ENEE 408E Optical System Design

Design Projects #1, 9/16/03

Due 9/25/03

(1) A thick lens with $n = 1.55$ has $R_1 = 100$ mm, $R_2 =$ -140mm with $d =$ 30mm. Calculate the position of the principal planes and the paraxial focal length of the lens.

(2) For the lens in question (1) write a computer program using Matlab, Mathcad, or whatever, to carry out exact ray tracing through the lens in a 2-D model. Make the lens aperture (the full diameter over which rays can enter) 20mm. Show a family of rays entering the lens traveling parallel to the axis and leaving though a blurred focus. Confirm that the focal length for rays near the axis agrees with the paraxial focal length calculated in question (1)

(3) An optical system has

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 3 & -4 \\ -0.5 & 1 \end{pmatrix}$$

Calculate:
(i) the focal length
(ii) the position of the principal planes
(iii) the output ray parameters if $r_1 = 0.002m$, $r_1' = 2^o$.
(iv) the input ray parameters if $r_2 = 0.003m$, $r_2' = 3^o$.

(4) A ray with parameters r_1, r_1' strikes a plane mirror. Determine a ray transfer matrix for the plane mirror that gives the ray parameters after reflection. Use your matrix to find the reflected ray parameters when r_1=0.002m, $r_1' = 3^o$

(5) for question (3) part (iii) calculate the parameters of the ray when it returns to the input plane if a plane mirror is placed perpendicular to the axis and a distance of 10mm after the original output plane. Draw a diagram to illustrate your answer

(6) A spherical surface of radius of curvature 200mm has air to the left and glass (n=1.5) to the right. A ray of light enters traveling from left to right. Where does it cross the axis? A ray of light traveling parallel to the axis enters traveling from right to left. Where does it cross the axis?

(7) Where are the principal planes in question (6)?

(8) A point source of light is placed 200mm from a concave mirror of radius of curvature 100mm. Calculate the blurring (spherical aberration) of the image at the paraxial image plane. The blurring is the diameter of the illuminated region at the paraxial image plane. Do an exact calculation using the precise geometry and Snell's law. Do not use the small angle approximation for the calculation of the blurring. The open aperture of the concave mirror is 75mm. This is the diameter of the mirror perpendicular to its axis.

ENEE 4O8E 2003
Design Problems No. 1

Done using Mathcad

(1) Declare variables:

$n := 1.55$

$R1 := 100 \cdot 10^{-3}$

units - meters

$R2 := -140 \cdot 10^{-3}$

$d := 30 \cdot 10^{-3}$

$D1 := \frac{n-1}{R1}$ powers of surfaces $D1 = 5.5$

$D2 := \frac{1-n}{R2}$ $D2 = 3.92857$

$h1 := \frac{d}{n \cdot \left[1 + \left(\frac{D1}{D2}\right) - \left(d \cdot \frac{D1}{n}\right)\right]}$ $h1 = 8.43882 \times 10^{-3}$ h1=8.44mm

$h2 := \frac{d}{n \cdot \left[1 + \left(\frac{D2}{D1}\right) - \left(d \cdot \frac{D2}{n}\right)\right]}$ $h2 = 0.01181$ h2=11.8mm

$x := \left(D1 + D2 - d \cdot D1 \cdot \frac{D2}{n}\right)$ $f := \frac{1}{x}$ $f = 0.11098$ focal length 111mm

(2) Start first surface at x=0, so its center is at x=0.1 $n := 1.55$
Second surface is clearly centered at x=-0.110

$x_1 := 0.1$ $x_2 := -0.110$ $R_1 := 0.1$ $R_2 := -0.14$

Equation of first surface is

$\left(x - x_1\right)^2 + y^2 = R_2^{\,2}$

Equation of second surface is

$\left(x - x_2\right)^2 + y^2 = R_2^{\,2}$

Input ray is

$y = r_1$ Parallel to axis

$i := 1, 2 .. 50$

$r_{1_i} := \frac{i}{2000}$ rays at different distances from axis

Find where input ray intersects first circle

$\left(x - x_1\right)^2 + y^2 = R_1^{\,2}$

$$\begin{bmatrix} x_1 + \left(R_1^{\,2} - y^2\right)^{\left(\frac{1}{2}\right)} \\ x_1 - \left(R_1^{\,2} - y^2\right)^{\left(\frac{1}{2}\right)} \end{bmatrix}$$

solving for x, choose lower solution
since this is to left of center of circle

$$x_i := x_1 - \left[R_1^2 - \left(r1_i\right)^2\right]^{\left(\frac{1}{2}\right)}$$ intersection points

Vector from intersection point to center of cicle is

$$a_i := \begin{pmatrix} x_1 - x_i \\ -r1_i \end{pmatrix}$$

Input ray vector is $b_i := \begin{pmatrix} r1_i \\ 0 \end{pmatrix}$

dot product is $ab_i := a_i \cdot b_i$

$$\theta 1_i := \operatorname{acos}\left(\frac{ab_i}{|a_i|\,|b_i|}\right)$$

angles of incidence is

angles of refraction $\theta 1_i := \operatorname{asin}\left(\frac{\sin\left(\theta 1_i\right)}{n}\right)$

Angle that surface normal makes with axis is

$$\phi_i := \operatorname{asin}\left(\frac{r1_i}{R_1}\right)$$

angle of refracted ray with respect to axis is

$$\theta 2_i := \phi_i - \theta 1_i$$

equation of refracted ray is $y = m \cdot x + b$

$$m_i := -\tan\left(\theta 2_i\right)$$

$$b_i := r1_i - m_i \cdot x_i$$

Find intersection of refracted ray and second circle

$$\left(x - x_2\right)^2 + (m \cdot x + b)^2 = R_2^2$$

$$\begin{bmatrix} \frac{1}{\left[2\cdot\left(m^2+1\right)\right]}\cdot\left[-2\cdot m\cdot b + 2\cdot x_2 + 2\cdot\left(-2\cdot m\cdot b\cdot x_2 - m^2\cdot x_2^2 + m^2\cdot R_2^2 - b^2 + R_2^2\right)^{\left(\frac{1}{2}\right)}\right] \\ \frac{1}{\left[2\cdot\left(m^2+1\right)\right]}\cdot\left[-2\cdot m\cdot b + 2\cdot x_2 - 2\cdot\left(-2\cdot m\cdot b\cdot x_2 - m^2\cdot x_2^2 + m^2\cdot R_2^2 - b^2 + R_2^2\right)^{\left(\frac{1}{2}\right)}\right] \end{bmatrix}$$

Solutions for intersection point, choose greater value for correct intersection point

$$x3_i := \frac{1}{\left[2\cdot\left[\left(m_i\right)^2+1\right]\right]}\cdot\left[-2\cdot m_i\cdot b_i + x_2\cdot 2 + 2\cdot\left[-2\cdot m_i\cdot b_i\cdot x_2 - \left(m_i\right)^2\cdot\left(x_2\right)^2 + \left(m_i\right)^2\cdot R_2^2 - \left(b_i\right)^2 + R_2^2\right]^{\left(\frac{1}{2}\right)}\right]$$

$$y3_i := m_i \cdot x3_i + b_i$$

Vector from intersection point to center of sphere is $a2_i := \begin{pmatrix} x3_i - x_2 \\ y3_i \end{pmatrix}$

Vector of ray incident on second surface is $b2_i := \begin{pmatrix} x3_i - x_i \\ y3_i - r1_i \end{pmatrix}$

dot product is $ab2_i := a2_i \cdot b2_i$

Angles of incidence $\theta 2_i := \operatorname{acos}\left(\frac{ab2_i}{|a2_i| \cdot |b2_i|}\right)$

Angles of refraction are $\theta 3_i := \operatorname{asin}\left(n \cdot \sin\left(\theta 2_i\right)\right)$

Angles that surface normals make with axis

$$\phi 2_i := \operatorname{atan}\left(\frac{y3_i}{-R_2}\right)$$

Angles of refracted rays are

$$\theta 4_i := \theta 3_i - \phi 2_i$$

Equation of refracted ray is $y = m_2 \cdot x + b_2$

$$m2_i := -\tan\left(\theta 4_i\right)$$

$$b2_i := y3_i - m2_i \cdot x3_i$$

This ray intersects the x axis at $x4_i := \frac{-b2_i}{m2_i}$

Draw rays through lens

$j := 1, 2 .. 4$ Dummy variable for plotting the straight line segments

$X_{1,i} := -0.01$ $Y_{1,i} := r1_i$ $X_{2,i} := x_i$ $Y_{2,i} := r1_i$

$X_{3,i} := x3_i$ $Y_{3,i} := y3_i$ $X_{4,i} := x4_i$ $Y_{4,i} := 0$

Draw circles

$k := 1, 2 .. 50$ variable for different points on circle

$(x - x_1)^2 + y^2 = R_1^2$ $yy1_k := k \cdot \frac{33}{50000}$ $yy2_k := -k \cdot \frac{33}{50000}$

$$\begin{bmatrix} x_1 + \left(R_1^2 - y^2\right)^{\left(\frac{1}{2}\right)} \\ x_1 - \left(R_1^2 - y^2\right)^{\left(\frac{1}{2}\right)} \end{bmatrix}$$

solutions for x as a function of y, choose lower value

$$xx1_k := x_1 - \left[R_1^2 - \left(yy1_k\right)^2\right]^{\left(\frac{1}{2}\right)}$$

$$xx2_k := x_1 - \left[R_1^2 - \left(yy2_k\right)^2\right]^{\left(\frac{1}{2}\right)}$$

Draw second arc of circle

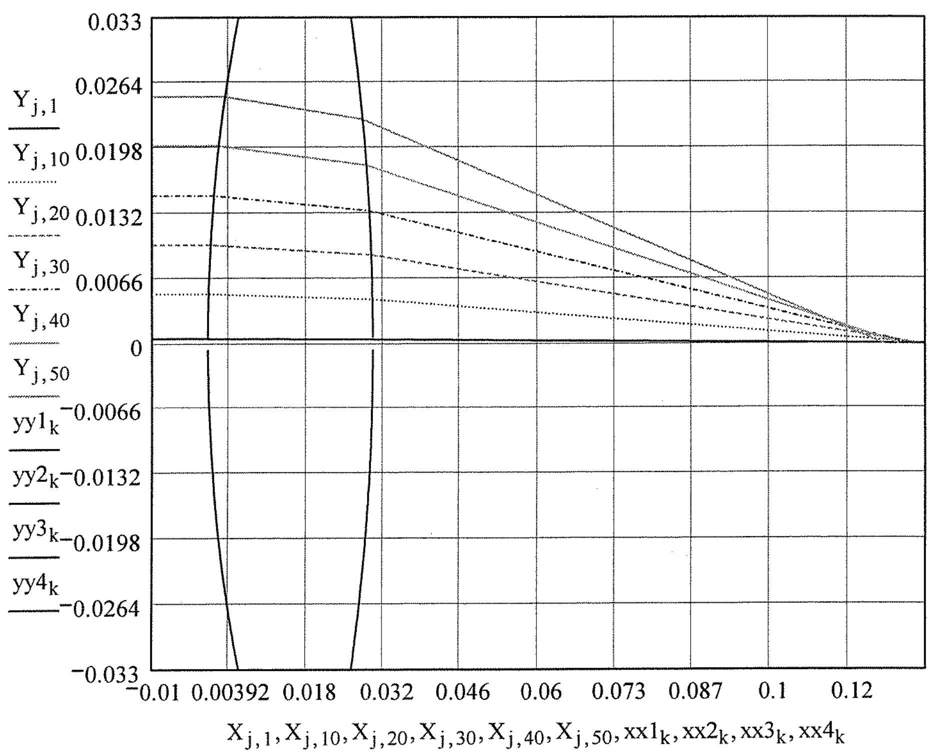
$yy3_k := k \cdot \frac{33}{50000}$ $\quad$ $yy4_k := -k \cdot \frac{33}{50000}$

$$xx3_k := x_2 + \left[R_2^{\,2} - \left(yy3_k\right)^2\right]^{\left(\frac{1}{2}\right)}$$

coordinates on upper and lower halves of circle

$$xx4_k := x_2 + \left[R_2^{\,2} - \left(yy4_k\right)^2\right]^{\left(\frac{1}{2}\right)}$$

Rays drawn passing through lens, observe the spherical aberration, the ray close to the axis goes almost through the paraxial focus



(3)

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} := \begin{pmatrix} 3 & -4 \\ -0.5 & 1 \end{pmatrix}$$

$$\left|\begin{pmatrix} A & B \\ C & D \end{pmatrix}\right| = 1$$

Determinant is 1. Therefore media on both side of system are the same

$h_1 := \frac{D-1}{C}$ $\quad$ $h_1 = 0$

$f := \frac{-1}{C}$ $\quad$ $f = 2$

$h_2 := \frac{A-1}{C}$ $\quad$ $h_2 = -4$

First PP is at input plane, second PP is to right of output plane

$$\begin{pmatrix} r_1 \\ r1_1 \end{pmatrix} := \begin{pmatrix} 0.002 \\ 2 \cdot \frac{2 \cdot \pi}{360} \end{pmatrix}$$

$$\begin{pmatrix} r2 \\ rl_2 \end{pmatrix} := \begin{pmatrix} A & B \\ C & D \end{pmatrix} \cdot \begin{pmatrix} r1 \\ rl_1 \end{pmatrix}$$

$r_2 = -0.13363$

$rl_2 = 0.03391$ radians

$rl_2 \cdot \frac{360}{2 \cdot \pi} = 1.9427$ degrees

$r_2 := 0.003$ $\qquad rl_2 := 3 \cdot \frac{2 \cdot \pi}{360}$

$$\begin{pmatrix} r_1 \\ rl_1 \end{pmatrix} := \begin{pmatrix} A & B \\ C & D \end{pmatrix}^{-1} \cdot \begin{pmatrix} r_2 \\ rl_2 \end{pmatrix}$$

$r_1 = 0.21244$ $\qquad rl_1 = 0.15858$ radians $\qquad rl_1 \cdot \frac{360}{2 \cdot \pi} = 9.08594$ degrees

(4) There are two approaches

Unfold the system -- equivalent to assuming that the mirror does nothing

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} := \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$ mirror matrix

Alternative: Make mirror matrix $\begin{pmatrix} A & B \\ A & D \end{pmatrix} := \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

After mirror reflection angle has changed sign because ray is traveling in opposite direction.

$\begin{pmatrix} r_1 \\ rl_1 \end{pmatrix} := \begin{pmatrix} 0.002 \\ 3 \cdot deg \end{pmatrix}$ $\qquad 3 \cdot deg = 0.05236$ radians

After reflection $\qquad r_2 := r_1 \qquad rl_2 := rl_1$ $\qquad$ But ray is traveling in the oppositedirection

A matrix for a constant distance d would now be $\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & -d \\ 0 & 1 \end{pmatrix}$

(5)

In the unfolded model the overall ray transfer matrix, up to the second pass through the system, is with d=10mm

$d := 0.01$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} := \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 3 & -4 \\ -0.5 & 1 \end{pmatrix}$$

$\begin{pmatrix} r_1 \\ rl_1 \end{pmatrix} := \begin{pmatrix} 0.002 \\ 3 \cdot deg \end{pmatrix}$ $\qquad \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 2.99 & -3.98 \\ -0.5 & 1 \end{pmatrix}$

$$\begin{pmatrix} r_3 \\ rl_3 \end{pmatrix} := \begin{pmatrix} A & B \\ C & D \end{pmatrix} \cdot \begin{pmatrix} r_1 \\ rl_1 \end{pmatrix}$$

$$\begin{pmatrix} r_3 \\ rl_3 \end{pmatrix} = \begin{pmatrix} -0.20241 \\ 0.05136 \end{pmatrix}$$

note that r13 is a positive angle, which must be changed to a negative angle for the second pass through the system

$r1_3 := -r1_3 \qquad r1_3 = -0.05136$

$$\begin{pmatrix} r_4 \\ r1_4 \end{pmatrix} := \begin{pmatrix} 3 & -4 \\ -0.5 & 1 \end{pmatrix}^{-1} \cdot \begin{pmatrix} r_3 \\ r1_3 \end{pmatrix}$$

$r_4 = -0.40785 \qquad r1_4 = -0.25529 \qquad \frac{r1_4}{deg} = -14.6268$

The ray is 408 mm below the axis on return, and is rising from right to left at 14.6 degrees at the input plane

(6) $R := 200 \cdot 10^{-3} \qquad n := 1.5$

$D_1 := \frac{n-1}{R}$ power of surface

$D_1 = 2.5$

The ray transfer matrix across a single spherical surface with air on the left side and index n on the other, when the power of the surface is D_1 is

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} := \begin{pmatrix} 1 & 0 \\ \frac{-D_1}{n} & \frac{1}{n} \end{pmatrix}$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -1.66667 & 0.66667 \end{pmatrix}$$

The ray parameters for a ray traveling parallel to the axis could be

$$\begin{pmatrix} r_1 \\ r1_1 \end{pmatrix} := \begin{pmatrix} 0.001 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} r_2 \\ r1_2 \end{pmatrix} := \begin{pmatrix} A & B \\ C & D \end{pmatrix} \cdot \begin{pmatrix} r_1 \\ r1_1 \end{pmatrix}$$

$r_2 = 1 \times 10^{-3}$ same as input

$r1_2 = -1.66667 \times 10^{-3}$ radians, negative angle because ray is heading in towards axis

If f is the distance from the interface where the ray crosses the axis

$\frac{r_2}{f} = -r1_2$ negative sign to make angle positive

$f := \frac{-r_2}{r1_2}$

$f = 0.6$

same answers

$\frac{-1}{C} = 0.6$

When ray travels in reverse direction the ray transfer matrix is found in a similar way

$D_2 := \frac{1-n}{-R} \qquad D_2 = 2.5$ power is still the same

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} := \begin{pmatrix} 1 & 0 \\ -D_2 & n \end{pmatrix}$$

$$\begin{pmatrix} r_2 \\ r1_2 \end{pmatrix} := \begin{pmatrix} A & B \\ C & D \end{pmatrix} \cdot \begin{pmatrix} r_1 \\ r1_1 \end{pmatrix}$$

$r_2 = 1 \times 10^{-3}$

$r1_2 = -2.5 \times 10^{-3}$ radians

$$f := \frac{-r_2}{r1_2}$$

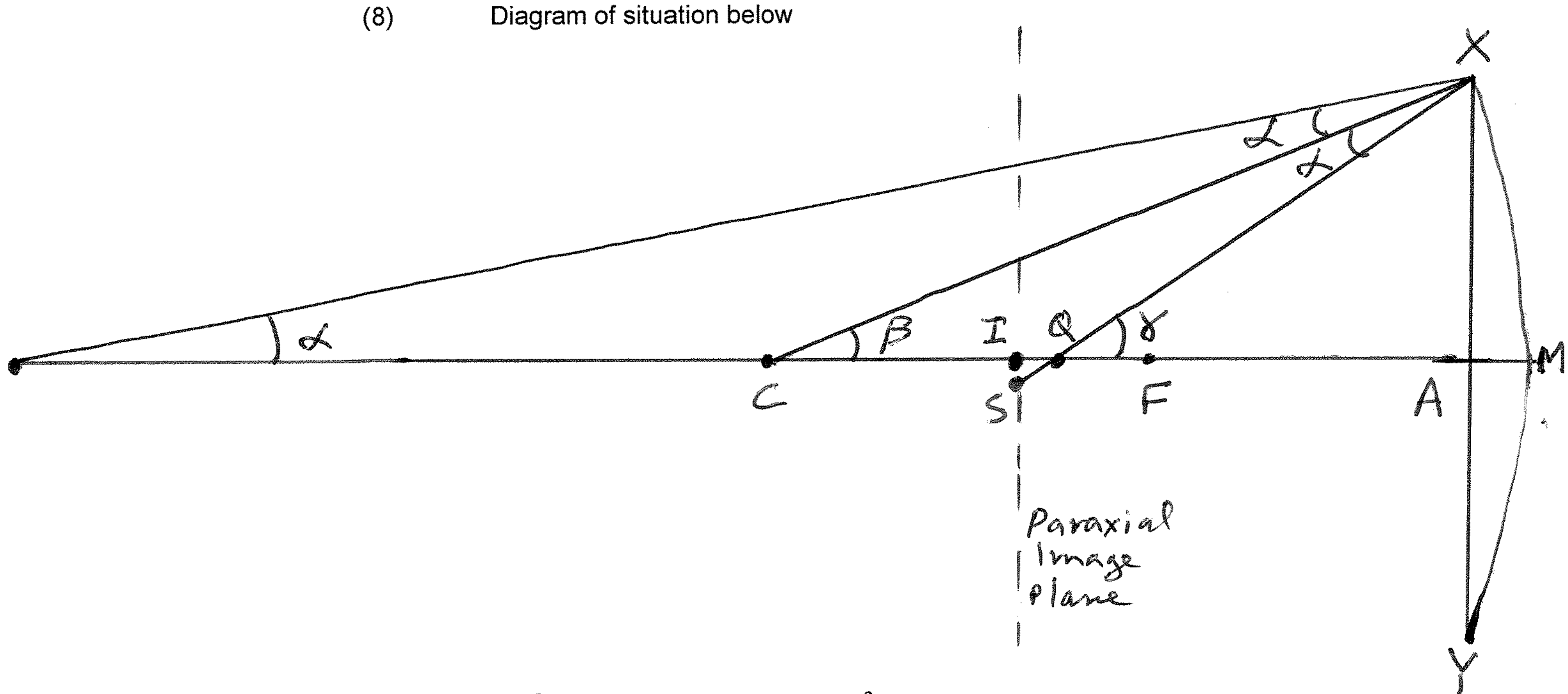
$f = 0.4$

same answers

$$\frac{-1}{C} = 0.4$$

(7) Clearly the input and output planes, and both principal planes are all co-incident and lie at the apex of the surface and the axis

(8) Diagram of situation below



$R := 100 \cdot 10^{-3}$ therefore $f := 50 \cdot 10^{-3}$

$u := 200 \cdot 10^{-3}$ paraxial image distance from mirror is v

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \qquad \text{so} \qquad v := \frac{1}{\frac{1}{f} - \frac{1}{u}}$$

$v = 0.06667$

In the diagram

$AX := 37.5 \cdot 10^{-3}$ semi-aperture of mirror

$CX := R$

$\beta := \operatorname{asin}\left(\frac{AX}{CX}\right)$ $\beta = 0.3844$ radians $\frac{\beta}{deg} = 22.02431$ degrees

$\alpha := \frac{\beta}{2}$ angle of incidence and reflection at mirror $\frac{\alpha}{deg} = 11.01216$ degrees

$\gamma := 3 \cdot \alpha$

Therefore $QA := \frac{37.5 \cdot 10^{-3}}{\tan(\gamma)}$ $QA = 0.05766$

$CA := CX \cdot \cos(\beta)$ $\quad$ $CA = 0.0927$

Therefore $\quad AM := R - CA$ $\quad$ $AM = 7.29752 \times 10^{-3}$

Therefore $\quad IA := v - AM$ $\quad$ $IA = 0.05937$

Therefore $\quad IQ := IA - QA$ $\quad$ $IQ = 1.7046 \times 10^{-3}$

$IS := IQ \cdot \tan(\gamma)$ $\quad$ $IS = 1.10852 \times 10^{-3}$

The spherical aberration is $\quad SA := 2 \cdot IS$

$SA = 2.21705 \times 10^{-3}$ $\quad$ The spherical aberration (diameter) is 2.217mm