ENEE 381 First Examination Solutions

(1) $\mathbf{P} := 10^{25}$ Power of point source in W

$\mathbf{R} := 10^{12}$

$\mathbf{S_{avg}} := \dfrac{\mathbf{P}}{4\cdot\pi\cdot\mathbf{R}^2}$ Average Poynting vector calculated using surface area of sphere

$\mathbf{S_{avg}} = 0.796$ W/m^2

$\varepsilon_0 := 8.854\cdot 10^{-12}$

$\mu_0 := 4\cdot\pi\cdot 10^{-7}$

$\mathbf{Z_0} := \sqrt{\dfrac{\mu_0}{\varepsilon_0}}$ Characteristic impedance of free space

$\mathbf{Z_0} = 376.734$

$\mathbf{S_{avg}} = \dfrac{(|\mathbf{E}|)^2}{2\cdot\mathbf{Z_0}}$ $\mathbf{S_{avg}} = \mathbf{Z_0}\cdot\dfrac{(|\mathbf{H}|)^2}{2}$

$\mathbf{E} := \sqrt{2\cdot\mathbf{Z_0}\cdot\mathbf{S_{avg}}}$ $\mathbf{E} = 24.487$ E-field amplitude in V/m

$\mathbf{H} := \sqrt{\dfrac{2\cdot\mathbf{S_{avg}}}{\mathbf{Z_0}}}$ $\mathbf{H} = 0.065$ H-field amplitude in A/m

$\varepsilon_r := 10$ $\mu_r := 5$

$\mathbf{Z} := \sqrt{\dfrac{\mu_0\cdot\mu_r}{\varepsilon_0\cdot\varepsilon_r}}$ $\mathbf{Z} = 266.391$

$\mathbf{E} := \sqrt{2\cdot\mathbf{Z}\cdot\mathbf{S_{avg}}}$ $\mathbf{E} = 20.591$ E-field amplitude in V/m

$\mathbf{H} := \sqrt{\dfrac{2\cdot\mathbf{S_{avg}}}{\mathbf{Z}}}$ $\mathbf{H} = 0.077$ H-field amplitude in A/m

With a 45 degree phase shift

$\mathbf{H_{new}} := \mathbf{H}\cdot\mathbf{e}^{i\cdot 45\cdot\mathbf{deg}}$ $45\cdot\mathbf{deg} = 0.785$ radians

$\mathbf{S_{avg}} := 0.5\cdot\mathbf{Re}\left(\mathbf{E}\cdot\overline{\mathbf{H_{new}}}\right)$ $\overline{\mathbf{H_{new}}}$ is complex conjugate

$\mathbf{S_{avg}} = 0.563$

(2) $\sigma := 3 \cdot 10^{7}$ $\quad \mathbf{r} := 10^{-2}$

$\nu := 10 \cdot 10^{9}$ Frequency

$$\delta := \frac{1}{\sqrt{\pi \cdot \mu_0 \cdot \nu \cdot \sigma}}$$

$\delta = 9.189 \times 10^{-7}$ Skin depth in meters

$\mathbf{R_s} := \frac{1}{\sigma \cdot \delta}$ $\quad \mathbf{R_s} = 0.036$ surface resistance in ohms

Resistance of wire

$\mathbf{R} := \frac{\mathbf{R_s}}{2 \cdot \pi \cdot \mathbf{r}}$ $\quad \omega\mathbf{L} := \mathbf{R}$

$\mathbf{Z} := \mathbf{R} + i \cdot \omega\mathbf{L}$

$\mathbf{Z} = 0.577 + 0.577i$ Impedance of 1m of wire in ohms

The current in the wire is

$\mathbf{I} := \frac{1}{\mathbf{Z}}$

$\mathbf{I} = 0.866 - 0.866i$

$\mathbf{H} := \frac{\mathbf{I}}{2 \cdot \pi \cdot \mathbf{r}}$

The real current is

$\mathbf{I_{real}} := |\mathbf{I}|$

$\mathbf{I_{real}} = 1.225$

$\mathbf{H_{real}} := \frac{\mathbf{I_{real}}}{2 \cdot \pi \cdot \mathbf{r}}$ From Amperes Law

$\mathbf{H_{real}} = 19.492$

Note that ohmic dissipation in wire is

$\mathbf{P} := 0.5 \cdot \mathbf{Re}\left(1 \cdot \overline{\mathbf{I}}\right)$

$\mathbf{P} = 0.433$

Poynting vector flux into wire

$\mathbf{P_S} := 2 \cdot \pi \cdot \mathbf{r} \cdot 0.5 \cdot \mathbf{Re}\left(1 \cdot \overline{\mathbf{H}}\right)$

$\mathbf{P_S} = 0.433$ same as ohmic dissipation

(3) $Z_0 := 100$ Characteristic impedance

$R := 50$ $C := 10 \cdot 10^{-9}$

$\omega := 2 \cdot \pi \cdot 10^8$

$$Z_L := R + \frac{1}{i \cdot \omega \cdot C}$$

$Z_L = 50 - 0.159i$

$$\rho := \frac{Z_L - Z_0}{Z_L + Z_0} \qquad \tau := \frac{2 \cdot Z_L}{Z_L + Z_0}$$

$\rho = -0.333 - 1.415i \times 10^{-3}$

$|\rho| = 0.333$

$\frac{\arg(\rho)}{\deg} = -179.757$ Phase angle in degrees

$$S := \frac{1 + |\rho|}{1 - |\rho|}$$

$S = 2$

Power fraction into load is

$F := 1 - (|\rho|)^2$

$F = 0.889$

(4) The motional electric field is **Em=v x B.** This is the apparent electric field that comes from the Lorentz force

$\omega := \frac{3000}{60} \cdot 2 \cdot \pi$ angular velocity in radians per second

The tangential velocity at radius r is v(r)=ωr, in m/s

$$v(r) = \frac{3000}{50} \cdot 2 \cdot \pi \cdot r$$

$$Em(r) = v(r)\,B$$

$B := 1$

$R := 0.5$ radius of disk

$$V := \int_0^R \frac{3000}{60} \cdot 2 \cdot \pi \cdot B \cdot r \, dr$$

$V = 39.27$ Volts