ENEE 408E 2003 First Examination Solutions

(1) $\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{(1-n)}{n \cdot R} & \frac{1}{n} \end{bmatrix}$ For proof see notes

$R := 10^{-2}$ radius of curvature

$r1 := 2 \cdot 10^{-3}$ $r11 := 1 \cdot deg$ Input ray parameters

$n := 1.5$ ref. index $d := 10^{-2}$ distance

The easiest way to solve the problem is to "unfold " the system
The ray transfer matrix for the "unfolded" system is:

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} := \begin{pmatrix} 1 & 0 \\ \frac{n-1}{-R} & n \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \cdot d \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ \frac{1-n}{n \cdot R} & \frac{1}{n} \end{pmatrix}$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 0.333 & 0.013 \\ -66.667 & 0.333 \end{pmatrix}$$

$\left| \begin{pmatrix} A & B \\ C & D \end{pmatrix} \right| = 1$ Check on determinant

$$\begin{pmatrix} r2 \\ r12 \end{pmatrix} := \begin{pmatrix} A & B \\ C & D \end{pmatrix} \cdot \begin{pmatrix} r1 \\ r11 \end{pmatrix}$$

$r2 = 8.994 \times 10^{-4}$ emerging ray is 0.8994mm above axis

$\frac{r12}{deg} = -7.306$ emerging ray is falling from right to left at 7.306 degrees

(2)

The effective focal length f=-1/C is the distance from the principal planes to the focal point.
The front and back focal points are the distances from the input and output planes to the focal points. These are all for paraxial rays.

For an imaging system, B=0 because all rays throuhg the top of the object go through the top of the image. A=m because this is the ratio of r2/r1

C=-1/f by definition. D=1/m=m'

For the system given

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} := \begin{pmatrix} 2 & 5 \\ -0.1 & 0.25 \end{pmatrix}$$

$$\left|\begin{pmatrix} A & B \\ C & D \end{pmatrix}\right| = 1$$

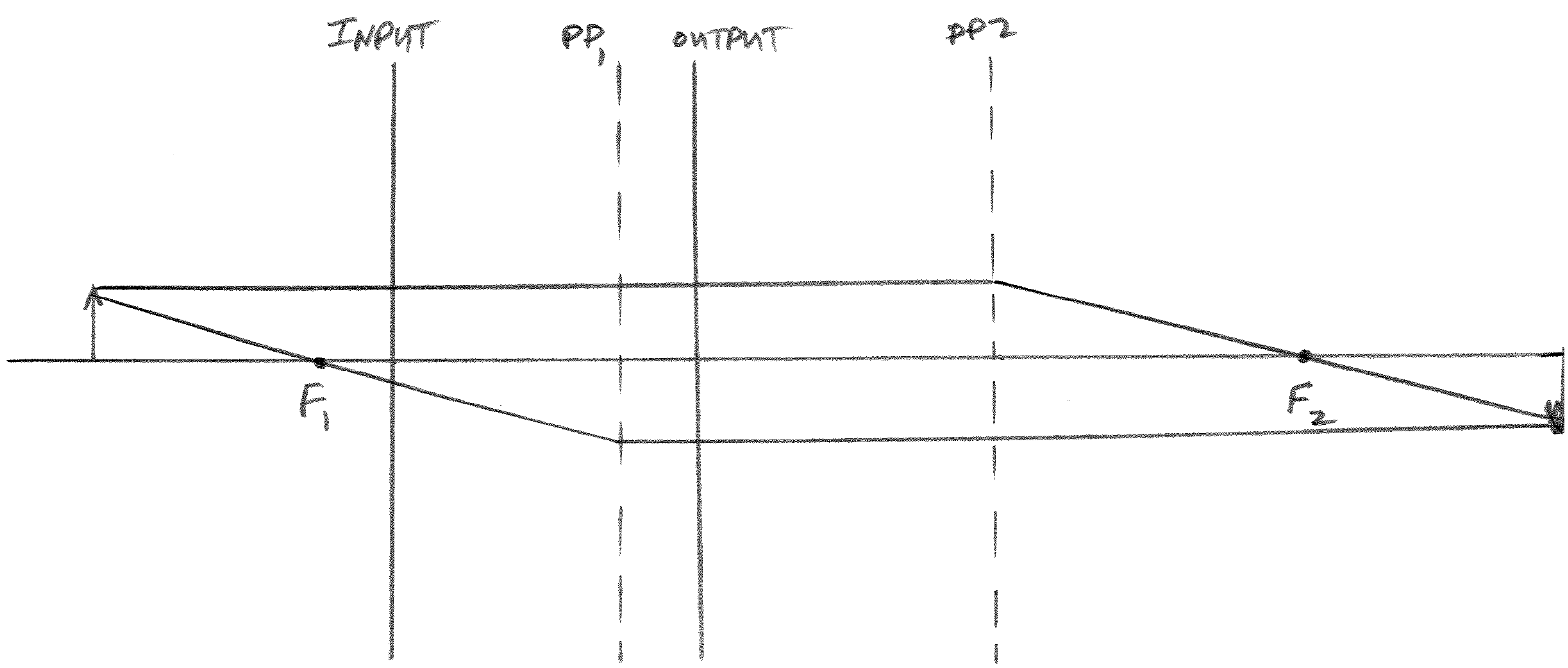
Check. Media to left of input plane and to right of ouput plane are the same.

$$h1 := \frac{D-1}{C} \qquad h2 := \frac{A-1}{C}$$

$$f := \frac{-1}{C}$$

$f = 10$ $h1 = 7.5$ $h2 = -10$

Second principal plane is to right of ouput plane. See ray tracing diagram below



(3) Aperture stop is the limiting aperture that determines which rays make it through ther system. Entrance pupil is the image of the aperture stop through all of system to left of aperture stop. Exit pupil is image of aperture stop through of all of system to right of aperture stop.

For the system shown use imaging equation to find pupils

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$v(u,f) := \frac{f \cdot u}{u - f}$ image distance

$m(u,f) := \frac{-f}{u - f}$ linear magnification

For entrance pupil $u := 15$ $f := 10$

$v(u,f) = 30$ $m(u,f) = -2$

$EPD := 2 \cdot |m(u,f)|$

$EPD = 4$

Entrance pupil is 30mm to left of input plane. EPD=4mm

For exit pupil $u := 10$ $f := 20$

$v(u,f) = -20$ $m(u,f) = 2$

Exit pupil is 20mm to left of ouput plane. Exit PD=4mm

The point source is far from the entrance pupil. Its distance from the entrance pupil is

$L := 10 - 30 \cdot 10^{-3}$ $EPD := 4 \cdot 10^{-3}$

We can use the solid angle approximation for large distances

$$\Delta\omega := \frac{\frac{\pi \cdot EPD^2}{4}}{L^2}$$

The power through is for 1W $P := \frac{1 \cdot \Delta\omega}{4 \cdot \pi}$

$P = 1.006 \times 10^{-8}$ 10nW goes through

(4) Use Snell's law to calculate angles of refraction θ2 at first surface and ange of incidence θ3 on second surface.

$n := 1.5$ $\quad$ $Z0 := 376.7$ $\quad$ $\theta 1 := 30 \cdot deg$

$$\theta 2 := \operatorname{asin}\left(\frac{\sin(\theta 1)}{n}\right)$$

$\frac{\theta 2}{deg} = 19.471$ angle of refraction at first prism face

Use simple geometry to find angle of incidence on second prism face

$$\theta 3 := 90 - \left[180 - \left(90 - \frac{\theta 2}{deg}\right) - 60\right]$$

$\theta 3 = 40.529$ angle of incidence on second prism face

$\theta 3 := \theta 3 \cdot deg$ $\quad$ $\theta 3 = 0.707$ in radians

$$\theta 4 := \operatorname{asin}(n \cdot \sin(\theta 3))$$

$\frac{\theta 4}{deg} = 77.096$

For P-wave component of input light

$Z11 := Z0 \cdot \cos(\theta 1)$ $\quad$ $Z21 := Z0 \cdot \frac{\cos(\theta 2)}{n}$ $\quad$ $Z31 := Z0 \cdot \frac{\cos(\theta 3)}{n}$ $\quad$ $Z41 := Z0 \cdot \cos(\theta 4)$

$Z11 = 326.232$ $\quad$ $Z21 = 236.771$ $\quad$ $Z31 = 190.881$ $\quad$ $Z41 = 84.125$

Calculate reflection coefficients at each face of the prism

$\rho 1 := \frac{Z21 - Z11}{Z21 + Z11}$ $\quad$ $\rho 2 := \frac{Z41 - Z31}{Z41 + Z31}$

$\rho 1 = -0.159$ $\quad$ $\rho 2 = -0.388$

$T1 := 1 - (|\rho 1|)^2$ $\quad$ $T2 := 1 - (|\rho 2|)^2$

$T1 = 0.975$ $\quad$ $T2 = 0.849$

$TP := T1 \cdot T2$

$TP = 0.828$

For S-wave component of input light

$$Z11 := \frac{Z0}{\cos(\theta 1)} \qquad Z21 := \frac{Z0}{n \cdot \cos(\theta 2)} \qquad Z31 := \frac{Z0}{n \cdot \cos(\theta 3)} \qquad Z41 := \frac{Z0}{\cos(\theta 4)}$$

$$Z11 = 434.976 \qquad Z21 = 266.367 \qquad Z31 = 330.404 \qquad Z41 = 1.687 \times 10^{3}$$

Calculate reflection coefficients at each face of the prism

$$\rho 1 := \frac{Z21 - Z11}{Z21 + Z11} \qquad \rho 2 := \frac{Z41 - Z31}{Z41 + Z31}$$

$$\rho 1 = -0.24 \qquad \rho 2 = 0.672$$

$$T1 := 1 - (|\rho 1|)^2 \qquad T2 := 1 - (|\rho 2|)^2$$

$$T1 = 0.942 \qquad T2 = 0.548$$

$$TS := T1 \cdot T2$$

$$TS = 0.516$$

Average transmittance is $T := \frac{TP + TS}{2}$

$T = 0.672$ 67.2% of light gets through