

ENEE 381 Spring 2004
SOLUTIONS TO FIRST EXAMINATION QUESTIONS

(1) The negative sign in the equation

$$\frac{E_y}{H_x} = -\sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}}$$

comes from the fact that (y,x,z) is not a right-handed coordinate system. In free space $\mathbf{j}=0$ and the result comes from the equation

$$\text{curl} H_y = \frac{d}{dz} H_x = \epsilon_0 \epsilon_r \frac{d}{dt} E_y \quad \text{which gives}$$

$$-k \cdot H_0 \cdot \cos(\omega \cdot t - k \cdot z) = \epsilon_0 \epsilon_r \cdot \omega \cdot E_0 \cdot \cos(\omega \cdot t - k \cdot z)$$

The required answer follows because

$$\frac{\omega}{k} = c = \frac{1}{\sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}}$$

The time-averaged Poynting vector is

$$\frac{(E_0)^2}{2 \cdot Z} \quad \text{or} \quad \frac{Z \cdot (H_0)^2}{2}$$

$$\text{with} \quad \epsilon_0 := 8.854 \cdot 10^{-12} \quad \mu_0 := 4 \cdot \pi \cdot 10^{-7} \quad \epsilon_r := 2 \quad \mu_r := 10 \quad H_0 := 10$$

$$Z := \sqrt{\frac{\mu_0 \cdot \mu_r}{\epsilon_0 \cdot \epsilon_r}}$$

$$Z = 842.404 \quad \text{ohms} \quad E_0 = Z \cdot H_0 = 8424.04 \quad \text{V/m}$$

$$S_{\text{avg}} := \frac{Z \cdot H_0^2}{2}$$

$$S_{\text{avg}} = 4.212 \times 10^4 \quad \text{W/m}^2$$

Write the given E_y and H_x as phasors

$$E_y := 10 \cdot e^{i \cdot \left(\frac{\pi}{7}\right)} \quad H_x := \frac{10}{Z} \cdot e^{-i \cdot \left(\frac{\pi}{5}\right)}$$

$$S_{\text{avg}} := \frac{1}{2} \cdot \text{Re}(E_y \cdot \overline{H_x}) \quad \text{overbar indicates complex conjugate}$$

$$S_{\text{avg}} = 0.028 \quad \text{W/m}^2$$

- (2) The derivation required is called Poynting's theorem. the energy flow goes into increasing stored electromagnetic energy and ohmic dissipation, plus work done on electric and magnetic dipoles

The resistance per unit length of the wire is

$$R = \frac{1}{\sigma \cdot \pi \cdot r^2} \quad \text{which gives for } \sigma := 5 \cdot 10^7 \quad r := 10^{-2}$$

$$R := \frac{1}{\sigma \cdot \pi \cdot r^2} \quad R = 6.366 \times 10^{-5}$$

At the surface of the wire the electric field is in the axial (z) direction

$$E_z := 2 \quad \text{V/m}$$

The current I in the wire is

$$I := \frac{E_z}{R}$$

At the surface of the wire the magnetic field is in the azimuthal (ϕ) direction

$$H_\phi := \frac{I}{2 \cdot \pi \cdot r}$$

The Poynting vector flux into the wire per unit length is

$$W := E_z \cdot H_\phi \cdot 2 \cdot \pi \cdot r$$

$$W = 6.283 \times 10^4$$

The ohmic dissipation is

$$W_{\text{ohmic}} := I^2 \cdot R$$

$$W_{\text{ohmic}} = 6.283 \times 10^4 \quad \text{same answer}$$

- (3) The skin depth comes from the standard derivation and is

$$\delta = \frac{1}{\sqrt{\pi \cdot \nu \cdot \mu_0 \cdot \mu_r \cdot \sigma}}$$

For a low loss dielectric the propagation constant of the wave is

$$k = \omega \cdot \sqrt{\mu_0 \cdot \mu_r \cdot \epsilon_0 \cdot \epsilon_r}$$

In this case this is

$$k = \omega \cdot \sqrt{(\mu_0 \cdot \mu_r \cdot \epsilon_0) \cdot \sqrt{\epsilon_1 - j \cdot \epsilon_2}}$$

where the dielectric constant is written as $\epsilon_1 - j\epsilon_2$

For $\epsilon_1 \gg \epsilon_2$

$$k = \omega \cdot \sqrt{\mu_0 \cdot \mu_r \cdot \epsilon_0 \cdot \epsilon_1} \cdot \left(1 - j \cdot \frac{\epsilon_2}{\epsilon_1}\right)^{\frac{1}{2}} = \omega \cdot \sqrt{\mu_0 \cdot \mu_r \cdot \epsilon_0 \cdot \epsilon_1} \cdot \left(1 - j \cdot \frac{\epsilon_2}{2 \cdot \epsilon_1}\right)$$

The attenuation coefficient of the wave is found from the imaginary part of k

$$\alpha = \omega \cdot \sqrt{\mu_0 \cdot \mu_r \cdot \epsilon_0 \cdot \epsilon_1} \cdot \frac{\epsilon_2}{2 \cdot \epsilon_1}$$

The 1/e penetration depth is

$$\delta = \frac{1}{\alpha}$$

With $\epsilon_2 := 4$ $\epsilon_1 := 0.01$ $\mu_r := 1$

$$\left(\sqrt{\mu_0 \cdot \mu_r \cdot \epsilon_0 \cdot \epsilon_1} \cdot \frac{\epsilon_2}{2 \cdot \epsilon_1}\right)^{-1} = 1.499 \times 10^7$$

$$\delta = \frac{1.499 \cdot 10^7}{\omega}$$

$$(4) \quad H_{\text{total}} = H_i \cdot e^{j(\omega \cdot t - k \cdot z)} + H_r \cdot e^{j(\omega t + k \cdot z)} \quad \text{which gives}$$

$$H_{\text{total}} = \frac{E_i}{Z} e^{j\omega t} \cdot (e^{-jkz} - \rho \cdot e^{jkz}) \quad \rho = -1 \text{ so}$$

$$H_{\text{total}} = \frac{E_i}{Z} \cdot e^{j\omega t} \cdot (e^{-jkz} + e^{jkz}) = \frac{2 \cdot E_i}{Z} \cdot \cos(kz)$$

$$E_{\text{total}} = E_i \cdot e^{j\omega t} \cdot (e^{-jkz} - e^{jkz}) = -2 \cdot j \cdot E_i \cdot \sin(kz)$$

The magnetic field amplitude at the perfect conductor is twice that from the incident wave alone so

$$H \text{ at conductor} = 2A/m$$

The surface current is also 2A/m

The Lorentz force between electrons moving in the surface and the magnetic field is

$$F = -e \cdot (v \times B)$$

This force acts into the conductor so there is an effective "radiation" pressure on the conductor