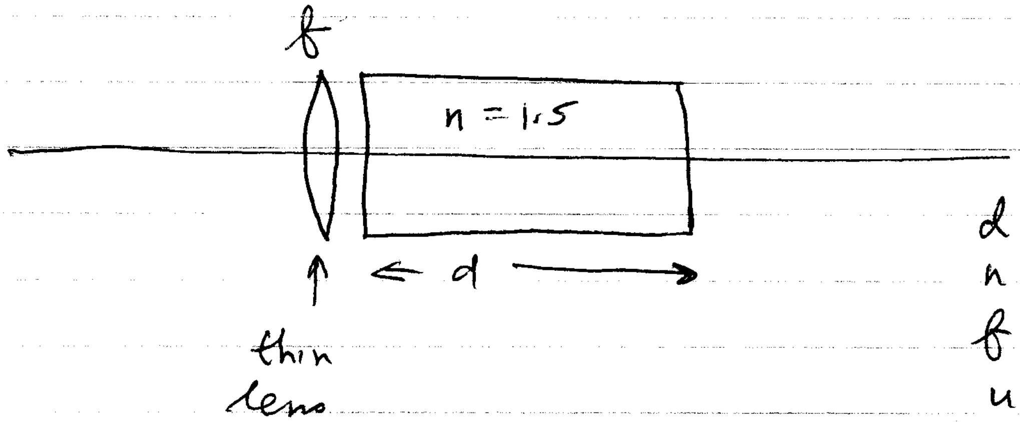
ENEE 408E FIRST EXAM

Fall 2000

SOLUTIONS

(1) For first part of question see section 14.4 of Lasers and Electro-optics



$d = 100\text{ mm}$
$n = 1.5$
$f = 100$
$u = 300$

The ray transfer matrix of the combination is

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & d/n \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} = \begin{pmatrix} 1-\frac{d}{nf} & \frac{d}{n} \\ -\frac{1}{f} & 1 \end{pmatrix}$$

Specifically $\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1-\frac{1}{1.5} & \frac{0.1}{1.5} \\ -10 & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & \frac{2}{30} \\ -10 & 1 \end{pmatrix}$

use $R_{out} = \dfrac{AR_{in} + B}{CR_{in} + D}$ $\qquad R_{in} = 0.3\text{ m}$

$$R_{out} = \frac{\frac{0.3}{3} + \frac{2}{30}}{-3 + 1} = -0.0833$$

Image is 83.3 mm from right hand face

overall matrix from object to image is

$$\begin{pmatrix} 1 & 0.0833 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \frac{1}{3} & \frac{2}{30} \\ -10 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0.3 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -0.5 & 0 \\ -10 & -2 \end{pmatrix}$$

Note that $B = 0$, this is an imaging system

Linear magnification is -0.5
Angular magnification is -2

(2) In a paraxial system the equation of light rays is

$$\frac{d}{dz}\left(n \frac{d\underline{r}}{dz}\right) = \text{grad } n$$

For $n(r) = n_0 e^{-r^2/a^2}$ $\qquad$ $\text{grad } n = \frac{-2r^2}{a^2} n \, \hat{\underline{r}}$

$$\underline{r} = r\hat{\underline{r}} + z\hat{\underline{z}}$$

$$\frac{d\underline{r}}{dz} = \frac{dr}{dz}\hat{\underline{r}} + \hat{\underline{z}}$$

$$\frac{d^2\underline{r}}{dz^2} = \frac{d^2 r}{dz^2}\hat{\underline{r}}$$

The differential equation is now

$$\frac{d^2 r}{dz^2} = \frac{-2r^2}{a^2}$$

Solution is $\qquad r = A\cos\frac{\sqrt{2}}{a}z + B\sin\frac{\sqrt{2}}{a}z$

$$r' = -\frac{\sqrt{2}}{a}A\sin\frac{\sqrt{2}}{a}z + \frac{\sqrt{2}}{a}B\cos\frac{\sqrt{2}}{a}z$$

Clearly $\quad A = r_{in} \quad . \quad \frac{\sqrt{2}}{a}B = r'_{in}$

Ray transfer matrix is

$$\begin{pmatrix} \cos\frac{\sqrt{2}}{a}z & \frac{a}{\sqrt{2}}\sin\frac{\sqrt{2}}{a}z \\ -\frac{\sqrt{2}}{a}\sin\frac{\sqrt{2}}{a}z & \cos\frac{\sqrt{2}}{a}z \end{pmatrix}$$

For $n_0 = 1.46$

$$1.455 = 1.46\, e^{-10^{-6}/a^2}$$

gives $a = 1.707 \cdot 10^{-2}$ m

So for $z = 20$ mm

$$\frac{1}{f} = \frac{\sqrt{2}}{a} \sin\left(\frac{\sqrt{2} \cdot 20\times10^{-3}}{1.707\times10^{-2}}\right) \qquad f = 417.4 \text{ mm}$$

The shortest focal length possible is

$$-\frac{1}{f} = -\frac{\sqrt{2}}{a} \qquad f = \frac{a}{\sqrt{2}} = 12.07 \text{ mm}$$

(3) $\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ -10 & 0.2 \end{pmatrix}$

Note $B=0$. This could be an imaging system

If this were a thin lens system

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & v \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} 1 & u \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1-\frac{v}{f} & v \\ -\frac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} 1 & u \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1-\frac{v}{f} & u\left(1-\frac{v}{f}\right)+v \\ -\frac{1}{f} & -\frac{u}{f}+1 \end{pmatrix}$$

$f = 0.1 \quad v = -0.4 \quad , \quad u = 0.08$

with the object 1m from the left of this system

$u = 1.08$

Using $\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = 10 - \frac{1}{1.08}$

$v = 0.1102$ m to right of thin lens.

The input angle is $r' = 5 \times 10^{-3}$

$r_{out}' = A r_{in} + B r_{in}' = 5 . 5 . 10^{-3}$

$= 25$ mrad.

Input curvature of spherical wave from point source is $R_{in} = 1\,m$

Output spherical curvature is $R_{out} = -(0.1102 + 0.4)$
$= -(0.5102)$

Check $R_{out} = \dfrac{5 + 0}{-10 + 0.2} = -0.5102$

QED

(4) For zero P-wave reflection

$$n_1 \cos\theta_2 = n_2 \cos\theta_1 \qquad (a)$$

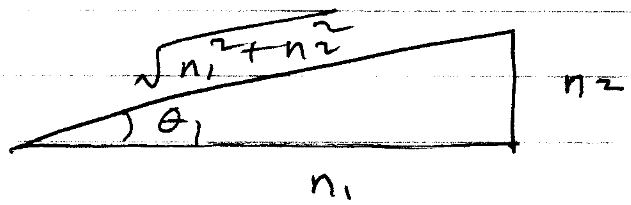
Snell's law $n_2 \sin\theta_2 = n_1 \sin\theta_1$ (b)

from (a) $n_1^2(1-\sin^2\theta_2) = n_2^2(1-\sin^2\theta_1)$

Subst. from (b) $n_1^2\left(1-\frac{n_1^2}{n_2^2}\sin^2\theta_1\right) = n_2^2(1-\sin^2\theta_1)$

$$\sin^2\theta_1 = \frac{(n_1^2-n_2^2)}{\frac{n_1^4}{n_2^2}-n_2^2} = \frac{n_2^2(n_1-n_2^2)}{n_1^4-n_2^2}$$

$$\sin^2\theta_1 = \frac{n_2^2}{n_1^2+n_2^2} \qquad \sin\theta_1 = \frac{n_2}{\sqrt{n_1^2+n_2^2}}$$



$$\tan\theta_1 = \frac{n_2}{n_1}$$

$$\rho = \frac{\frac{Z_0}{n_2}\cos\theta_2 - \frac{Z_0}{n_1}\cos\theta_1}{\frac{Z_0}{n_2}\cos\theta_2 + \frac{Z_0}{n_1}\cos\theta_1}$$

At 0° $\rho = \frac{n_1 - n_2}{n_1 + n_2}$

for $n_1 = 1$, $n_2 = n$ $\rho = \frac{1-n}{1+n}$

$$\left(\frac{1-n}{1+n}\right)^2 = 0.5 \qquad \frac{1-n}{1+n} = \pm\sqrt{0.5} \qquad 1-n = \pm\sqrt{0.5} \pm \sqrt{0.5}\,n$$

$n = 5.828$

At grazing incidence

$\theta_1 \to 90^\circ$ $\cos\theta_1 \to 0$

$\theta_2 = 9.88^\circ$

$\rho = 1$

If unpolarized light strikes at Brewster's angle the P polarization is completely transmitted. Therefore, the reflected light is S-polarized.