

ENEE 408E First Examination Solutions

(1) $f := 120 \cdot 10^{-3}$ focal length

$$l_o = 200 \cdot 10^{-3} + h_1.$$

$$l_i = 300 \cdot 10^{-3} + h_2$$

$$h_2 := f - 100 \cdot 10^{-3} \quad \text{from ray parallel to axis}$$

$$h_2 = 0.02 \quad h_2 \text{ is 20mm}$$

$$l_i := 300 \cdot 10^{-3} + h_2$$

$$h_1 := 20 \quad \text{Guess}$$

Given

$$\frac{1}{200 \cdot 10^{-3} + h_1} + \frac{1}{l_i} = \frac{1}{f}$$

$$h_1 := \text{Find}(h_1)$$

$$h_1 = -8 \times 10^{-3}$$

$$h_1 \text{ is -8mm}$$

(2) Unit cell consists of thin lens, a uniform length $d/4$, a medium of length $d/2$ and a second uniform length $d/4$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & \frac{d}{4} \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & \frac{d}{4} \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & \frac{d}{4} \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 - \frac{3 \cdot d}{4 \cdot f} & \frac{3 \cdot d}{4} \\ -\frac{1}{f} & 1 \end{pmatrix}$$

$$0.5 \cdot (A + D) = 0.5 \cdot \left(1 - \frac{3}{4} \cdot \frac{d}{f} + 1 \right)$$

$$f := 100 \cdot 10^{-3}$$

$$d := 100 \cdot 10^{-3} \quad \text{Guess}$$

Given

$$-1 = 0.5 \cdot \left(1 - \frac{3}{4} \cdot \frac{d}{f} + 1 \right) \quad \text{Check lower limit of -1}$$

$$d := \text{Find}(d)$$

$$d = 0.533$$

Given

$$1 = 0.5 \cdot \left(1 - \frac{3}{4} \cdot \frac{d}{f} + 1 \right)$$

$$d := \text{Find}(d)$$

$$d = 0$$

Stability condition is

$$0 \leq d \leq 533 \quad \text{in mm}$$

(3) Use aperture stop location as the image in the thin lens imaging equation

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$u = \frac{v \cdot f}{v - f}$$

$$f := 20 \cdot 10^{-3} \quad v := 50 \cdot 10^{-3}$$

$$d := \frac{v \cdot f}{v - f}$$

$$d = 0.033 \quad \text{Aperture Stop is 33mm to right of first lens}$$

$$m := \frac{v}{d} \quad \text{magnification}$$

$$D := \frac{20}{m}$$

D = 13.333 Aperture stop diameter in mm

Use thin lens imaging equation to find exit pupil

$$f := 50 \cdot 10^{-3}$$

$$d_1 := \frac{d \cdot f}{d - f}$$

$$d_1 = -0.1$$

$$m_1 := \left| \frac{d_1}{d} \right| \quad m_1 = 3$$

$$\text{Dep} := m_1 \cdot D$$

Dep = 40 Exit pupil diameter is 40mm in diameter
It is located 100mm to the left of lens number 2.

$$D_o := 20 \cdot 10^{-3} \quad \text{Entrance Pupil diameter}$$

Distance of point source from entrance pupil is

$$L := 1 - v$$

$$L = 0.95$$

The solid angle subtended by the entrance pupil is

$$\Delta\Omega := \frac{\pi \cdot D_o^2}{4 \cdot L^2} \quad \text{This is a good approximation in this case since point source is far away from small entrance pupil}$$

$$\Delta\Omega = 3.481 \times 10^{-4}$$

Power through is

$$P := \frac{10^{-2} \cdot \Delta\Omega}{4 \cdot \pi}$$

$$P = 2.77 \times 10^{-7} \quad 277\text{nW goes through}$$