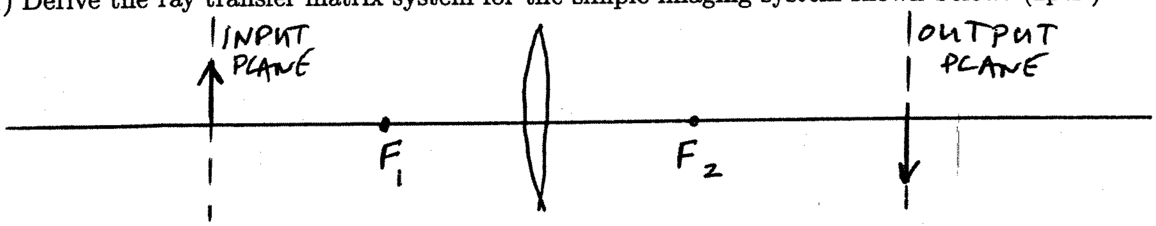
ENEE 408E Optical System Design 2004
Final Examination

Monday, December 13, 2004, 8:00 - 10:00 am

ANSWER 4 QUESTIONS

(if more than 4 are answered best 4 will count)

(1) Derive the ray transfer matrix system for the simple imaging system shown below. (2pts.)



Show that for this to be an imaging system the element B of the ray transfer matrix must be 0 (1pts.). By so doing prove the thin lens equation (2pts.)

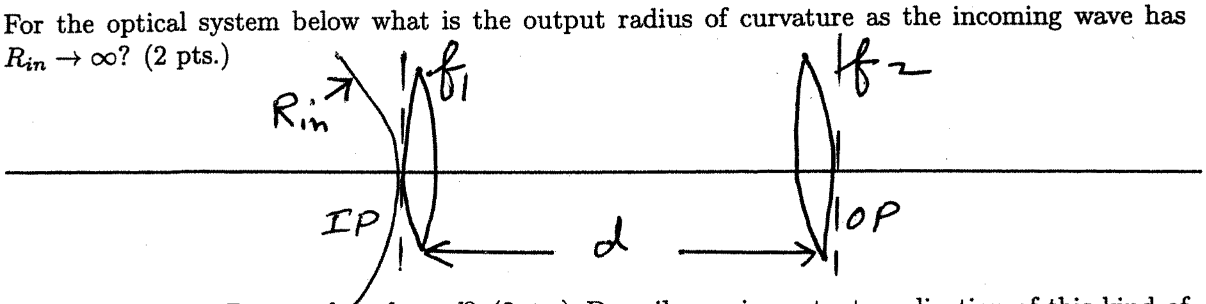
$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}.$$

What are the linear and angular magnifications of this system. (2pts.) For the following three situations draw a ray tracing diagram showing the formation of the image and calculate v, the linear magnfication m, and the angular magnfication m'. (3pts.)
(a) f=50mm, u=150mm
(b) f=-50mm, u=150mm
(c) f=50mm, u=30mm.

(2) Prove that for a spherical wave passing through an optical system with ray transfer matrix $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$, that the input and output radii of curvatures are related by (3pts.)

$$R_{out} = \frac{AR_{in} + B}{CR_{in} + D}.$$

For the optical system below what is the output radius of curvature as the incoming wave has $R_{in} \to \infty$? (2 pts.)



What happens to R_{out} as $f_1 + f_2 \to d$? (2pts.) Describe an important application of this kind of arrangement. (2pts.)

(3)The transformed impedance of a transparent window with air ($n = 1$) on both sides is

$$Z_3'' = Z_2' \left(\frac{Z_1' \cos k_2 d' + jZ_2' \sin k_2 d'}{Z_2' \cos k_2 d' + jZ_1' \sin k_2 d'} \right),$$

where the parameter d' is the effective thickness of the window.

Coherent light with 50% P-polarization, 50% S-polarization is incident on a window with $n = 3.5$ that has an effective thickness of $\frac{2}{5}\lambda$ at an angle of incidence of 40°. What is the overall transmittance of the window? (7pts.) What is the polarization state of the reflected light? (2pts.) At what angle of incidence is all the reflected light pure S-polarized? (1pt.)

Hint: For P-waves $Z' = Z_0 \cos\theta/n$.

(4) The Gaussian beam parameter q is usually defined as

$$\frac{1}{q} = \frac{1}{R} - j\frac{\lambda}{\pi w^2}$$

If the beam waist of a Gaussian beam is at $z = 0$ than prove that at distance z from the beamwaist (4pts.)

$$w^2(z) = w_0^2\left[1 + \left(\frac{\lambda z}{\pi w_0^2}\right)^2\right],$$

and

$$R(z) = z\left[1 + \left(\frac{\pi w_0^2}{\lambda z}\right)^2\right].$$

Use these results to show that the beam divergence angle is (2pts.)

$$\theta_{beam} = \frac{\lambda}{\pi w_0} \qquad \text{(2pts.)}$$

An elliptical Gaussian beam (a Gaussian beam with different minimum spotsizes in orthogonal planes) has minimum spotsize in the yz plane of 50μm, and in the xz plane 200μm. If the beam waists in both planes are at z=0, and a thin lens of focal length f= 5mm is placed at the beam waist, where is the focused spot circular? (4pts.) Take the wavelength as 1.55μm.

(5) Do three of the following

(a) Explain how a photomultiplier tube works . Prove that for the photocathode in such a device the responsivity R is

$$R = \frac{e\eta}{h\nu}$$

Explain the meaning of the quantities that you use.

(b) The concepts of radiant intensity, radiant emittance, and radiance, and the properties of a Lambertian surface.

(c) The design of a Cassegrain telescope. Explain the key differences between such a telescope and a standard terrestrial telescope.

(6) Prove that the numerical aperture of an optical fiber with core index n_1 and cladding index n_2 is

$$\sin\theta_0 = NA = \sqrt{n_1^2 - n_2^2}. \qquad \text{(2pts.)}$$

Prove also that the number of modes that can propagate in a step-index optical fiber can be estimated in a simple 2-D model as (3pts.)

$$m = \frac{2D(NA)}{\lambda_0}.$$

A fiber with n_1=1.46, n_2=1.455, and with core diameter 20μm has a point source placed on axis right against the end planar face of the core, but just outside the core. The power of the source is 1mW. What power gets to the far end of the fiber, which is 100m long? (4pts.) What is the minimum wavelength of the source that would make the fiber a *single-mode* fiber? (1pt.)

Hint: The solid angle correponding to a cone of semi-vertical angle θ is $\delta\omega = 2\pi(1 - \cos\theta)$.

ENEE 408E 2004 Final Exam Solutions

(1) The ray transfer matrix for the system from object to image is

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & v \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ \frac{-1}{f} & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & u \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{bmatrix} 1 - \frac{v}{f} & \left(1 - \frac{v}{f}\right) \cdot u + v \\ \frac{-1}{f} & \frac{-1}{f} \cdot u + 1 \end{bmatrix}$$

For imaging, any ray from a point on the obect must pass through a corresponding point on the mage. This must be idependent of the ray angle, so B must=0

$\left(1 - \frac{v}{f}\right) \cdot u + v = 0$ which gives $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

linear magnification is is element A of the matrix, angular magnfication is element D

$m = 1 - \frac{v}{f} = \frac{-v}{u}$ linear magnification

$m1 = 1 - \frac{u}{f} = \frac{-u}{v}$ angular magnification

$$m(u,f) := \frac{1}{1 - \frac{u}{f}} \qquad m1(u,f) := 1 - \frac{u}{f} \qquad v(u,f) := \frac{1}{\frac{1}{f} - \frac{1}{u}}$$

(a) $v(150,50) = 75$ $m(150,50) = -0.5$ $m1(150,50) = -2$

(b) $v(150,-50) = -37.5$ $m(150,-50) = 0.25$ $m1(150,-50) = 4$

(c) $v(30,50) = -75$ $m(30,50) = 2.5$ $m1(30,50) = 0.4$

(2) For an input spherical wave of radius R_{in} the relationship between ray height and ray angle is

$$r1_{in} = \frac{r_{in}}{R_{in}}$$

$$r_{out} = A \cdot r_{in} + B \cdot r1_{in}$$

$$r1_{out} = C \cdot r_{in} + D \cdot r1_{in}$$

Dividing the second of these equations by the first gives

$$\frac{r_{out}}{r1_{out}} = \frac{A \cdot r_{in} + B \cdot r1_{in}}{C \cdot r_{in} + r1_{in}} = \frac{A \cdot \dfrac{r_{in}}{r1_{in}} + B}{C \cdot \dfrac{r_{in}}{r1_{in}} + D} = \frac{(A \cdot R_{in} + B)}{C \cdot R_{in} + D}$$

For the system given

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \dfrac{-1}{f_2} & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ \dfrac{-1}{f_1} & 1 \end{pmatrix}$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{bmatrix} 1 - \dfrac{d}{f_1} & d \\ \dfrac{-1}{f_2} - \dfrac{\left(\dfrac{-1}{f_2} \cdot d + 1\right)}{f_1} & \dfrac{-1}{f_2} \cdot d + 1 \end{bmatrix}$$

which gives

$$R_{out} = \frac{\left(1 - \dfrac{d}{f_1}\right) \cdot R_{in} + d}{\left[\dfrac{-1}{f_2} - \dfrac{\left(\dfrac{-1}{f_2} \cdot d + 1\right)}{f_1}\right] \cdot R_{in} + 1 - \dfrac{d}{f_2}}$$

As R_{in} goes to infinity

$$R_{out} = \frac{1 - \dfrac{d}{f_1}}{\dfrac{d}{f_1 \cdot f_2} - \dfrac{1}{f_2} - \dfrac{1}{f_1}} = \frac{(f_1 - d) \cdot f_2}{d - (f_1 + f_2)}$$

As $f_1 + f_2$ approaches d, Rout goes to infinity. This is clearly a re-collimator or beam expander. One of the focal lengths can be negative to accomplish this

(3) $n := 3.5$

$k2d := \frac{2}{5}\cdot 2\cdot\pi$ This is the phase shift in the slab

$\theta_1 := 40\cdot deg$

$\theta_2 := \operatorname{asin}\left(\frac{\sin(\theta_1)}{n}\right)$

$\theta_2 = 0.185$ $\frac{\theta_2}{deg} = 10.583$

Since Z_0 cancels out we can write

$Z1P := \cos(\theta_1)$ $Z2P := \frac{\cos(\theta_2)}{n}$

$Z1S := \frac{1}{\cos(\theta_1)}$ $Z2S := \frac{1}{n\cdot\cos(\theta_2)}$

$$Z311P := Z2P\cdot\frac{(Z1P\cdot\cos(k2d) + i\cdot Z2P\cdot\sin(k2d))}{Z2P\cdot\cos(k2d) + i\cdot Z1P\cdot\sin(k2d)}$$

$Z311P = 0.238 + 0.267i$

$$Z311S := Z2S\cdot\frac{(Z1S\cdot\cos(k2d) + i\cdot Z2S\cdot\sin(k2d))}{Z2S\cdot\cos(k2d) + i\cdot Z1S\cdot\sin(k2d)}$$

$Z311S = 0.171 + 0.348i$

$\rho P := \frac{Z311P - Z1P}{Z311P + Z1P}$ $\rho S := \frac{Z311S - Z1S}{Z311S + Z1S}$

$\rho P = -0.426 + 0.379i$ $\rho S = -0.675 + 0.394i$

$TP := 1 - (|\rho P|)^2$ $TS := 1 - (|\rho S|)^2$

$TP = 0.675$ $TS = 0.389$

$T_{overall} := \frac{TP + TS}{2}$

$T_{overall} = 0.532$

The reflected light is clearly elliptically polarized, and contains more S than P

There is only an S-wave reflected at Brewster's angle $\frac{\operatorname{atan}(n)}{deg} = 74.055$ degrees

(5) $\lambda_0 := 1.55 \cdot 10^{-6}$

$f := 5 \cdot 10^{-3}$

$w01 := 50 \cdot 10^{-6}$ $\qquad qin1 := i \cdot \frac{\pi \cdot w01^2}{\lambda_0}$

$w02 := 200 \cdot 10^{-6}$

$A := 1 \qquad B := 0$ $\qquad qin2 := i \cdot \frac{\pi \cdot w02^2}{\lambda_0}$

$C := \frac{-1}{f} \qquad D := 1$

$$qout1 := \frac{A \cdot qin1 + B}{C \cdot qin1 + D}$$

$$qout2 := \frac{A \cdot qin2 + B}{C \cdot qin2 + D}$$

$qout1 = -2.533 \times 10^{-3} + 2.5i \times 10^{-3}$ . .his focus is at 2.533mm

$qout2 = -4.981 \times 10^{-3} + 3.072i \times 10^{-4}$ This focus is at 4.981 mm

$d := 4 \cdot 10^{-3}$ Guess

Given

$$\mathrm{Im}\left(\frac{1}{qout1 + d}\right) = \mathrm{Im}\left(\frac{1}{qout2 + d}\right)$$

$d_{circ} := \mathrm{Find}(d)$

$d_{circ} = 4.011 \times 10^{-3}$

Beam is circular at 4.011mm

(6) $n_1 := 1.46$ $n_2 := 1.455$

$$NA := \sqrt{n_1^2 - n_2^2}$$

$NA = 0.121$

Max acceptance angle is

$\theta_0 := \mathrm{asin}(NA)$

$$\frac{\theta_0}{deg} = 6.934$$

The fraction of the power from the source that is within a cone of this semiangle is

$$F := 2\cdot\pi\cdot\frac{\left(1 - \cos\left(\theta_0\right)\right)}{4\cdot\pi}$$

$$F = 3.657 \times 10^{-3}$$

Only light within this angle gets guided and will go a 100m because the fiber has negligible loss over this distance

$P_{out} := 10^{-3}\cdot F$

$P_{out} = 3.657 \times 10^{-6}$ 3.66 microwatts

$D := 20\cdot 10^{-6}$

For single mode m has to be less than 1

$\lambda_0 := 2\cdot D\cdot NA$

$\lambda_0 = 4.829 \times 10^{-6}$ Minimum wavelength is 4.83 micrometers