ENEE 381 Problem Set #6, 2004. SOLUTIONS

(1) $a := 10 \cdot 10^{-3}$ conductor spacing

$c_0 := 2.998 \cdot 10^{8}$ velocity of light in a vacuum

The formula for the cutoff frequency is

$\nu_c = \frac{m \cdot c}{2 \cdot a}$ c is velocity of light in material between planes

The TE_m and TM_m have the same cutoff frequency, which is determined by the integer m

In air, for m=1

$$\nu_c := \frac{c_0}{2 \cdot a}$$

$\nu_c = 1.499 \times 10^{10}$ 15GHz

For m=2

$$\nu_c := \frac{2 \cdot c_0}{2 \cdot a}$$

$\nu_c = 2.998 \times 10^{10}$ 30GHz

For m=3

$$\nu_c := \frac{3 \cdot c_0}{2 \cdot a}$$

$\nu_c = 4.497 \times 10^{10}$ 50GHz

(2) $\nu := 10 \cdot 10^{9}$ $a := 50 \cdot 10^{-3}$ $\varepsilon_r := 2.25$

$c := \frac{c_0}{\sqrt{\varepsilon_r}}$ $\omega := 2 \cdot \pi \cdot \nu$

For the TEM mode

$\beta := \frac{\omega}{c}$ $\beta = 314.369$

$v_p := c$

$v_g := c$

$Z_z := \frac{376.7}{\sqrt{\varepsilon_r}}$ $Z_z = 251.133$

For TM_1 mode

$$K := \frac{\pi}{a}$$

$$\omega_c := \frac{\pi c}{a}$$

$$\nu_c := \frac{\omega_c}{2\cdot\pi} \qquad \nu_c = 1.999 \times 10^9$$

$$ZTM := Z_Z\cdot\sqrt{1 - \left(\frac{\nu_c}{\nu}\right)^2} \qquad ZTM = 246.066$$

$$\beta := \frac{\omega}{c}\cdot\sqrt{1 - \left(\frac{\nu_c}{\nu}\right)^2} \qquad \beta = 308.026$$

$$v_p := \frac{\omega}{\beta} \qquad v_p = 2.04 \times 10^8$$

$$v_g := c\cdot\sqrt{1 - \left(\frac{\nu_c}{\nu}\right)^2} \qquad v_g = 1.958 \times 10^8$$

$$\lambda_g := 2\cdot\frac{\pi}{\beta} \qquad \lambda_g = 0.02$$

For TM_2 mode

$$K := \frac{2\cdot\pi}{a}$$

$$\omega_c := \frac{2\cdot\pi c}{a}$$

$$\nu_c := \frac{\omega_c}{2\cdot\pi} \qquad \nu_c = 3.997 \times 10^9$$

$$ZTM := Z_Z\cdot\sqrt{1 - \left(\frac{\nu_c}{\nu}\right)^2} \qquad ZTM = 230.197$$

$$\beta := \frac{\omega}{c}\cdot\sqrt{1 - \left(\frac{\nu_c}{\nu}\right)^2} \qquad \beta = 288.16$$

$$v_p := \frac{\omega}{\beta} \qquad v_p = 2.18 \times 10^8$$

$$v_g := c \cdot \sqrt{1 - \left(\frac{\nu_c}{\nu}\right)^2}$$

$$v_g = 1.832 \times 10^8$$

$$\lambda_g := 2 \cdot \frac{\pi}{\beta}$$

$$\lambda_g = 0.022$$

(3) The formula for the cutoff frequency for a $TE_{m,n}$ or $TM_{m,n}$ mode is

$$\nu_{c_{m,n}} = \frac{c}{2 \cdot \pi} \cdot \sqrt{\left(\frac{m \cdot \pi}{a}\right)^2 + \left(\frac{n \cdot \pi}{b}\right)^2}$$

$a := 50 \cdot 10^{-3}$ $b := 20 \cdot 10^{-3}$

$\nu := 10 \cdot 10^9$

For an air-filled guide $c := c_0$

$m := 1, 2 .. 3$ $n := 0, 1 .. 3$ different mode subscripts

$$\nu_{c_{m,n}} := \frac{c}{2 \cdot \pi} \cdot \sqrt{\left(\frac{m \cdot \pi}{a}\right)^2 + \left(\frac{n \cdot \pi}{b}\right)^2}$$

$\nu_{c_{1,0}} = 2.998 \times 10^9$ $\nu_{c_{1,1}} = 8.072 \times 10^9$ $\nu_{c_{1,2}} = 1.529 \times 10^{10}$ $\nu_{c_{2,3}} = 2.327 \times 10^{10}$

$\nu_{c_{2,1}} = 9.598 \times 10^9$ $\nu_{c_{2,2}} = 1.614 \times 10^{10}$ $\nu_{c_{2,0}} = 5.996 \times 10^9$

With a dielectric with ε_r=4 $c := \frac{c_0}{2}$

$$\nu_{c_{m,n}} := \frac{c}{2 \cdot \pi} \cdot \sqrt{\left(\frac{m \cdot \pi}{a}\right)^2 + \left(\frac{n \cdot \pi}{b}\right)^2}$$

$\nu_{c_{1,0}} = 1.499 \times 10^9$ $\nu_{c_{1,1}} = 4.036 \times 10^9$ $\nu_{c_{1,2}} = 7.643 \times 10^9$ $\nu_{c_{2,3}} = 1.164 \times 10^{10}$

$\nu_{c_{2,1}} = 4.799 \times 10^9$ $\nu_{c_{2,2}} = 8.072 \times 10^9$ $\nu_{c_{2,0}} = 2.998 \times 10^9$

(4) For the TE_{10} mode the cutoff frequency is $\nu_c = \frac{c_0}{2 \cdot a}$

In air $\nu_c := \frac{c_0}{2 \cdot a}$ $a := 50 \cdot 10^{-3}$ $b := 20 \cdot 10^{-3}$ guide dimensions

$\nu_c = 2.998 \times 10^9$ $\nu := 1.1 \cdot \nu_c$ input frequency

$\nu = 3.298 \times 10^{9}$ $\lambda_0 := \dfrac{c_0}{\nu}$

Note that $\dfrac{c_0}{2 \cdot b} = 7.495 \times 10^{9}$ so mode only propagates in one orientation

The guide wavelength in the air-filled section is

$$\lambda_g := \frac{\lambda_0}{\sqrt{1 - \left(\frac{\lambda_0}{2 \cdot a}\right)^2}} \qquad \omega := 2 \cdot \pi \cdot \nu$$

$\lambda_g = 0.218$ $\lambda_0 = 0.091$

The TE mode impedances are, using Z_0=376.7 ohm $Z_0 := 376.7$

$$ZTEair := \frac{376.7}{\sqrt{1 - \left(\frac{\lambda_0}{2 \cdot a}\right)^2}}$$

$ZTEair = 904.229$ ohms

alternative formula

$$ZTEair := \frac{Z_0}{\sqrt{1 - \left(\frac{\nu_c}{\nu}\right)^2}}$$

$ZTEair = 904.229$

$$\beta := \frac{\omega}{c_0} \cdot \sqrt{1 - \left(\frac{\nu_c}{\nu}\right)^2}$$

$\beta = 28.793$

$$v_p := \frac{\omega}{\beta}$$

$v_p = 7.196 \times 10^{8}$

$$v_g := c_0 \cdot \sqrt{1 - \left(\frac{\nu_c}{\nu}\right)^2}$$

$v_g = 1.249 \times 10^{8}$

$$\lambda_g := 2 \cdot \frac{\pi}{\beta}$$

$\lambda_g = 0.218$

(5)

$a := 20 \cdot 10^{-3}$ $b := 10 \cdot 10^{-3}$ guide dimensions

For the TM_{11} mode the cutoff frequency is

$$\omega_c := c_0 \cdot \sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2}$$

$$\nu_c := \frac{\omega_c}{2 \cdot \pi}$$

$\nu_c = 1.676 \times 10^{10}$

$\nu_c = 1.676 \times 10^{10}$ $\nu := 1.2 \cdot \nu_c$ input frequency

$\nu = 2.011 \times 10^{10}$ $\lambda_0 := \dfrac{c_0}{\nu}$

Note that $\dfrac{c_0}{2 \cdot b} = 1.499 \times 10^{10}$ so mode only propagates in one orientation

The guide wavelength in air is

$$\lambda_g := \frac{\lambda_0}{\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}}$$

$\omega := 2 \cdot \pi \cdot \nu$

$\lambda_g = 0.027$ $\lambda_0 = 0.015$

The TE mode impedances are, using Z_0=376.7 ohm $Z_0 := 376.7$

$$ZTM := 376.7 \cdot \sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}$$

$ZTM = 208.229$ ohms

$$\beta := \frac{\omega}{c_0} \cdot \sqrt{1 - \left(\frac{\nu_c}{\nu}\right)^2}$$

$\beta = 232.987$

$v_p := \dfrac{\omega}{\beta}$ $v_p = 5.424 \times 10^8$

$$v_g := c_0 \cdot \sqrt{1 - \left(\frac{\nu_c}{\nu}\right)^2}$$

$v_g = 1.657 \times 10^8$

$\lambda_g := 2 \cdot \dfrac{\pi}{\beta}$

$\lambda_g = 0.027$

(6) For the TE_{10} mode the cutoff frequency in air is $\nu_c = \dfrac{c_0}{2 \cdot a}$

In air $\nu_c := \dfrac{c_0}{2 \cdot a}$ $a := 50 \cdot 10^{-3}$ $b := 20 \cdot 10^{-3}$ guide dimensior

$\nu_c = 2.998 \times 10^9$ $\nu := 1.1 \cdot \nu_c$ input frequency

$\nu = 3.298 \times 10^9$ $\lambda_0 := \dfrac{c_0}{\nu}$

In dielectric $c := \dfrac{c_0}{\sqrt{3}}$

$\nu_{cd} := \dfrac{c}{2 \cdot a}$ cutoff frequency in dielectric

$\nu_{cd} = 1.731 \times 10^9$

The TE mode impedances are, using Z_0=376.7 ohm $Z_0 := 376.7$

$$ZTE_{air} := \frac{Z_0}{\sqrt{1 - \left(\frac{\nu_c}{\nu}\right)^2}}$$

$$ZTEair = 904.229$$

$$ZTE_{diel} := \frac{Z_0}{\sqrt{3} \cdot \sqrt{1 - \left(\frac{\nu_{cd}}{\nu}\right)^2}}$$

$$ZTE_{diel} = 255.511$$

$$\rho := \frac{ZTE_{diel} - ZTE_{air}}{ZTE_{diel} + ZTE_{air}}$$

$$\rho = -0.559$$

$$|\rho| = 0.559 \qquad \frac{\arg(\rho)}{deg} = 180$$ reflection coefficient

$$S := \frac{1 + |\rho|}{1 - |\rho|}$$

$$S = 3.539$$ VSWR

(7)

$$a := 20 \cdot 10^{-3} \qquad b := 10 \cdot 10^{-3}$$

$$K := \frac{\pi \cdot c_0}{a}$$

$$\nu_c := \frac{c_0}{2 \cdot a} \qquad \nu_c = 7.495 \times 10^9$$

$$\nu := 10 \cdot 10^9$$

$$Z_0 := 376.7$$

Transverse fields are

$$E_y = E_0 \cdot \sin\left(\frac{\pi \cdot x}{a}\right)$$

$$H_x = \frac{-E_0}{Z_{TE}} \cdot \sin\left(\frac{\pi \cdot x}{a}\right)$$

$$Z_{TE} := \frac{Z_0}{\sqrt{1 - \left(\frac{\nu_c}{\nu}\right)^2}} \qquad Z_{TE} = 569.029$$

$$P = \int_0^b \int_0^a \frac{{E_0}^2}{2 \cdot Z_{TE}} \cdot \sin\left(\frac{\pi \cdot x}{a}\right)^2 dx\, dy \qquad P = \frac{1}{4} \cdot b \cdot a \cdot \frac{{E_0}^2}{Z_{TE}}$$

$$P := 1$$ Power in W

$$E_0 := \sqrt{4 \cdot \frac{P \cdot Z_{TE}}{b \cdot a}}$$

$$E_0 = 3.374 \times 10^3$$ V/m

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