**ENEE 381 Problem Set #4**

**10/14/04 due 10/26/04**

**Questions like (1) -(6) and (9) -(14) could easily be on the first examination**

(1) A length of loss-less transmission line is first short-circuited at one end and then open-circuited. The impedance measured at the other end in the first case is $Z_1$ and $Z_2$ in the second. Prove that $Z_1 Z_2 = Z_0^2$. This is a convenient way for measuring the characteristic impedance of an unknown line.

(2) A 50 ohm transmission line is terminated with an impedance of 20-j30. What is the magnitude and phase of the reflection coefficient?

(3) Repeat question (2) using the Smith chart

(4) A 75 ohm transmission line is terminated with a load of $150 + j50$ ohm. Compute $\rho$ in terms of both amplitude $|\rho|$ and $\phi$. What fraction of incident power is absorbed in the load?

(5) Use Smith chart. A line with $Z_0 = 100\Omega$ is terminated with an unknown load. The SWR is found to be 3. A current maximum is observed $0.1\lambda$ from the load. What are:

(a) the load?

(b) the reflection coefficient $\rho$, magnitude and angle?

(c) how would you match the line without changing the load at the end of the line?

(6) Use Smith chart. A transmission line of characteristic impedance 75 ohm is terminated with an impedance 50+j125 ohm. $0.1\lambda$ from the load a 150ohm shorted stub $0.2\lambda$ long is connected in shunt to the main line. What are:

(a) The reflection coefficient in magnitude and phase at this point?

(b) The standing wave ratio?

(c) Where is the nearest current minimum that is greater than $0.1\lambda$ from the load?

(d) Where is the nearest point greater than $0.1\lambda$ from the load where the line can be matched with an open 75 ohm stub?

(7) RWvD 5.2b

(8) RWvD 5.5b

(9) RWvD 5.7f Do with and without the Smith Chart

(10) RWvD 5.8a Use Smith Chart

(11) RWvD 5.8c Use Smith Chart

(12) RWvD 5.10a Use Smith Chart

(13) RWvD 5.10b Use Smith Chart

(14) RWvD 5.10g Use Smith Chart

# ENEE 381 Problem Set 4. SOLUTIONS

(1) The transformed impedance equation is

$$Z_i = Z_0 \cdot \frac{\left(Z_L \cdot \cos(kl) + j \cdot Z_0 \cdot \sin(kl)\right)}{Z_0 \cdot \cos(kl) + j \cdot Z_L \cdot \sin(kl)}$$

For a shorted line $Z_L$=0, so

$$Z_{short} = j \cdot Z_0 \cdot \tan(kl)$$

For an open line $Z_L$ is infinite, so

$$Z_{open} = -j \cdot Z_0 \cdot \cot(kl)$$

$$Z_{short} \cdot Z_{open} = Z_0^{\,2}$$ Q.E.D.

(2) $Z_0 := 50$ $\quad$ $j := i$

$Z_L := 20 - j \cdot 30$

$$\rho := \frac{Z_L - Z_0}{Z_L + Z_0}$$

$\rho = -0.207 - 0.517i$

$|\rho| = 0.557$ magnitude

$\dfrac{\arg(\rho)}{\deg} = -111.801$ phase angle in degrees

(3) On chart the normalized impedance is

$$\zeta_L := \frac{Z_L}{Z_0}$$

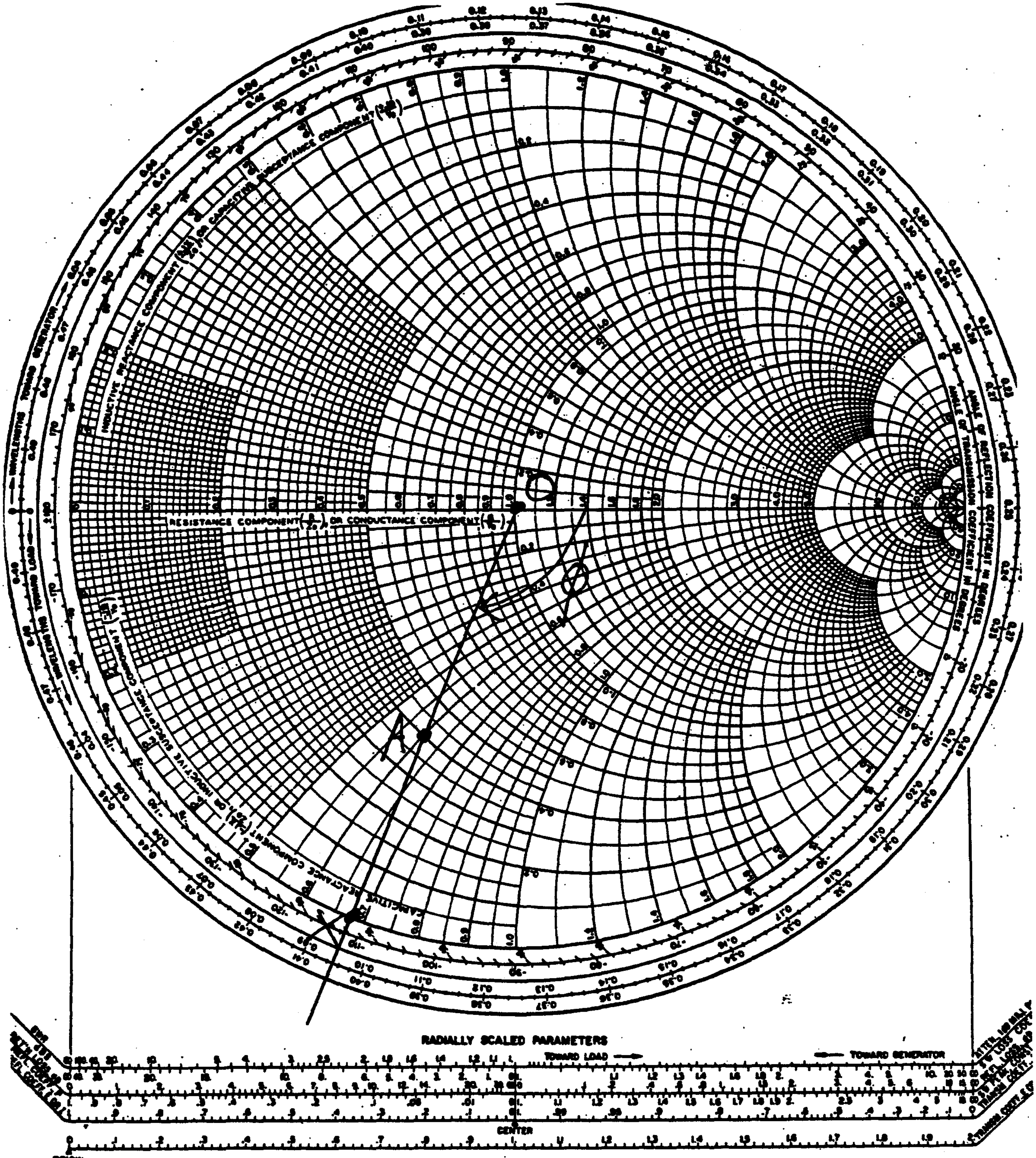
$\zeta_L = 0.4 - 0.6i$ Point A on chart

(4) $Z_0 := 75$ $\quad$ $Z_L := 150 + j \cdot 50$

$$\rho := \frac{Z_L - Z_0}{Z_L + Z_0}$$

$(|\rho|)^2 = 0.153$ Fraction of power reflected

$1 - (|\rho|)^2 = 0.847$ Fraction of power absorbed in load




(3) $z_L = 0.4 - j0.6$

$|\rho| = \dfrac{OA}{OX} = 0.56 \quad \phi = -111^\circ$

(5)

$Z_0 := 100$ $\quad S := 3$

On chart mark point on real axis for S=3, draw circel abour center of chart $I_{max}$ is on left hand side. Load is 0.1λ away at point 0.5-j0.6

$|\rho| = 0.49$ $\quad \phi = 108$ degrees

Sanity check

$\zeta_L := 0.5 - j\cdot 0.6$

$Z_L := Z_0 \cdot \zeta_L$

$$\rho := \frac{Z_L - Z_0}{Z_L + Z_0}$$

$|\rho| = 0.483$ $\quad \frac{\arg(\rho)}{deg} = -108.004$

Matching points are at B or C. You can match with series or parallel components but the distances from the load vary depending on whether you use the chart in impedance or admittance

(6)

$Z_0 := 75$ $\quad Z_L := 50 + j\cdot 125$

$\zeta_L := \frac{Z_L}{Z_0}$ Normalized load

$\zeta_L = 0.667 + 1.667i$ point A

$y_L := \frac{1}{\zeta_L}$

$y_L = 0.207 - 0.517i$ Normalized admittance of load, point B

Shorted stub 0.2λ long starts at point C and has its normalized admittance at point D

$y_{stub150} := -j\cdot 0.32$

Renormalized $\quad Y_{stub} := \frac{1}{150} \cdot y_{stub150}$ multiplying by admittance of 150 ohm line

Normalize again but this time to 75 ohm line

$$y_{stub75} := \frac{Y_{stub}}{\frac{1}{75}}$$

$y_{stub75} = -0.16i$

0.1λ from the load is point E, where

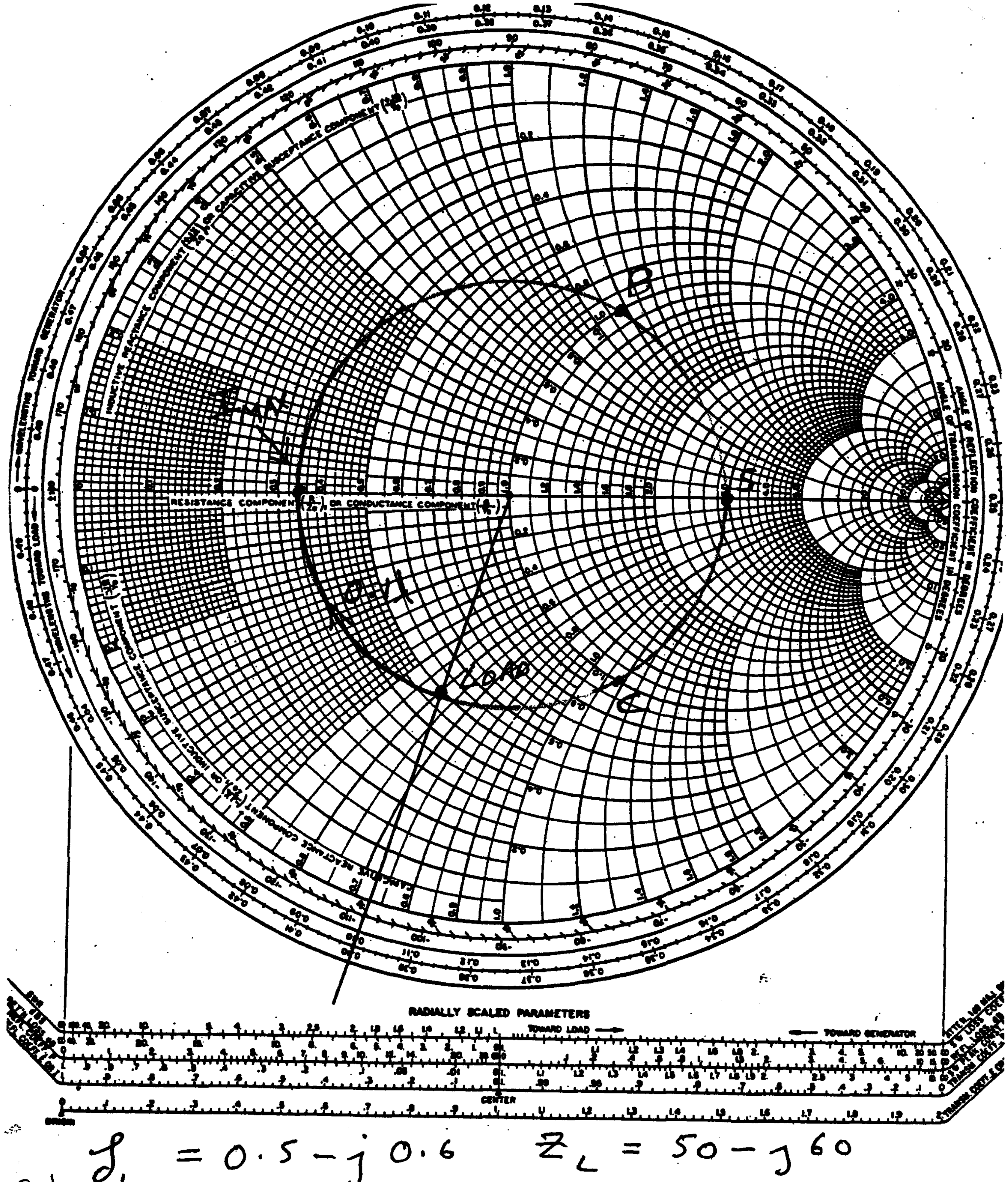
$y_i := 0.17 + j \cdot 0.14$

After adding stub

$y_{total} := y_i + y_{stub75}$

$y_{total} = 0.17 - 0.02i$ Point F

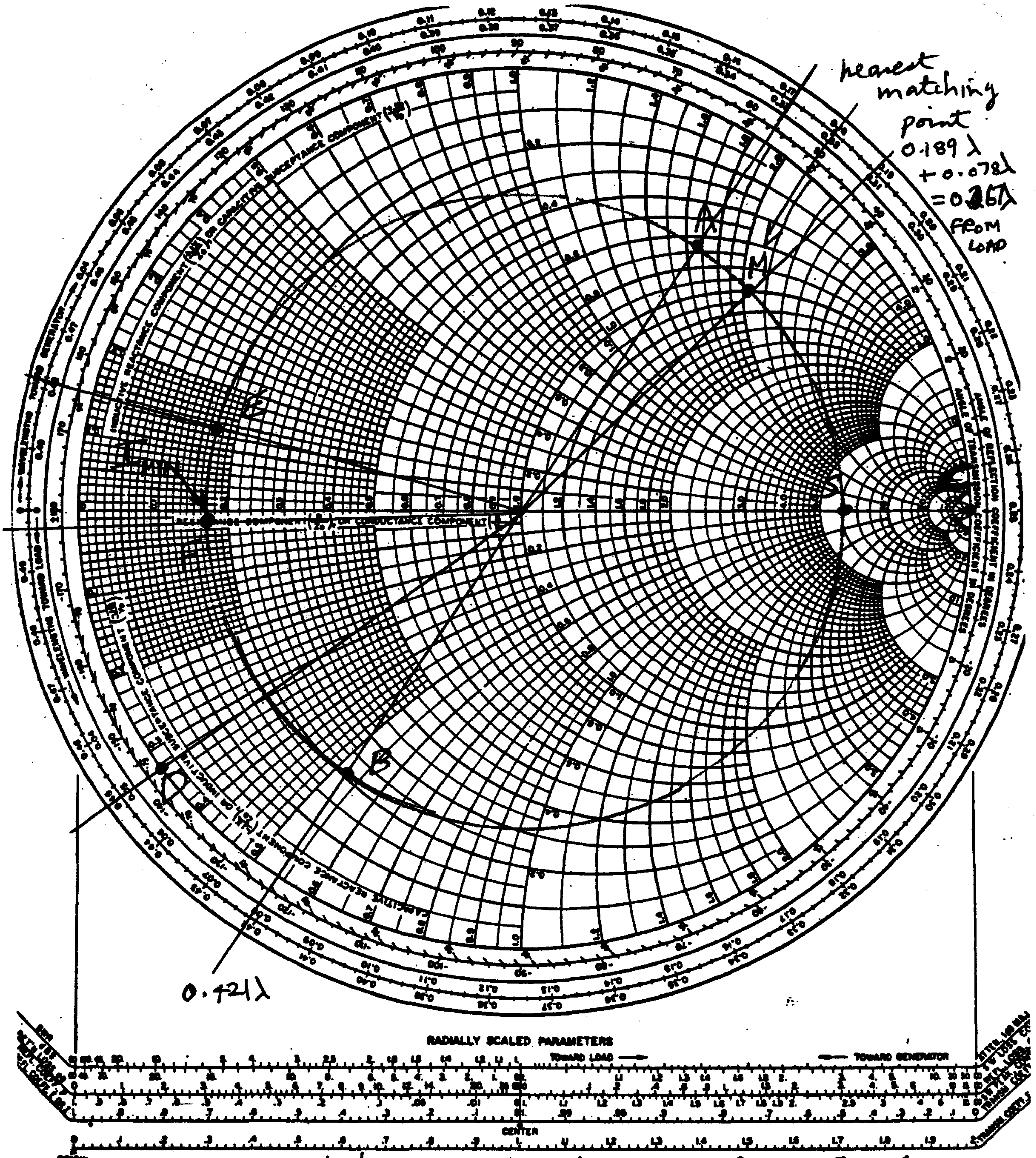
See chart for rest of problem




$z_L = 0.5 - j0.6$ $Z_L = 50 - j60$

(5) $|\rho| = 0.49$ $\phi = -108°$. MATCH AT B or C

EXAMPLE $B = 1 + j1.16$ ADD normalized $-j1.16$ IN SERIES




At F $|\rho| = 0.71$ $\phi = -178°$ $S = 6$

(6) S is also the location of $I_{MAX}$, $I_{MIN}$ is just past point F. The nearest $I_{MIN}$ > 0.1λ from load is 0.078λ + 0.5λ = 0.578λ from load

(7) <u>5.2b:</u> From eqs 1.9(3) and 2.5(3), for parallel plane line

(i) $C = \frac{\epsilon w}{a}$ F/m and $L = \frac{\mu a}{w}$ H/m, so $Z_0 = \frac{a}{w}\sqrt{\frac{\mu}{\epsilon}}$, $v = \frac{1}{\sqrt{\mu\epsilon}}$

(ii) For $w = 5\mu m$, $a = 1\mu m$, $\mu = \mu_0$, $\epsilon = 2.5\epsilon_0$

$$Z_0 = \frac{1}{5}\sqrt{\frac{4\pi\times10^{-7}}{2.5\times8.854\times10^{-12}}} = 47.7\Omega, \quad v = \frac{c}{\sqrt{2.5}} \approx 1.9\times10^{8} \text{ m/sec.}$$

(iii) For $a = .5\mu m$, other values as above,

$Z_0 =$ half of above $= 23.8\,\Omega$, $v =$ unchanged in this model.

(8) <u>5.5b:</u> Time to propagate 200 m,

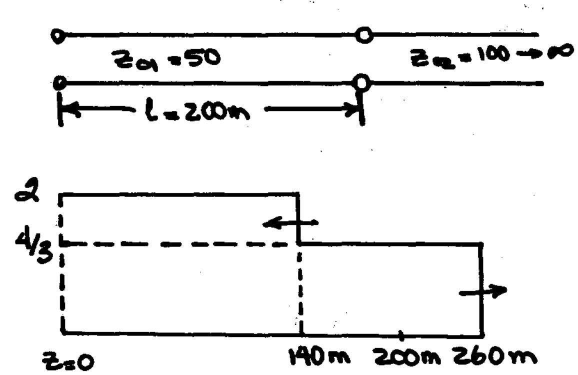
$$t_1 = \frac{200}{2\times10^{8}} = 10^{-6} \text{ sec} = 1\,\mu\text{sec}.$$

So reflection occurs after $1\mu$sec

$$\frac{V_-}{V_+} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}} = \frac{50}{150} = 1/3$$

but $\frac{I_-}{I_+} = -1/3$, $I_+ = \frac{V_+}{Z_{01}} = \frac{100}{50} = 2A$

Note current continuous at $z = 200$ after reflection. So plot at $t = 1.3\mu$sec to right




$W_{T+} = V_+ I_+ = 100\times2 = 200W$

$W_{T-} = V_- I_- = \frac{100}{3}\times 2/3 = \frac{200}{9}$ W

$(W_T)_2 = V_2 I_2 = \frac{200}{150}\times100\times4/3 = \frac{1600}{9}$ W

but $200 - 200/9 = \frac{1600}{9}$

so there is a power balance.

(9) <u>5.7f:</u> $Y_L = .05 + j2\pi\times10^{10}\times10^{-12} = .05 + j.0628$ S, $Y_0 = .05$ S,

$$\rho = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Y_0 - Y_L}{Y_0 + Y_L} = \frac{-j6.28}{10 + j6.28} = -.30 - j.47 = .56\angle 237.4^\circ$$

$$Y_i = Y_0\left[\frac{Y_L\cos\beta l + jY_0\sin\beta l}{Y_0\cos\beta l + jY_L\sin\beta l}\right] = .05\left[\frac{(5 + j6.28)\sqrt{2}/2 + j5\times\sqrt{2}/2}{5\times\sqrt{2}/2 + j(5 + 6.28)\times\sqrt{2}/2}\right]$$

$$Y_i = (-9.42 + j7.44)10^{-2} = .12\angle 141.7^\circ$$

(10) <u>5.8a:</u> $$\rho = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(100 + j100) - 50}{(100 + j100) + 50} = \frac{7 + 4j}{13} = .62\angle 29.7^\circ$$

$$S = \frac{1 + |\rho|}{1 - |\rho|} = \frac{1 + .62}{1 - .62} = 4.27.$$

(11) 5.8c: This problem is readily done on the Smith chart, but numerical solution given here:

$|\rho| = \frac{s-1}{s+1} = \frac{1.5-1}{1.5+1} = .2$

$\beta z_{max} = \beta z_{min} + \frac{\pi}{2} = -\frac{2\pi}{\lambda}(.1\lambda) + \frac{\pi}{2} = .3\pi$

so $\varphi = -2(\beta z)_{max} = -.6\pi = -108°$, so $\rho = .2 \angle -108°$

$Z_i = Z_0 \left[\frac{1+\rho e^{-2j\beta l}}{1-\rho e^{-2j\beta l}}\right] = Z_0 \frac{1+.2e^{-j(.6+.8)\pi}}{1-.2e^{-j(.6+.8)\pi}}$

$Z_i = Z_0 \left[\frac{1+.2e^{j.6\pi}}{1-.2e^{j.6\pi}}\right] = [.827 + j.328] Z_0$

(12) 5.10a: $\frac{Z_L}{Z_0} = \frac{75-j69}{50} = 1.50 - j1.38$ point A; Smith chart on next page.

$\frac{l}{\lambda} = \frac{35 \times 50 \times 10^6}{3 \times 10^8} = .583$

we subtract a half wave and add .083 to .3145 getting .3975.

Read $\frac{Z_i}{Z_0} = .46 - j.65$

$Z_i = (23 - j32.5)\,\Omega$

SWR - read from C:

$S = 3.3$

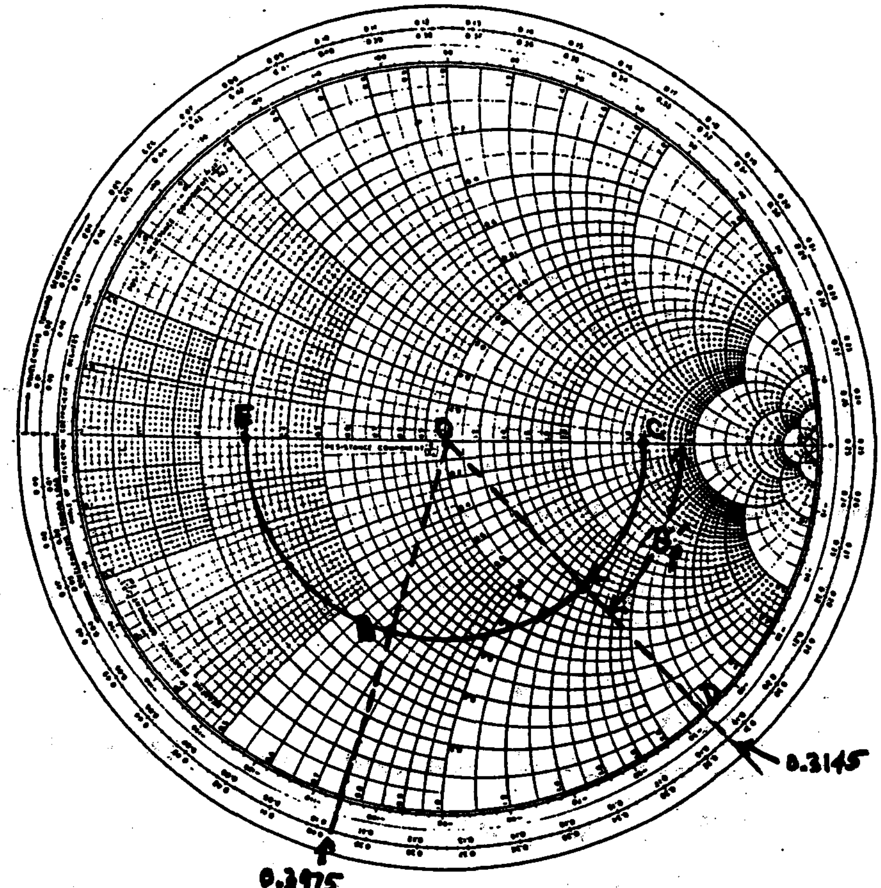
$|\rho| = \frac{0A}{0D} = \frac{2.9}{5.3} = .55$

check $|\rho| = \frac{S-1}{S+1} = \frac{2.3}{4.3} = .54$ (fair)

$\varphi_\rho = -\left(\frac{.3145 - .25}{.5}\right) \times 360° = -46.4°$ (continue.)

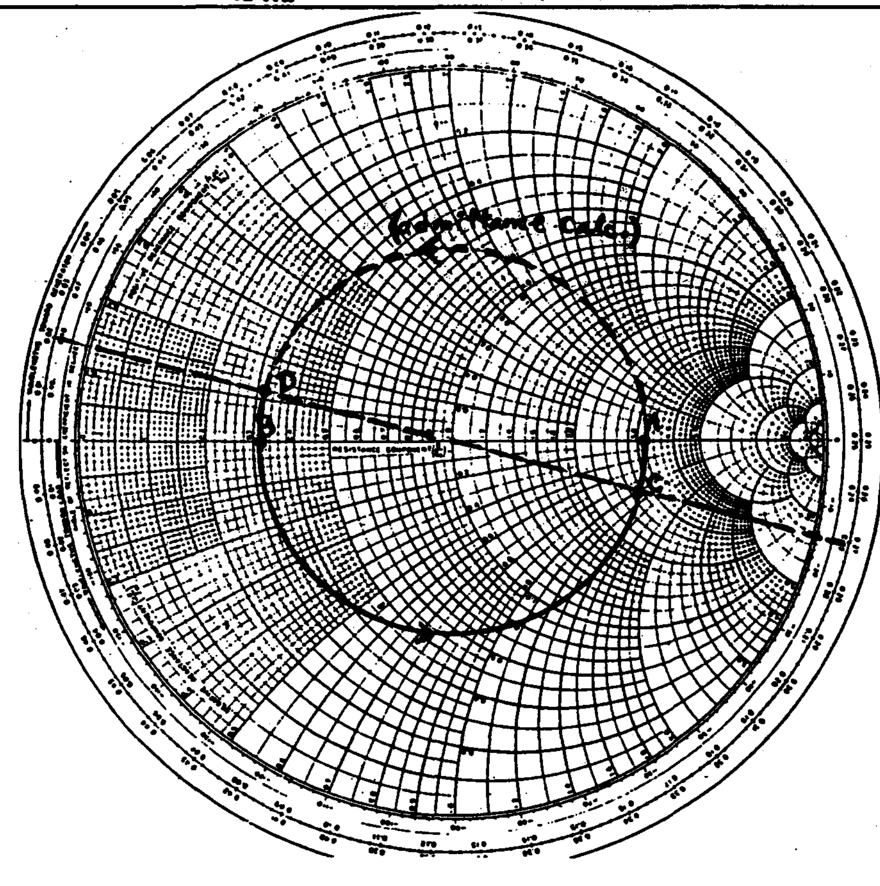
(12) continued

Position of voltage minimum is at E, (.5 − .3145)λ From load or 1.11 meters in front of load. (check from equations gives slightly different numbers because of inaccuracies in reading chart.



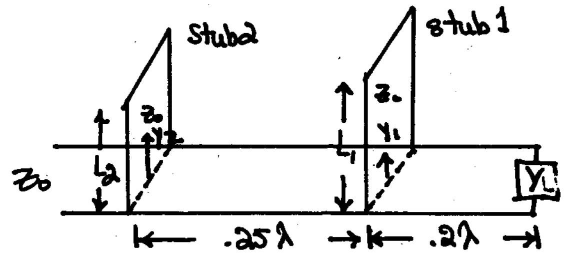

5.10b: Enter S = 3.2 at A voltage minimum, at same radius, pt. B Then move .23 λ from B toward load to get load impedance, pt. C Read $\frac{z_L}{z_0} = 2.7 - j1.0$ or $z_L = 70(2.7 - j1.0) = 189 - j70$

(13)



(14)

5.10g: Since shorted stubs are in parallel with main line, we use admittances.




$\frac{Y_L}{Y_0} = \frac{50}{20+j30} = .77 - j1.15$ (point A of chart)

Moving .2λ toward generator we reach point B, $.30 + j.27$, the shorted stub can add susceptance only. We want to add enough so that when transformed λ/4 we end on the circle $1 + jb$. Then stub 2 can be adjusted to cancel b and provide a match. It is thus helpful to transform the circle K (locus of $1+jb$) λ/4 to give circle K′, the transformed locus. Then stub 1 should add enough susceptance to go from B to C on K′, addition of $Y_1 = j.20$ gives C, $.30 + j.47$. Moving λ/4 toward load then gives D, $1.0 - j1.5$ and addition of $Y_2 = j1.5$ gives match, O. Then length $L_1$ of stub 1, $-jY_{01} \cot \beta L_1 = jY_0(.20)$, $Y_0 = Y_{01} = 1/50$, so $\beta L_1 = 101.3° \Rightarrow L_1 = .281\lambda$. For stub 2, $-jY_{02} \cot \beta L_2 = jY_0(1.5)$, $Y_0 = Y_{02} = 1/50 \Rightarrow \beta L_2 = 146.3 \Rightarrow L_2 = .406\lambda$. (alternatively could have added negative susceptance to get to point C′ – then $L_1$ & $L_2$ each shorter than above by λ/4.)