**ENEE 408E Optical System Design 2003**
**First Examination**

Thursday, October 30, 2003, 11:00 - 12:15 pm

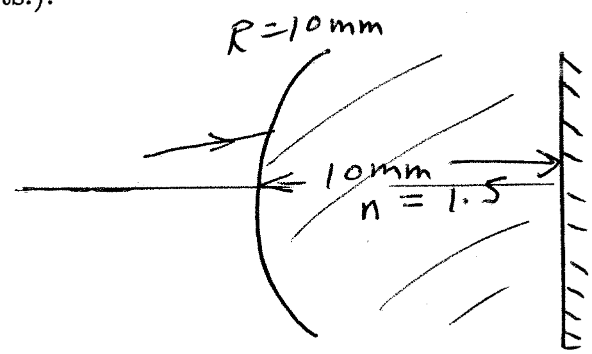
**ANSWER 3 QUESTIONS**

*(if more than 3 are answered best 3 will count)*

(1) Prove that the ray transfer matrix for a convex spherical interface between air and a glass of refractive index $n$ is

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{1-n}{nR} & 1/n \end{pmatrix} \quad \text{(5pts.)}$$

In the following system a ray enters at an angle of 1°, 2mm above the axis. The ray enters the glass, reflects off the back mirror surface and re-emerges. What are its ray parameters when it re-emerges (5pts.).




The transfer matrix for a uniform length $d$ of medium is

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$$

(2) Explain the concepts of effective focal length, back focal length and front focal length for a general optical system and how these are related to the position of the principal planes. (3pts.) Assume that the media in the object and image spaces are the same.

Prove that for an imaging system the ray transfer matrix is

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} m & 0 \\ -1/f & m' \end{pmatrix} \quad \text{(3pts.)}$$

For a particular optical system

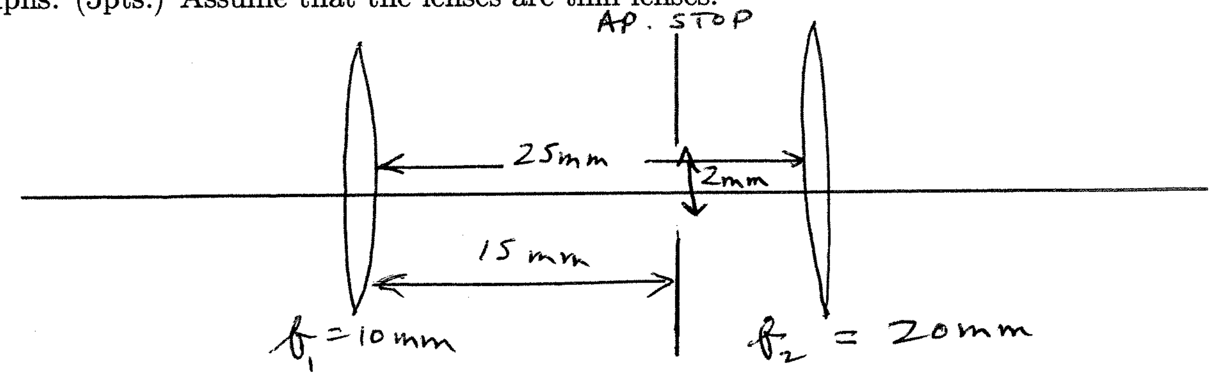
$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 2 & 5 \\ -0.1 & 0.25 \end{pmatrix}$$

An object is placed 10m to the left of the input plane of this system. Draw a ray tracing diagram to show where the image is formed. (4pts.)

Hint: $h_1 = (D-1)C$; $h_2 = (A-1)/C$.

(3) Explain the concepts of aperture stop, entrance pupil, and exit pupil in an optical system. (3pts.)

In the system below, calculate the position and diameter of both the entrance and exit pupils. (5pts.) Assume that the lenses are thin lenses.

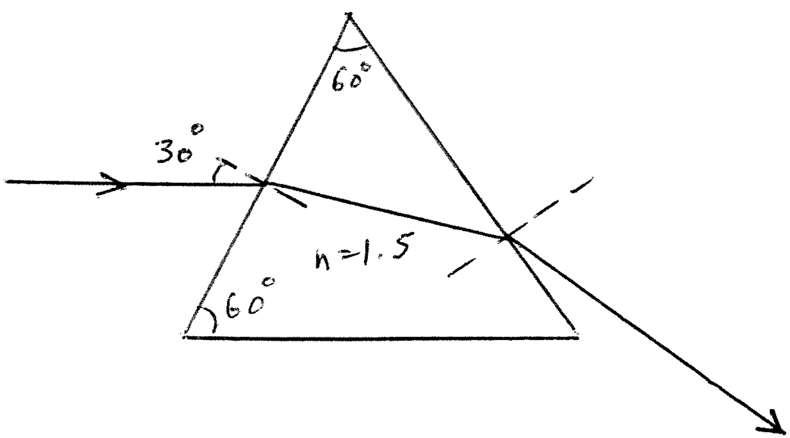



In the system above a point source of light of power 1W is placed 10m to the left of the input plane. What is the total power emerging from the output plane? (2pts.)

(4) Unpolarized light of wavelength $\lambda_0$=488nm strikes an equilateral prism at an angle of incidence of 30° as shown below and refracts again at the second face. If the index of refraction of the prism is $n$=1.5, what fraction of the incident energy emerges from the second face of the prism?

Hint: For P-waves $Z' = Z_0 \cos\theta / n$.

Don't worry about bouncing waves.



**ENEE 408E Optical System Design 2003**
**Second Examination**

Tuesday, November 25, 2003, 11:00 - 12:15 pm

**ANSWER 3 QUESTIONS**
*(if more than 3 are answered best 3 will count)*

(1) The transformed impedance of a transparent window with air ($n = 1$) on both sides is

$$Z_3'' = Z_2'\left(\frac{Z_1' \cos k_2 d' + jZ_2' \sin k_2 d'}{Z_2' \cos k_2 d' + jZ_1' \sin k_2 d'}\right).$$

**Use this result** to show that there is zero reflectance for P-waves at Brewster's angle. (5pts.)

The parameter $d'$ is the effective thickness of the window.

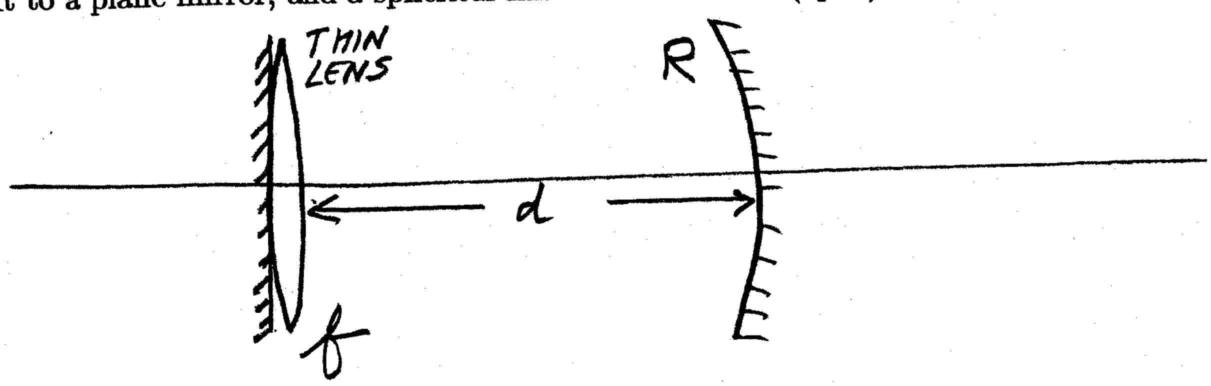
For P-waves $Z_1' = Z_0 \cos(\theta_1)$, where $\theta_1$ is the angle of incidence. A window of thickness $2\mu$m and refractive index 2 is illuminated with S-polarized *coherent* light of wavelength $\lambda_0$=1.3$\mu$m, at angle of incidence $\theta_1$. All the incident light passes through the window. What is the angle of incidence? (This is not Brewster's angle) (5pts.)

Hint: For P-waves $Z' = Z_0 \cos\theta / n$.

Brewster's angle is $\theta_B = \arctan(n_2/n_1)$.

(2) Explain exactly what it means for an optical sequence to be *stable* for input paraxial rays. (1pt.)

Derive the stability condition for the resonator shown below, which contains a thin lens next to a plane mirror, and a spherical mirror of radius $R$. (5pts.)




If for the resonator above R=1m, f=0.5m, what is the maximum value of $d$ for which the resonator is stable? (4pts.)

Hint: Sylvester's theorem provides a way of writing a $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ matrix raised to the *nth* power in terms of an angular factor $\phi$ for which $\cos\phi = (A + D)/2$.

(3) A graded index medium has

$$n(r) = n_0 \cos(\alpha r),$$

and it can be assumed that for all practical values of $r$ for which a ray remains paraxial $\alpha r \ll 1$. Derive its ray transfer matrix. (6pts.)

For such a medium of length 10mm, $n_0$=1.46, and the index 1mm from the axis is 1.45. What is the focal length of the medium. (2pts.) What is the shortest length of medium ($l$ >0) that will not change the ray parameters of an input wave? (2pts.)

Hint: The simplified equation for light rays is

$$\frac{d^2r}{dz^2}\hat{\rho} = \frac{1}{n}\text{grad}n.$$

(4) The Gaussian beam parameter $q$ is usually defined as

$$\frac{1}{q} = \frac{1}{R} - j\frac{\lambda}{\pi w^2}$$

and $q$ varies linearly in a uniform medium according to $q(z) = q(z=0) + z$. Explain what $q$ tells us about the characteristics of the Gaussian beam. (2pts.) If the beam waist of a Gaussian beam is at $z = 0$ prove that at distance $z$ from the beamwaist

$$w^2(z) = w_0^2\left[1 + \left(\frac{\lambda z}{\pi w_0^2}\right)^2\right],$$

and

$$R(z) = z\left[1 + \left(\frac{\pi w_0^2}{\lambda z}\right)^2\right]. \quad \text{(4pts.)}$$

A Gaussian beam with $w_0$=1mm and $\lambda_0$=1.55$\mu$m has its beamwaist at the input plane of an optical system with

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ -20 & 41 \end{pmatrix}$$

Where is the beamwaist of the emerging Gaussian beam relative to the output plane of the system, and what is the new minimum spotsize? (4pts.) You can assume that the image and object spaces both have $n = 1$.