

## ENEE 408E Optical System Design

### Design Projects #1, 9/14/04

Due 9/23/04

- (1) A thick lens with  $n = 1.55$  has  $R_1 = 100$  mm,  $R_2 = -140$ mm with  $d = 30$ mm. Calculate the position of the principal planes and the paraxial focal length of the lens.
- (2) For the lens in question (1) write a computer program using Matlab, Mathcad, or whatever, to carry out exact ray tracing through the lens in a 2-D model. Make the lens aperture (the full diameter over which rays can enter) 20mm. Show a family of rays entering the lens traveling parallel to the axis and leaving through a blurred focus. Confirm that the focal length for rays near the axis agrees with the paraxial focal length calculated in question (1)

- (3) An optical system has

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 3 & -4 \\ -0.5 & 1 \end{pmatrix}$$

Calculate:

- (i) the focal length
- (ii) the position of the principal planes
- (iii) the output ray parameters if  $r_1 = 0.002m$ ,  $r'_1 = 2^\circ$ .
- (iv) the input ray parameters if  $r_2 = 0.003m$ ,  $r'_2 = 3^\circ$ .
- (4) A ray with parameters  $r_1, r'_1$  strikes a plane mirror. Determine a ray transfer matrix for the plane mirror that gives the ray parameters after reflection. Use your matrix to find the reflected ray parameters when  $r_1=0.002m$ ,  $r'_1 = 3^\circ$
- (5) for question (3) part (iii) calculate the parameters of the ray when it returns to the input plane if a plane mirror is placed perpendicular to the axis and a distance of 10mm after the original output plane. Draw a diagram to illustrate your answer
- (6) A spherical surface of radius of curvature 200mm has air to the left and glass ( $n=1.5$ ) to the right. A ray of light enters traveling from left to right. Where does it cross the axis? A ray of light traveling parallel to the axis enters traveling from right to left. Where does it cross the axis?
- (7) Where are the principal planes in question (6)?
- (8) A point source of light is placed 200mm from a concave mirror of radius of curvature 100mm. Calculate the blurring (spherical aberration) of the image at the paraxial image plane. The blurring is the diameter of the illuminated region at the paraxial image plane. Do an exact calculation using the precise geometry and Snell's law. Do not use the small angle approximation for the calculation of the blurring. The open aperture of the concave mirror is 75mm. This is the diameter of the mirror perpendicular to its axis.

(1) Declare variables:

$$n := 1.55$$

$$R1 := 100 \cdot 10^{-3}$$

units - meters

$$R2 := -140 \cdot 10^{-3}$$

$$d := 30 \cdot 10^{-3}$$

$$D1 := \frac{n-1}{R1}$$

powers of surfaces

$$D1 = 5.5$$

$$D2 := \frac{1-n}{R2}$$

$$D2 = 3.92857$$

$$h1 := \frac{d}{n \cdot \left[ 1 + \left( \frac{D1}{D2} \right) - \left( d \cdot \frac{D1}{n} \right) \right]}$$

$$h1 = 8.43882 \times 10^{-3}$$

$$h1 = 8.44 \text{ mm}$$

$$h2 := \frac{d}{n \cdot \left[ 1 + \left( \frac{D2}{D1} \right) - \left( d \cdot \frac{D2}{n} \right) \right]}$$

$$h2 = 0.01181$$

$$h2 = 11.8 \text{ mm}$$

$$x := \left( D1 + D2 - d \cdot D1 \cdot \frac{D2}{n} \right)$$

$$f := \frac{1}{x}$$

$$f = 0.11098$$

focal length 111mm

(2) Start first surface at  $x=0$ , so its center is at  $x=0.1$   
Second surface is clearly centered at  $x=-0.110$

$$n := 1.55$$

$$x1 := 0.1$$

$$x2 := -0.110$$

$$R1 := 0.1$$

$$R2 := -0.14$$

Equation of first surface is

Equation of second surface is

$$(x - x1)^2 + y^2 = R1^2$$

$$(x - x2)^2 + y^2 = R2^2$$

Input ray is

$$y = r1$$

Parallel to axis

$$i := 1..50$$

$$r1_i := \frac{i}{2000}$$

rays at different distances from axis

Find where input ray intersects first circle

$$(x - x1)^2 + y^2 = R1^2$$

$$\left[ \begin{array}{l} x1 + (R1^2 - y^2)^{\frac{1}{2}} \\ x1 - (R1^2 - y^2)^{\frac{1}{2}} \end{array} \right]$$

solving for x, choose lower solution  
since this is to left of center of circle

$$x_i := x_1 - \left[ R_1^2 - (r_{1_i})^2 \right]^{\left(\frac{1}{2}\right)} \quad \text{intersection points}$$

Vector from intersection point to center of circle is

$$a_i := \begin{pmatrix} x_1 - x_i \\ -r_{1_i} \end{pmatrix}$$

Input ray vector is

$$b_i := \begin{pmatrix} r_{1_i} \\ 0 \end{pmatrix}$$

dot product is

$$ab_i := a_i \cdot b_i$$

$$\theta_{1_i} := \arccos\left(\frac{ab_i}{|a_i| |b_i|}\right)$$

angles of incidence is

$$\theta_{1_i} := \arcsin\left(\frac{\sin(\theta_{1_i})}{n}\right)$$

angles of refraction

Angle that surface normal makes with axis is

$$\phi_i := \arcsin\left(\frac{r_{1_i}}{R_1}\right)$$

angle of refracted ray with respect to axis is

$$\theta_{2_i} := \phi_i - \theta_{1_i}$$

equation of refracted ray is

$$y = m \cdot x + b$$

$$m_i := -\tan(\theta_{2_i})$$

$$b_i := r_{1_i} - m_i \cdot x_i$$

Find intersection of refracted ray and second circle

$$(x - x_2)^2 + (m \cdot x + b)^2 = R_2^2$$

$$\left[ \begin{array}{l} \frac{1}{[2 \cdot (m^2 + 1)]} \cdot \left[ -2 \cdot m \cdot b + 2 \cdot x_2 + 2 \cdot \left( -2 \cdot m \cdot b \cdot x_2 - m^2 \cdot x_2^2 + m^2 \cdot R_2^2 - b^2 + R_2^2 \right)^{\left(\frac{1}{2}\right)} \right] \\ \frac{1}{[2 \cdot (m^2 + 1)]} \cdot \left[ -2 \cdot m \cdot b + 2 \cdot x_2 - 2 \cdot \left( -2 \cdot m \cdot b \cdot x_2 - m^2 \cdot x_2^2 + m^2 \cdot R_2^2 - b^2 + R_2^2 \right)^{\left(\frac{1}{2}\right)} \right] \end{array} \right]$$

Solutions for intersection point, choose greater value for correct intersection point

$$x_{3_i} := \frac{1}{[2 \cdot [(m_i)^2 + 1]]} \cdot \left[ -2 \cdot m_i \cdot b_i + x_2 + 2 \cdot \left[ -2 \cdot m_i \cdot b_i \cdot x_2 - (m_i)^2 \cdot (x_2)^2 + (m_i)^2 \cdot R_2^2 - (b_i)^2 + R_2^2 \right]^{\left(\frac{1}{2}\right)} \right]$$

$$y_{3_i} := m_i \cdot x_{3_i} + b_i$$

Vector from intersection point to center of sphere is

$$a2_i := \begin{pmatrix} x3_i - x2 \\ y3_i \end{pmatrix}$$

Vector of ray incident on second surface is

$$b2_i := \begin{pmatrix} x3_i - x1 \\ y3_i - r1_i \end{pmatrix}$$

dot product is

$$ab2_i := a2_i \cdot b2_i$$

$$\text{Angles of incidence } \theta_{2_i} := \text{acos}\left(\frac{ab2_i}{|a2_i| \cdot |b2_i|}\right)$$

Angles of refraction are

$$\theta_{3_i} := \text{asin}(n \cdot \sin(\theta_{2_i}))$$

Angles that surface normals make with axis

$$\phi_{2_i} := \text{atan}\left(\frac{y3_i}{-R2}\right)$$

Angles of refracted rays are

$$\theta_{4_i} := \theta_{3_i} - \phi_{2_i}$$

Equation of refracted ray is

$$y = m2 \cdot x + b2$$

$$m2_i := -\tan(\theta_{4_i})$$

$$b2_i := y3_i - m2_i \cdot x3_i$$

This ray intersects the x axis at

$$x4_i := \frac{-b2_i}{m2_i}$$

Draw rays through lens

$$j := 1, 2..4$$

Dummy variable for plotting the straight line segments

$$X_{1,i} := -0.01$$

$$Y_{1,i} := r1_i$$

$$X_{2,i} := x_i$$

$$Y_{2,i} := r1_i$$

$$X_{3,i} := x3_i$$

$$Y_{3,i} := y3_i$$

$$X_{4,i} := x4_i$$

$$Y_{4,i} := 0$$

Draw circles

$$k := 1, 2..50$$

variable for different points on circle

$$(x - x1)^2 + y^2 = R1^2$$

$$yy1_k := k \cdot \frac{33}{50000}$$

$$yy2_k := -k \cdot \frac{33}{50000}$$

$$\begin{bmatrix} x1 + (R1^2 - y^2)^{\frac{1}{2}} \\ x1 - (R1^2 - y^2)^{\frac{1}{2}} \end{bmatrix}$$

solutions for x as a function of y, choose lower value

$$f := \frac{-1}{C} \quad f = 2 \quad h_1 := \frac{D-1}{C} \quad h_1 = 0$$

$$h_2 := \frac{A-1}{C} \quad h_2 = -4$$

First PP is at input plane, second PP is to right of output plane

$$\begin{pmatrix} r_1 \\ r1_1 \end{pmatrix} := \begin{pmatrix} 0.002 \\ 2 \cdot \frac{2 \cdot \pi}{360} \end{pmatrix}$$

$$\begin{pmatrix} r_2 \\ r1_2 \end{pmatrix} := \begin{pmatrix} A & B \\ C & D \end{pmatrix} \cdot \begin{pmatrix} r_1 \\ r1_1 \end{pmatrix}$$

$$r_2 = -0.13363$$

$$r1_2 = 0.03391 \quad \text{radians}$$

$$r1_2 \cdot \frac{360}{2 \cdot \pi} = 1.9427 \quad \text{degrees}$$

$$r_2 := 0.003 \quad r1_2 := 3 \cdot \frac{2 \cdot \pi}{360}$$

$$\begin{pmatrix} r_1 \\ r1_1 \end{pmatrix} := \begin{pmatrix} A & B \\ C & D \end{pmatrix}^{-1} \cdot \begin{pmatrix} r_2 \\ r1_2 \end{pmatrix}$$

$$r_1 = 0.21244 \quad r1_1 = 0.15858 \quad \text{radians} \quad r1_1 \cdot \frac{360}{2 \cdot \pi} = 9.08594 \quad \text{degrees}$$

(4) There are two approaches

Unfold the system -- equivalent to assuming that the mirror does nothing

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} := \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{mirror matrix}$$

$$\text{Alternative: Make mirror matrix} \quad \begin{pmatrix} A & B \\ A & D \end{pmatrix} := \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

After mirror reflection angle has changed sign because ray is traveling in opposite direction.

$$\begin{pmatrix} r_1 \\ r1_1 \end{pmatrix} := \begin{pmatrix} 0.002 \\ 3 \cdot \text{deg} \end{pmatrix} \quad 3 \cdot \text{deg} = 0.05236 \quad \text{radians}$$

After reflection  $r_2 := r_1$   $r1_2 := r1_1$  But ray is traveling in the oppositedirection

A matrix for a constant distance d would now be

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & -d \\ 0 & 1 \end{pmatrix}$$

(5)

In the unfolded model the overall ray transfer matrix, up to the second pass through the system, is with  $d=10\text{mm}$

$$d := 0.01$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} := \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 3 & -4 \\ -0.5 & 1 \end{pmatrix}$$

$$\begin{pmatrix} r_1 \\ r_{11} \end{pmatrix} := \begin{pmatrix} 0.002 \\ 3 \cdot \text{deg} \end{pmatrix} \quad \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 2.99 & -3.98 \\ -0.5 & 1 \end{pmatrix}$$

$$\begin{pmatrix} r_3 \\ r_{13} \end{pmatrix} := \begin{pmatrix} A & B \\ C & D \end{pmatrix} \cdot \begin{pmatrix} r_1 \\ r_{11} \end{pmatrix}$$

$$\begin{pmatrix} r_3 \\ r_{13} \end{pmatrix} = \begin{pmatrix} -0.20241 \\ 0.05136 \end{pmatrix}$$

note that  $r_{13}$  is a positive angle, which must be changed to a negative angle for the second pass through the system

$$r_{13} := -r_{13} \quad r_{13} = -0.05136$$

$$\begin{pmatrix} r_4 \\ r_{14} \end{pmatrix} := \begin{pmatrix} 3 & -4 \\ -0.5 & 1 \end{pmatrix}^{-1} \cdot \begin{pmatrix} r_3 \\ r_{13} \end{pmatrix}$$

$$r_4 = -0.40785 \quad r_{14} = -0.25529 \quad \frac{r_{14}}{\text{deg}} = -14.6268$$

The ray is 408 mm below the axis on return, and is rising from right to left at 14.6 degrees at the input plane

$$(6) \quad R := 200 \cdot 10^{-3} \quad n := 1.5$$

$$D_1 := \frac{n-1}{R} \quad \text{power of surface}$$

$$D_1 = 2.5$$

The ray transfer matrix across a single spherical surface with air on the left side and index  $n$  on the other, when the power of the surface is  $D_1$  is

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} := \begin{pmatrix} 1 & 0 \\ -D_1 & 1 \\ n & n \end{pmatrix}$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -1.66667 & 0.66667 \end{pmatrix}$$

The ray parameters for a ray traveling parallel to the axis could be

$$\begin{pmatrix} r_1 \\ r_{11} \end{pmatrix} := \begin{pmatrix} 0.001 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} r_2 \\ r_{12} \end{pmatrix} := \begin{pmatrix} A & B \\ C & D \end{pmatrix} \cdot \begin{pmatrix} r_1 \\ r_{11} \end{pmatrix}$$

$$r_2 = 1 \times 10^{-3} \quad \text{same as input}$$

$$r_{12} = -1.66667 \times 10^{-3} \quad \text{radians, negative angle because ray is heading in towards axis}$$

If  $f$  is the distance from the interface where the ray crosses the axis

$$\frac{r_2}{f} = -r_{12} \quad \text{negative sign to make angle positive}$$

$$f := \frac{-r_2}{r_{12}}$$

$$f = 0.6$$

same answers

$$\frac{-1}{C} = 0.6$$

When ray travels in reverse direction the ray transfer matrix is found in a similar way

$$D_2 := \frac{1-n}{-R} \quad D_2 = 2.5 \quad \text{power is still the same}$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} := \begin{pmatrix} 1 & 0 \\ -D_2 & n \end{pmatrix}$$

$$\begin{pmatrix} r_2 \\ r_{12} \end{pmatrix} := \begin{pmatrix} A & B \\ C & D \end{pmatrix} \cdot \begin{pmatrix} r_1 \\ r_{11} \end{pmatrix}$$

$$r_2 = 1 \times 10^{-3}$$

$$r_{12} = -2.5 \times 10^{-3} \quad \text{radians}$$

$$f := \frac{-r_2}{r_{12}}$$

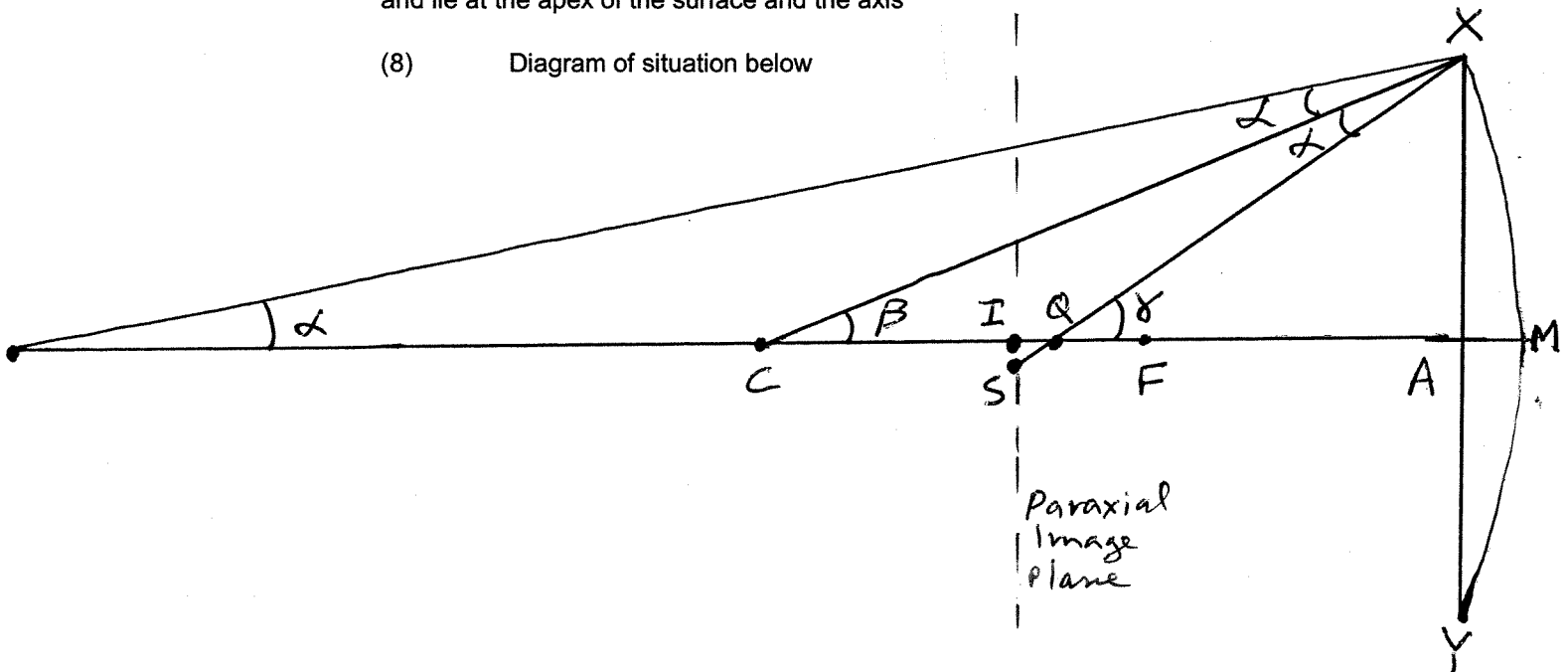
$$f = 0.4$$

same answers

$$\frac{-1}{C} = 0.4$$

(7) Clearly the input and output planes, and both principal planes are all co-incident and lie at the apex of the surface and the axis

(8) Diagram of situation below



$$R := 100 \cdot 10^{-3} \quad \text{therefore} \quad f := 50 \cdot 10^{-3}$$

$$u := 200 \cdot 10^{-3} \quad \text{paraxial image distance from mirror is } v$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad \text{so} \quad v := \frac{1}{\frac{1}{f} - \frac{1}{u}}$$

$$v = 0.06667$$

In the diagram

$$AX := 37.5 \cdot 10^{-3} \quad \text{semi-aperture of mirror}$$

$$CX := R$$

$$\beta := \text{asin}\left(\frac{AX}{CX}\right) \quad \beta = 0.3844 \quad \text{radians} \quad \frac{\beta}{\text{deg}} = 22.02431 \quad \text{degrees}$$

$$\alpha := \frac{\beta}{2} \quad \text{angle of incidence and reflection at mirror} \quad \frac{\alpha}{\text{deg}} = 11.01216 \quad \text{degrees}$$

$$\gamma := 3 \cdot \alpha$$

$$\text{Therefore} \quad QA := \frac{37.5 \cdot 10^{-3}}{\tan(\gamma)} \quad QA = 0.05766$$

$$CA := CX \cdot \cos(\beta) \quad CA = 0.0927$$

$$\text{Therefore} \quad AM := R - CA \quad AM = 7.29752 \times 10^{-3}$$

$$\text{Therefore} \quad IA := v - AM \quad IA = 0.05937$$

$$\text{Therefore} \quad IQ := IA - QA \quad IQ = 1.7046 \times 10^{-3}$$

$$IS := IQ \cdot \tan(\gamma) \quad IS = 1.10852 \times 10^{-3}$$

$$\text{The spherical aberration is} \quad SA := 2 \cdot IS$$

$$SA = 2.21705 \times 10^{-3} \quad \text{The spherical aberration (diameter) is 2.217mm}$$