**ENEE 381 Problem Set #1**

9/9/04 - due 9/16/04

(1)(380 Review) The current density in a certain region is

$$\mathbf{J} = 0.1e^{-10^6 t}\hat{\mathbf{r}}/r$$

in spherical coordinates. At t=1$\mu$s how much current is crossing the surface $r$=5?

(2) (380 review) A current density $\mathbf{J} = 5\hat{\mathbf{j}}\mathrm{A/m}^2$ exists wherever $|z| < 2$m. (a) Find $\mathbf{H}$ for $|z| < 2$ and $|z| > 2$. Find the magnetic vector potential $\mathbf{A}$ for $|z| < 2$ if $\mathbf{A} = 0$ at the origin.

(3) A circular coil of 100 turns of radius 50mm, total resistance 1 ohm, and no self inductance is rotated about a vertical diameter with uniform angular velocity 100 rad/s in a horizontal magnetic flux of 0.2 Tesla. Calculate the average power needed to keep the coil in motion.

The mean power required to keep the coil in motion is

$$W = n^2\pi^2 a^4 b^2 \omega^2/(2R)$$

What is the ohmic power dissipated in the coil?

(4) A small magnetic needle, which is free to turn slowly in a horizontal plane, is placed at the center of the coil in question (3). Calculate the angle with respect to $\mathbf{B}$ at which it reaches equilibrium.

Show that it will set at an angle $\phi$ to $B$ where

$$\cot\phi = 4R/(\pi n^2 \mu_o \omega a)$$

(5) A charged particle starts from rest at the origin of ooordinates in a region where there is a uniform electric field $\mathbf{E}$ parallel to the $x$-axis, and a uniform magnetic flux density $\mathbf{B}$ parallel to the $z$-axis. Show that the coordinates of the particle at a time $t$ later will be

$$x = (\frac{E}{\omega B})(1 - \cos(\omega t),$$

$$y = -(\frac{E}{\omega B})(\omega t - \sin\omega t),$$

$$z = 0,$$

where $\omega = eB/m$. ($E = |\mathbf{E}|$, $B = |\mathbf{B}|$)(This path is called a cycloid.)

(6) Electrons are liberated with zero velocity from the negative plate of a parallel plate capacitor, to which is applied a magnetic flux density $\mathbf{B}$ parallel to the plates. Prove that these electrons will not reach the positive plate if the plate separation $d$ is greater than $2mE/eB^2$, where $E = |\mathbf{E}|$ is the field between the plates.

(7) A plane circular disk of radius $a$ rotates at a speed of $2\pi f$ rad/s about an axis through the center of the disk perpendicular to the plane of the disk. There is a uniform magnetic flux $\mathbf{B}$ parallel to the axis of rotation. Prove that the $emf$ between the center of the disk and its rim is of magnitude $V = fB\pi a^2$. ($B = |\mathbf{B}|$.)

# ENEE 381 PROBLEM SET #1

SOLUTIONS

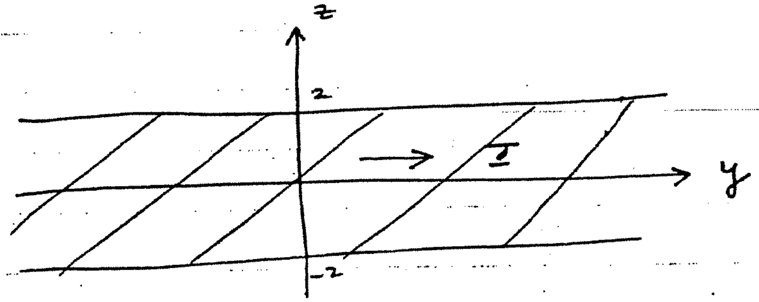
(1) $\underline{J} = \dfrac{0.1 e^{-10^6 t}\,\hat{r}}{r}$

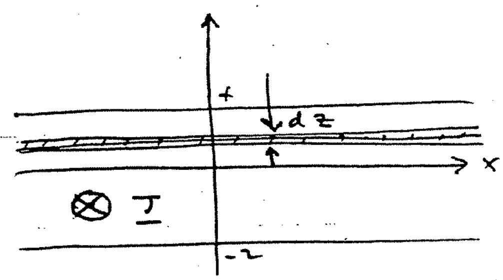
At $t = 1\,\mu s$ $\quad \underline{J} = \dfrac{0.1\, e^{-1}\,\hat{r}}{r}$

$\underline{J}$ is independent of $\theta$ or $\phi$. Therefore at $r = 5$

$$I = 4\pi r^2 \left(\frac{0.1 e^{-1}}{r}\right) = 4\pi r \times 0.1 e^{-1} = \underline{2.311\,A}$$

(2) $\underline{J} = 5\hat{\underline{j}}\ A/m^2$ for $|z| < 2$




(a) The field from a surface current $\underline{K}$ is $\underline{H} = \frac{1}{2}\underline{K} \times \hat{n}$, where $\hat{n}$ is a surface normal.

For $|z| > 2$ the observation point is on one side or the other of the region of current flow. Therefore the effective

$\underline{K} = 20\,\hat{\underline{j}}\ A/m$

For $z > 2 \quad \underline{H} = 10\hat{\underline{j}} \times \hat{\underline{k}} = \underline{10\hat{\underline{i}}}$

For $z < 2 \quad \underline{H} = 10\hat{\underline{j}} \times (-\hat{\underline{k}}) = \underline{-10\hat{\underline{i}}}$

If observation point has $|z| < 2$, then it is inside the current region.

At point $(x, y, z)$ the surface current ~~above~~ the point is $5(2-z)$ A/m, the surface current below is $5(2+z)$ A/m

The overall field is $-\frac{5}{2}(2-z)\hat{\underline{i}} + \frac{5}{2}(2+z)\hat{\underline{i}} = 5z\hat{\underline{i}}$

(b) For $|z| < 2$ $\quad \underline{H} = 5z\hat{\underline{i}} \quad \underline{B} = 5\mu z\hat{i}$

$\underline{B} = \text{curl}\,\underline{A} \Rightarrow B_{x\hat{i}} = (\text{curl}\,\underline{A})_{x\hat{i}}$

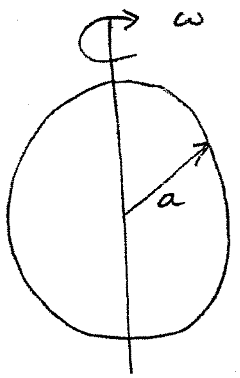
Therefore $\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{i} = 5\mu z\hat{i}$

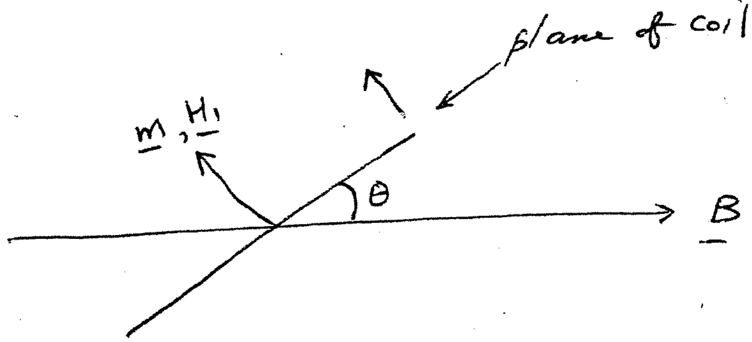
Because $\underline{J}$ is in the $y$ direction $\underline{A}$ is only in $y$ direction, and only varies with $z$

$$\frac{\partial A_y}{\partial z} = -5\mu z$$

$$A_y = -\frac{5\mu z^2}{2}$$

(3)




Flux through coil when it makes an angle $\theta$ with $\underline{B}$ is

$$\Phi = \underbrace{\pi a^2}_{\text{area}} \underset{\uparrow \text{ number of turns}}{N} B \sin\theta$$

$$\frac{\partial \Phi}{\partial t} = -\pi a^2 N B \cos\theta \frac{d\theta}{dt} = -\pi a^2 N B \omega \cos\theta$$

Now $\dfrac{\partial \Phi}{\partial t} = -\oint \underline{E} \cdot d\underline{L} = V$ potential difference across coil

Therefore $V = \pi a^2 N B \omega \cos\theta$

Current in coil is $I = \dfrac{V}{R} = \dfrac{\pi a^2 N B \omega \cos\theta}{R}$

Ohmic heating is $VI = \dfrac{N^2 \pi^2 a^4 B^2 \omega^2 \cos^2\theta}{R}$

$$= \frac{N^2 \pi^2 a^4 B^2 \omega^2}{2R}(1 + \cos 2\theta)$$

Average ohmic heating is $\dfrac{N^2 \pi^2 a^4 B^2 \omega^2}{2R}$

The current in the coil produces a magnetic dipole $\underline{m}$ that is oriented as shown - according to Lenz's law it acts to oppose the change in external flux

$$\underline{m} = \mu I N \underset{\substack{\uparrow \\ \text{area of} \\ \text{coil}}}{\underline{A}} = \mu \pi a^2 N \underline{I}$$

The torque that acts is $\underline{\Gamma} = \underline{m} \times \underline{H} = \underline{m} \times \dfrac{\underline{B}}{\mu}$

Therefore $\Gamma = \mu \pi a^2 N . \dfrac{\pi a^2 N B \omega \cos\theta}{R} \dfrac{B}{\mu} \sin(90 - \theta)$

$$= \frac{\pi^2 a^4 N^2 B^2 \omega}{R} \cos^2\theta$$

The average work done against this torque in one revolution is $2\pi \bar{\Gamma}$

$$\bar{\Gamma} = \frac{\pi^2 a^4 N^2 B^2 \omega}{2R}$$

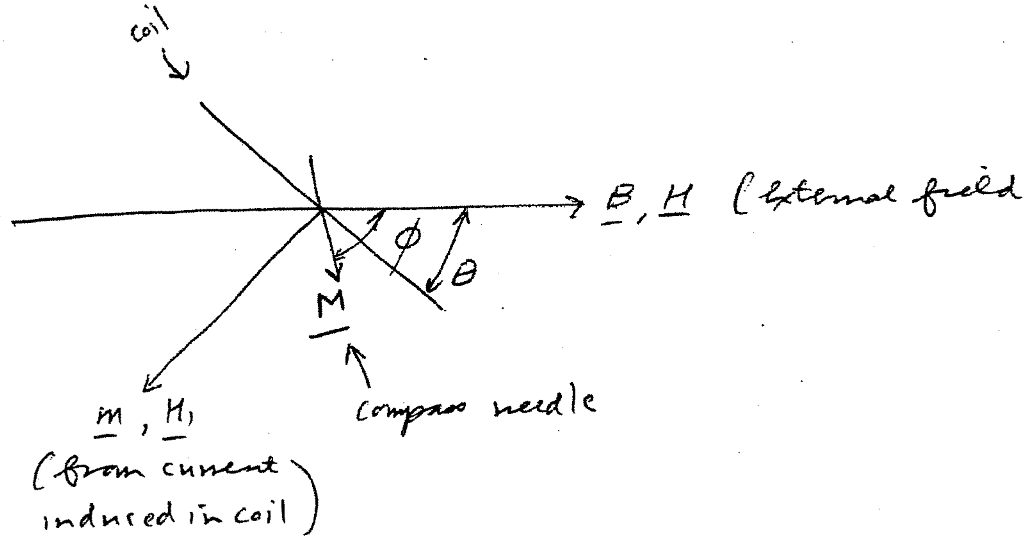
Work done per second is $2\pi \bar{\Gamma} . \underbrace{\dfrac{\omega}{2\pi}}_{\substack{\text{No of revolutions} \\ \text{per second}}}$

$$\text{Power} = \frac{\pi^2 a^4 N^2 B^2 \omega^2}{2R} = \text{ohmic heating}$$

For $N = 100$, $a = 50\,\text{mm}$, $B = 0.2\text{T}$, $\omega = 100$ rad/s, $R = 1$

power = ohmic heating = $\underline{2467 \text{ Watts}}$

(4)




Two magnetic fields act on the small magnetic needle — the external field $\underline{H}$ and the field that results from the current induced in the coil, $\underline{H_1}$

$$H_1 = \frac{NI}{2a} = \frac{\pi a N^2 B w \cos\theta}{2R}$$ (field at center of circular coil)

$$B_1 = \mu H_1 = \frac{\mu \pi a N^2 w B \cos\theta}{2R} = k B \cos\theta$$

$$\left( k = \frac{\mu \pi a N^2 w}{2R} \right)$$

Two torques act on the magnetic needle.

In equilibrium

$$\underline{M} \times H_1 = \underline{M} \times \underline{H}$$

$$\underline{M}\,\frac{\underline{B_1}}{\mu} \sin\left(90 - (\phi - \theta)\right) = \frac{\underline{M}\underline{B}}{\mu} \sin\phi$$

Therefore

$$k \cos\theta \left(\cos\phi \cos\theta + \sin\phi \sin\theta\right) = \sin\phi$$

$$\overline{k\cos\phi\,(\cos^2\theta) + k\sin\phi\,\sin\theta\cos\theta} = \sin\phi$$

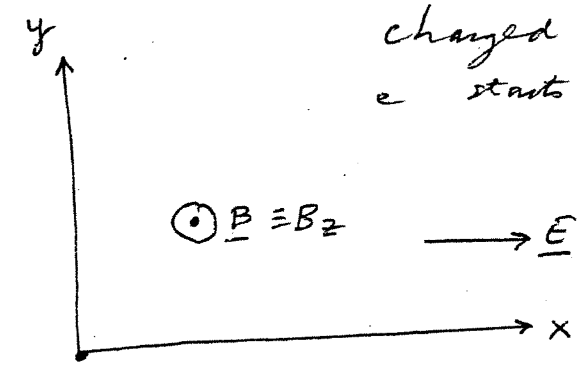
gives $\quad \dfrac{k\cos\phi}{2} = \sin\phi$

$$\cot\phi = \frac{2}{k}\ , \quad \tan\phi = \frac{k}{2} = \frac{\pi N^2 \mu \omega a}{4R}$$

with $N = 100$, $\omega = 100$ rad/s, $R = 1$, $a = 50$ mm, $\mu = 4\pi \times 10^{-7}$

$$\underline{\phi = 2.83^\circ}$$

(5)




charged particle with charge $e$ starts from origin

Force on particle is $\underline{F} = e(\underline{E} + \underline{v} \times \underline{B})$

$$\underline{F} = e(E_x \hat{\underline{i}} + \underline{v} \times B_z) = e(E_x \hat{\underline{i}} + v_x \hat{\underline{i}} \times B\hat{\underline{k}} + v_y \hat{\underline{j}} \times B \hat{\underline{k}})$$

$$\underline{F} = e(E_x \hat{i} - V_x B \hat{\underline{j}} + v_y B \hat{i})$$

$$F_x = e(E_x + V_y B) = e(E_x + \frac{dy}{dt} B)$$

$$F_y = -eV_x B = -eB\frac{dx}{dt} \qquad F_z = 0$$

If particle mass is $m$ then we have

$$m\frac{d^2x}{dt^2} = e(E_x + \frac{dy}{dt}B) \qquad m\frac{dV_x}{dt} = e(E_x + V_y B) \quad (1)$$

$$m\frac{d^2y}{dt^2} = -eB\frac{dx}{dt} \qquad m\frac{dv_y}{dt} = -eBV_x \quad (2)$$

From (1) $$m\frac{d^2V_x}{dt^2} = eB\frac{dv_y}{dt} = -\frac{(eB)^2}{m}V_x$$

Therefore $$\frac{d^2V_x}{dt^2} = -\left(\frac{eB}{m}\right)^2 V_x = -\omega^2 V_x \qquad \omega = \frac{eB}{m}$$

Therefore $V_x = A\sin\omega t + B\cos\omega t$

At $t=0$, $V_x = 0$ therefore $B = 0$

$V_x = A\sin\omega t \Rightarrow \frac{dx}{dt} = A\sin\omega t \qquad x = -A\cos\omega t + \text{Constant}$

At $t=0, x=0$, therefore $x = A(1-\cos\omega t)$

$$\frac{d^2x}{dt^2} = +A\omega^2\cos\omega t \qquad \left(\frac{d^2x}{dt^2}\right)_{t=0} = +A\omega^2 = \frac{eE}{m}$$

Therefore $A = +\frac{eE}{m\omega^2}$ $\qquad x = \frac{eE}{m\omega^2}\left[1-\cos\omega t\right) = \underline{\frac{E}{\omega B}(1-\cos\omega t)}$

Therefore $\frac{m\,d^2y}{dt^2} = -\frac{eE}{\omega}(\omega\sin\omega t)$

$$\frac{d^2y}{dt^2} = -\frac{eE}{m}\sin\omega t \qquad \frac{dy}{dt} = +\frac{eE}{m\omega}\cos\omega t + D$$

At $t=0$ $\frac{dy}{dt} = 0$ Therefore $D = -\frac{eE}{m\omega}$

$$\frac{dy}{dt} = -\frac{eE}{m\omega}(1-\cos\omega t) \qquad y = -\frac{eE}{m\omega}\left(t - \frac{\sin\omega t}{\omega}\right) + E$$

At $t=0, y=0$ therefore $E=0$

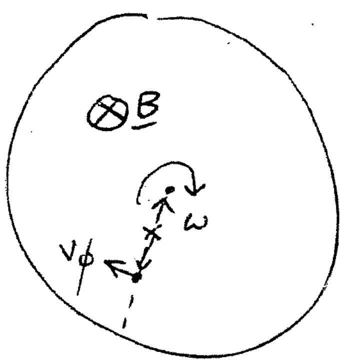
$$y = \frac{eE}{m\omega^2}(\omega t - \sin\omega t) = \frac{-E}{\omega B}(\omega t - \sin\omega t)$$

(6) From (5) $x_{max} = \frac{2E}{\omega B}$

$\omega = \frac{eB}{m}$ $\quad x_{max} = \frac{2mE}{eB^2}$

Therefore charge will not reach opposite capacitor plate if $d > \frac{2mE}{eB^2}$

(7)




$\omega = 2\pi f$

Along a radius of the rotating disc the motional electric field is : $(\underline{v} \times \underline{B}) = \mathcal{E}_m$

At radius $x$ $\quad v_\phi = \omega x$

$\mathcal{E}_m = B\omega x \, \hat{\underline{r}}$

$V = -\int \underline{\mathcal{E}} \cdot d\ell = \int B\omega x \, dx = \frac{B\omega a^2}{2}$

$= \frac{B \, 2\pi f a^2}{2}$

$\underline{V = f B \pi a^2}$