}$$

$\text{Power} := 1$

$$E_0 := \sqrt{\frac{4\cdot\text{Power}\cdot Z_{TE}}{a\cdot b}}$$

$E_0 = 3.375\cdot 10^{3}$ peak electric field