

ENEE 497 First Examination

Wednesday, October 2, 1996, 9:30 - 10:45 am

ANSWER 3 QUESTIONS

(if more than 3 are answered best 3 will count)

(1) Derive the ray transfer matrix for a spherical interface between two media of refractive indices n_1 , n_2 , respectively. Take the radius of the spherical surface as R . Explain the sign convention that you are using. (5pts.)

By using the ray transfer matrix you have found derive a version of the imaging equation that applies to the surface for object distance u and image distance v . (4pts.)

What is the image distance for $u=5\text{m}$, $R=1\text{m}$, $n_1 = 1$, $n_2 = 2$? (1pt.)

(2) Derive the ray transfer matrix for a thin lens. (1pt.)

Then derive the ray transfer matrix for two lenses of focal lengths f_1 and f_2 , separated by a distance d . (3pts.) What is the overall focal length of this new system? (1 pt.) What are its linear and angular magnifications? Explain what these terms mean. (2pts.)

Explain what happens to input rays of light that are parallel to the axis when $f_1 + f_2 = d$. (2pts.)

What do you think this arrangement does if $f_2 \ll f_1$ and both focal lengths are positive? (1pt.)

(3) What is the overall transmittance through a parallel-sided slab of refractive index 1.5 whose effective thickness is $3\lambda/8$ if the input light is an equal mixture of S and P polarizations and is incident at an angle of 45° . (7pts.) What is the polarization state of the reflected light? Be specific in describing the S and P reflected field components as phasors. (3pts.)

The transformed impedance formula is

$$Z_3'' = Z_2' \left(\frac{Z_3' \cos k_2 d_2' + j Z_2' \sin k_2 d_2'}{Z_2' \cos k_2 d_2' + j Z_3' \sin k_2 d_2'} \right).$$

(4) Do three of the following:

(a) Calculate the image distance for a concave mirror with $R=1\text{m}$ and $u=1.5\text{m}$. Draw a ray tracing diagram.

(b) Prove that the Brewster angle between air and glass (index n) is $\theta_B = \arctan(n)$.

(c) Explain the difference between aperture stops and field stops in an optical system.

(d) Explain qualitatively why diffraction occurs and when diffraction effects become important in an optical system.

(3pts. for each part plus a 1pt. bonus for three good answers.)

ENEE 408E OPTICAL SYSTEM DESIGN

First Examination, Thursday October 26, 2000, 11:00 - 12:15

ANSWER THREE (3) QUESTIONS - IF MORE THAN THREE ARE ANSWERED, BEST THREE WILL COUNT

(1) Explain how the principal planes and focal points of an optical system can be used for paraxial ray tracing (2pts.). Thereby prove the lensmakers equation

$$\frac{1}{l_o} + \frac{1}{l_i} = \frac{1}{f}. \quad (3\text{pts.})$$

A thin lens of focal length 100mm is followed immediately by a planar slab 100mm thick with refractive index 1.5. An object is placed 300mm to the left of the thin lens. Where is the image? (3 pts.) What are the linear and angular magnifications of the system? (2 pts.)

The ray transfer matrix for a thin lens is

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}.$$

(2) The differential equation for light rays is

$$\frac{d}{ds} \left(n \frac{d\mathbf{r}}{ds} \right) = \text{grad } n.$$

Use this equation to derive the ray transfer matrix for a length d of graded index medium with the following refractive index profile:

$$n(r) = n_0 e^{-r^2/\sigma^2}. \quad ((6\text{pts.}))$$

A cylindrically symmetric medium with such a Gaussian index profile has axial index 1.46 and index at 1mm from the axis of 1.455. What is the focal length of a medium length of 20mm? (3pts.) What is the shortest focal length that can be obtained by varying the length of this medium? (1pt.)

ENEE 488K First Examination

Monday, October 18, 1993, 9:30 - 10:45 am

ANSWER 3 QUESTIONS

(if more than 3 are answered best 3 will count)

(1) Explain the use of the principal planes and the focal points of an optical system to describe the path of principal rays through the system(2pts). Derive the ray transfer matrix for a thin lens and then use it to derive the imaging equation(3pts.)

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

For the following u, f combinations draw a ray diagram to show the location of the image(5pts.).

- (a) $u = 2, f = 1$
- (b) $u = 1, f = 2$
- (c) $u = 1, f = -2$.

Say whether each image is real or virtual.

(2) Prove that Brewster's angle θ_B for an interface between media of index n_1, n_2 , respectively, satisfies

$$\tan(\theta_B) = n_2/n_1.$$

(5pts.) Calculate the fraction of incident intensity of an S-wave that is reflected when the wave is incident on a surface at Brewster's angle. Take $n_1 = 2, n_2 = 4$ (5pts.).

Hint: $\rho = (Z'_2 - Z'_1)/(Z'_2 + Z'_1)$.

(3) The ray equation in a medium whose refractive index varies from point to point is

$$\frac{d}{ds} \left(n \frac{d\mathbf{r}}{ds} \right) = \text{grad } n.$$

A certain medium has an axially symmetric refractive index that as a function of radial distance r from the axis can be written as

$$n(r) = n_0 \exp(-r^2/\sigma^2),$$

where for paraxial rays $\sigma \gg r$ Derive an expression for the ray transfer matrix of a length d of such a medium(6pts.).

A ray of light traveling parallel to the axis enters a parallel-sided slab of a material whose index varies as described above with $n_0=1.5, \sigma=1\text{m}^2, d=20\text{mm}$. The ray strikes the first face of the slab normally. Where does the emerging ray cross the axis? (4pts.)

(4) A Gaussian beam is described by a beam parameter q where

$$\frac{1}{q} = \frac{1}{R} - \frac{i\lambda}{\pi w^2}$$

explain the significance of w and R (2pts.).

Derive expressions for the variation of R and w with distance from the beamwaist. Use the relation $q = q_0 + z$ where q_0 is the beam parameter at the beamwaist, where $R = \infty$. (4pts.)

What is the smallest radius of curvature of a Gaussian beam with $w_0 = 0.1\text{mm}, \lambda_0=810\text{nm}$?(3pts.)

What would be the smallest radius of curvature if the Gaussian beam was completely contained within a medium of refractive index=3.5?(1pt.)

ENEE 488K First Examination
 Monday, November 16, 1992, 9:30 - 10:45 am
ANSWER 3 QUESTIONS
(if more than 3 are answered best 3 will count)

(1) In a medium whose refractive index varies with position the path of a ray of light obeys the differential equation

$$\frac{d}{ds} \left(n \frac{d\mathbf{r}}{ds} \right) = \text{grad } n. \quad (1)$$

where s is distance measured along the ray and \mathbf{r} is the position vector relative to an appropriate origin. Use Eq.(1) to derive the ray transfer matrix in the paraxial approximation for a length l of a medium whose refractive index only depends on radial distance from the axis according to

$$n(r) = n_0 \exp(-\alpha r^2)$$

(2) The paraxial ray transfer matrix of a simple optical system is found to be

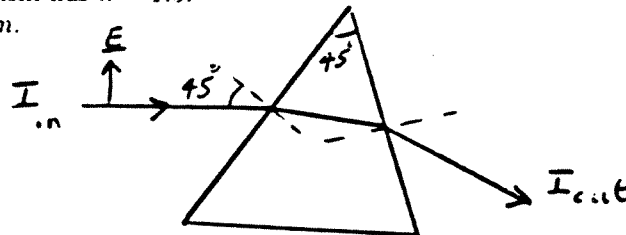
$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \equiv \begin{pmatrix} 1 & 2 \\ -0.25 & 0.5 \end{pmatrix}$$

An input ray enters this system at 10mm from the axis at a positive angle of 5° . What are the ray parameters at the output plane?

By considering what happens to rays both parallel to the axis and through the focal point, find the focal length of the system. What do you think is the simplest system that could be represented by the above ray transfer matrix?

(3) P-polarized light strikes a prism with a 45° apex angle at an angle of incidence of 45° . Calculate what fraction of the incident intensity emerges from the second face of the prism after two refractions, as shown below. The prism has $n = 1.5$.

$Z_0 = 376.7 \text{ ohm}$.

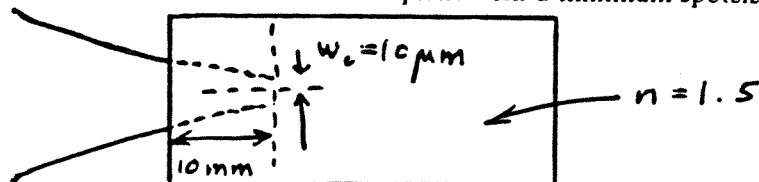


(4) A Gaussian beam is described by a beam parameter q where

$$\frac{1}{q} = \frac{1}{R} - \frac{i\lambda}{\pi w^2}$$

explain the significance of w and R .

A Gaussian beam with $\lambda_0 = 1.06 \mu\text{m}$ is being focused into a glass slab as shown below. Without the slab the beam would come to a focus 10mm from the reference plane with a minimum spotsize of $10 \mu\text{m}$.



Where is the focus, and what is the minimum spotsize when the slab is put into position?

Hint: the ray transfer matrix for an interface is:

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1/n \end{pmatrix}$$

for a ray crossing from air to glass.

ENEE 497 OPTICAL SYSTEM DESIGN

Second Examination, Tuesday, November 24, 1998, 11:00 am - 12:15 pm

ANSWER THREE (3) QUESTIONS - IF MORE THAN THREE ARE ANSWERED, BEST THREE WILL COUNT

(1) A Gaussian beam with $\lambda_0=980\text{nm}$ at a particular location in space has radius of curvature $R=-10\text{m}$, and spotsize $w=2\text{mm}$. Where is the beam waist of this beam and what is its minimum spotsize w_0 ? (6pts.) If the beam at the location where $R=-10\text{m}$ enters a medium of index $n=3$ through a planar interface where does the beam waist of the Gaussian beam in the slab move to? (4pts.)

Hint: The Gaussian beam parameter q is defined by the relation

$$\frac{1}{q} = \frac{1}{R} - j \frac{\lambda}{\pi w^2}.$$

(2) The differential equation for light rays is

$$\frac{d}{ds} \left(n \frac{d\mathbf{r}}{ds} \right) = \text{grad } n.$$

Use this equation to derive the ray transfer matrix for a length l of graded index medium with the following refractive index profile:

$$n(r) = 1 + n_1 e^{-r^2/a^2}. \quad (6\text{pts.})$$

A cylindrically symmetric medium with such a Gaussian index profile has axial index 1.6 and index at 1mm from the axis of 1.56. What is the focal length of a medium length of 10mm? (3pts.) What is the shortest focal length that can be obtained with this medium? (1pt.)

(3) Discuss three of the following:

- The Cassegrain astronomical telescope arrangement
- Connecting single-mode optical fibers together. Be specific about the problems that must be solved.
- How to make short optical pulses with a spinning prism. How would you make such pulses very short?
- The Goos-Hänchen shift.

Great mathematical detail is not required to get full marks.

(4) The variation of the radius $R(z)$ and spotsize $w(z)$ of a Gaussian beam with distance z from the beamwaist are given by

$$w^2(z) = w_0^2 \left[1 + \left(\frac{\lambda z}{\pi w_0^2} \right)^2 \right],$$
$$R(z) = z \left[1 + \left(\frac{\pi w_0^2}{\lambda z} \right)^2 \right].$$

Use these results to find the minimum spotsize and location of the beamwaist in a symmetrical resonator using 2 concave mirrors, both of radius R and spaced by distance d (6pts.). A thin lens of focal length $f = R/2$ is placed inside the resonator at its midpoint. What is now the minimum spotsize in this hybrid resonator? (4pts.)

ENEE 408E OPTICAL SYSTEM DESIGN

Second Examination, Thursday, November 30, 2000, 11-12:15

ANSWER THREE (3) QUESTIONS - IF MORE THAN THREE ARE ANSWERED, BEST THREE WILL COUNT

(1) Explain the significance of the Gaussian beam q parameter defined by

$$\frac{1}{q} = \frac{1}{R} - \frac{j\lambda}{\pi w^2}. \quad (2\text{pts})$$

Prove that for a Gaussian beam with minimum spot size w_0 at $z = 0$ that the spotsize and radius of curvature at z are

$$w^2(z) = w_0^2 \left[1 + \left(\frac{\lambda z}{\pi w_0^2} \right)^2 \right]$$

$$R(z) = z \left[1 + \left(\frac{\pi w_0^2}{\lambda z} \right)^2 \right]. \quad (4\text{pts.})$$

The minimum radius of curvature of the beam from a 780nm laser is found to be 1m. What are its minimum spot size and beam divergence? (4pts.)

(2) Given the ray transfer matrix condition $\cos \phi = \frac{1}{2}(A + D)$ derive the stability condition for a laser resonator with mirrors of radius of curvature R_1 , R_2 , and spacing d . (5pts.)

Design a confocal, symmetric resonator for $\lambda=530\text{nm}$ with a minimum spot size of 1mm. (5pts.) Hint: information given in question (1) may be useful.

(3) Explain why for a Gaussian beam passing through an optical system with ray transfer matrix

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

$$q_{out} = \frac{Aq_{in} + B}{Cq_{in} + D}. \quad (5\text{pts.})$$

If $w_0=1\text{mm}$, $\lambda_0=1\mu\text{m}$, find the focal length of a series of thin lenses that when equally spaced 500mm apart will preserve this minimum spotsize as the beam propagates (5pts.).

(4) Discuss three of the following:

- the aperture stop of an optical system, give a specific example,
- focusing a Gaussian beam with a lens,
- Fermat's principle,
- laser beam expanders.

(3) An optical system has ray transfer matrix

$$\begin{pmatrix} 5 & 0 \\ -10 & 0.2 \end{pmatrix}$$

What kind of system could this be (3 pts)?

A point source of light is placed 1m to the left of this optical system, which has an entrance pupil diameter of 10mm. The aperture stop diameter is also 10mm and is placed at the input plane of the system. Calculate the angle with respect to the axis of the extreme rays leaving the system. (3 pts.) Where is the image? (2 pts.)

Prove that these angles are consistent with the transformation of a spherical ray through the system according to

$$R_{out} = \frac{AR_{in} + B}{CR_{in} + D}. \quad (2pts.)$$

(4) Prove that when a P-wave strikes the boundary between two different transparent media of refractive indices n_1 and n_2 , respectively, that there is a certain angle of incidence θ_B for which there is no reflection. Show that this angle is given by

$$\tan \theta_B = \frac{n_2}{n_1}. \quad (4pts.)$$

A P-polarized wave strikes the boundary between 2 media of refractive indices 1 and n at 0° . For what value of n is the transmittance=0.5? (3pts.) Given the value of n that you have calculated, what is the reflectance for a grazing incidence ray $\theta_1 \rightarrow 90^\circ$? (2pts.) An unpolarized light beam strikes the interface at Brewster's angle. What is the polarization state of the reflected light? (1pt.)

ENEE 408E Final Examination

Friday, December 15, 2000. 8-10am

ANSWER 4 QUESTIONS

(if more than 4 are answered best 4 will count)

(1) Explain the use of the principal planes and the focal points of an optical system to describe the path of principal rays through the system(2pts). Derive the ray transfer matrix for a thin lens and then use it to derive the imaging equation(3pts.)

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}.$$

For the following u, f combinations draw a ray diagram to show the location of the image(5pts.).

- (a) $u = 3, f = 2$
- (b) $u = 1, f = 2$
- (c) $u = 1, f = -2$.

Say whether each image is real or virtual, and what the linear magnification is.

(2) The transformed impedance of a medium with impedance Z_2 sandwiched between media of impedances Z_1 and Z_3 is

$$Z_3'' = Z_2' \frac{Z_3' \cos(k_2 d') + j Z_2' \sin(k_2 d')}{Z_2' \cos(k_2 d') + j Z_3' \sin(k_2 d')}.$$

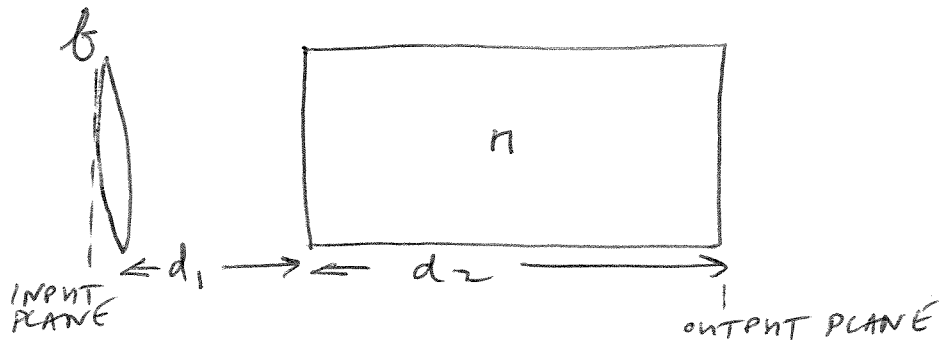
Explain the significance of the transformed impedances Z_2' and Z_3' . How is d' related to the actual thickness of medium 2. (2 pts.) Why? (1pt.)

Two media of refractive indices 1 (medium number 1) and 3.5 (medium number 3) have an antireflective layer placed on medium 3 to reduce the reflectance of the whole structure to zero at 500nm at an angle of incidence of 45° . What is the thickness and refractive index of this layer? (7pts.)

Hint: At a single boundary $\rho = (Z_2' - Z_1') / (Z_2' + Z_1')$.

(3) Using any lens or mirror combination you have worked with this semester, explain how a Code V analysis is set up. Explain the concepts of "fields", entrance pupil, entrance pupil diameter, spot diagram, and MTF. Explain how you improve the imaging performance of the optical system you have selected. (10 pts.) Careful detail must be provided to get full points.

(4) Derive the ray transfer matrix of the system shown below



Take $d_1=20\text{mm}$, $d_2 = 40\text{mm}$, $f=20\text{mm}$ $n=1.5$. If an object is placed 100 mm to the left of the input plane, where is the image? If a ray enters at the image plane with distance from the axis of 1mm, angle 2 degrees, what are the output ray parameters?

(5) Discuss three of the following

- (a) How a photomultiplier works
- (b) The detectivity of an optical detector
- (c) The Numerical Aperture (NA) of an optical fiber
- (d) The glass chart

(6) Explain how a photovoltaic detector works. Draw a simple circuit that could be used for such a detector in a reverse biased mode (where zero voltage appears across the photodiode).

The contributions to noise in a p-i-n photodiode are

Shot noise: $\langle i_n^2(f) \rangle = 2e\bar{i}\Delta f$.

Johnson Noise: $\langle i_n^2(f) \rangle = 4kT/R$.

A particular p-i-n diode has responsivity of 0.5 A/W at 780nm. What is its NEP ($\text{W}/\sqrt{\text{Hz}}$) when $T=300\text{K}$, $R=50\text{ ohm}$?

Useful information: Boltzmann's constant: $k=1.38 \times 10^{-23}\text{ J/K}$,

Planck's constant: $h=6.626 \times 10^{-34}\text{ Js}$, velocity of light: $c_0=3 \times 10^8\text{ m/s}$, magnitude of the electronic charge: $e=1.6 \times 10^{-19}\text{ C}$.

ENEE 408E OPTICAL SYSTEM DESIGN

Final Examination, Friday, December 14, 2001, 8:00 - 10:00 am

ANSWER FOUR(4) QUESTIONS - IF MORE THAN FOUR ARE ANSWERED, BEST FOUR WILL COUNT

(1) Explain the use of principal planes and focal points to trace principal rays through an optical system. (3pts.) Use these ideas to show that in an imaging system the following relation holds. (2pts.)

$$\frac{1}{l_o} + \frac{1}{l_i} = \frac{1}{f}$$

For the following thin lens combinations draw a ray tracing diagram to locate the image. In each case give the image distance v , say whether the image is real or virtual and give the angular magnification. (5pts.)

(a) $f=1, u=0.5$

(b) $f=-3, u=4$

(c) $f=2, u=3$

(2) Derive the ray transfer matrix for a single spherical surface of radius R between two media of refractive indices n' and n . (4pts.) What are the two focal lengths for the surface? (3pts.)

A system has ray transfer matrix

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}$$

A ray enters the system at a distance of 10mm from the axis at an angle of -6° . What are the ray parameters at the output plane? (3pts.)

(3) Discuss three of the following:

(a) Faraday isolators

(b) GRIN lenses

(c) Intensity, brightness, and irradiance

(d) Photomultiplier tubes

(4) A plane wave strikes the planar boundary between air and a glass with $n=1.6$ at an angle of incidence of 45° . The incident light contains equal amounts of S and P polarizations. Calculate:

(a) The S and P wave reflection coefficient, overall reflectance and overall transmittance. (5pts.)

(b) The polarization state of the reflected light. (3pts.)

(c) What would the angle of incidence need to be for all the reflected light to be S polarized? (2pts.)

(5) Prove that when a spherical wave passes through an optical system with ray transfer matrix

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

that

$$R_{out} = \frac{AR_{in} + B}{CR_{in} + D}. \quad 3\text{pts.}$$

Explain why the Gaussian beam parameter obeys a similar equation (1pt.).

A Gaussian beam has a minimum spotsize of w_0 at $z = 0$, and a wavelength λ . At a distance z from the beam waist

$$w^2(z) = w_0^2 \left[1 + \left(\frac{\lambda z}{\pi w_0^2} \right)^2 \right],$$

$$R(z) = z \left[1 + \left(\frac{\pi w_0^2}{\lambda z} \right)^2 \right].$$

Sketch the behavior of R with distance from the beamwaist (1pt.). At what distance from the beamwaist does the radius of curvature reach its minimum value? (2pts.) Design a hemispherical resonator with $d=100\text{mm}$ to give $w_0=2\text{mm}$ for a wavelength of 632.8nm (3pts.)

Hint: The Gaussian beam parameter q is defined by the relation

$$\frac{1}{q} = \frac{1}{R} - j \frac{\lambda}{\pi w^2}.$$

(6) Explain the difference between shot noise and Johnson noise (3pts.)

The contributions to noise in a photodiode are

$$\text{Shot Noise: } \langle i_{N_1}^2(f) \rangle = 2e\bar{i}\Delta f.$$

$$\text{Johnson Noise: } \langle i_{N_2}^2(f) \rangle = 4kT\Delta f/R.$$

A particular avalanche photodetector (APD) has a quantum efficiency of 0.8 at $1.3\mu\text{m}$, a current gain of 10, and a noise factor of 5. What is its NEP ($\text{WHz}^{-1/2}$) when $T=300\text{K}$, $R=50\text{ohm}$? Neglect dark current (4pts.)

If this APD is being used to detect an optical signal with a power of $1\mu\text{W}$ at $1.3\mu\text{m}$ with a bandwidth of 1GHz , what is the signal to noise ratio (3pts.).

Hint: Responsivity $R = e\eta/h\nu$

(7) Explain how the indicatrix of a uniaxial crystal can be used to describe the propagation of light through such a crystal (3pts.).

An E-ray is traveling at an angle of 45° to the optic axis in a crystal with $n_o=1.5$, $n_e=2.0$. What is the direction of the E-wave and what refractive index does it see? (5pts.) If the O and E-waves are parallel in the crystal, what is the minimum thickness of crystal used in this way that will constitute a $\lambda/4$ plate? (2pts.) Take $\lambda_0=1.3\mu\text{m}$.

The following result may be useful:

$$n_e(\theta) = \frac{n_o n_e}{(n_e^2 \cos^2 \theta + n_o^2 \sin^2 \theta)^{1/2}}.$$

Useful constants: $e=1.6 \times 10^{-19}$ Coulombs

$$h=6.626 \times 10^{-34} \text{ Js}$$

$$k=1.38 \times 10^{-23} \text{ J/K.}$$

ENEE 497 OPTICAL SYSTEM DESIGN

Final Examination, Tuesday, December 16, 1997, 1:30 - 3:30 pm

ANSWER FOUR (4) QUESTIONS - IF MORE THAN FOUR ARE ANSWERED, BEST FOUR WILL COUNT

(1) Explain how principal planes and focal points of an optical system allow the paths of paraxial rays through the system to be determined. (2pts.)

Derive the ray transfer matrix for a thin lens of focal length f followed immediately by a slab of length d , refractive index n , and then a second lens identical to the first. (3pts.)

What is the overall focal length of this combination? (1pt.) (Do not derive the transfer matrix for a thick lens - there is an easier way.)

An optical system has $n=1$ in both the object and image spaces, a ray enters this system at a distance of 10mm from the axis and at an angle of 5° . It emerges at a distance of 50mm from the axis and at an angle of -5° . An input ray at 10mm from the axis that is parallel to the axis emerges 20mm from the axis. What is the ray transfer matrix for the system? (4pts.)

(2) Prove that for a P-wave striking the planar boundary between two media of refractive indices n_1, n_2 , respectively, at angle of incidence θ_1 the reflection coefficient is (2pts.)

$$\rho = \frac{Z_2 \cos \theta_2 - Z_1 \cos \theta_1}{Z_2 \cos \theta_2 + Z_1 \cos \theta_1}$$

A linearly polarized wave that is 60% S-wave and 40% P-wave strikes such a boundary at 30° when $n_1=2, n_2=3$. What fraction of the input energy is transmitted? (4pts.) What is the polarization state of the reflected light? (2pts.)

What would be the characteristics of a single layer (thickness and refractive index) that could be placed on the surface to reduce the P-wave reflectance to zero? (2pts.)

(3) Explain the significance of the spotsize $w(z)$ and the radius of curvature $R(z)$ of a Gaussian beam (2pts.)

A Gaussian beam has a minimum spotsize of w_0 at $z = 0$, and a wavelength λ . At a distance z from the beam waist

$$w^2(z) = w_0^2 \left[1 + \left(\frac{\lambda z}{\pi w_0^2} \right)^2 \right],$$
$$R(z) = z \left[1 + \left(\frac{\pi w_0^2}{\lambda z} \right)^2 \right].$$

At what distance from the beamwaist does the radius of curvature reach its minimum value? (2pts.) Use the equations above to calculate the minimum spot size in a symmetrical resonator with two concave mirrors of radius 3m spaced by 2m when the wavelength is 633nm. (4pts.) Also use these results to calculate the maximum spacing at which the resonator would be stable. (2pts.) (Do not use the stability condition equation to solve this part).

(4) Do three of the following:

(a) An optical detector has a responsivity of 0.5A/W at 780nm . What is its quantum efficiency? $h=6.626\times 10^{-34}\text{ Js}$, $e=1.6\times 10^{-19}\text{C}$.

(b) Explain the concept of retardation produced by a birefringent crystal.

(c) Why does a photoemissive material have a response that peaks at a particular wavelength?

(d) What is the overall fraction of transmitted light in the following situation:

A linearly polarized wave strikes a linear polarizer 35° from its preferred direction. The wave then strikes a second linear polarizer that has its preferred direction 55° rotated from the first polarizer.

(5) What is the numerical aperture of a step index optical fiber with $n_1=1.53$, $n_2=1.5$? (3pts.) If such a fiber with a core diameter of $20\mu\text{m}$ is illuminated with light from a point source of power 10mW placed 30mm from the end of the fiber what power will be guided in the fiber? (3pts.) If a Gaussian beam of power 10mw is focused to a spot size of $20\mu\text{m}$ on the center of the core, what approximate power is guided into the fiber? (4pts.) You may assume that the Gaussian beam has a wavelength for which the fiber is single mode.

(6) Prove that the refractive index seen by an E-wave propagating at angle θ to the optic axis in a uniaxial crystal is (3pts.)

$$n_e(\theta) = \frac{n_o n_e}{(n_e^2 \cos^2 \theta + n_o^2 \sin^2 \theta)^{1/2}}$$

A uniaxial crystal has $n_o=1.7$, $n_e=1.5$. An unpolarized wave enters a slab of this crystal material at an angle of incidence of 0° . After entering the crystal the wave is traveling at an angle of 40° to the optic axis. The crystal is 20mm thick. Calculate the lateral separation of the emerging O- and E- rays. (7pts.)

Hint: First calculate the angle between the E-ray and E-wave vector.

The indicatrix of the crystal is

$$\frac{x^2 + y^2}{n_o^2} + \frac{z^2}{n_e^2} = 1$$

ENEE 488K FINAL EXAMINATION
Thursday, December 16, 1993. 1:30 - 3:30 pm

ANSWER 4 QUESTIONS

(if more than 4 are answered best 4 will count)

(1) An optical system has ray transfer matrix

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} .25 & 5 \\ -.15 & 1 \end{pmatrix}$$

What is the focal length of the system? (1 pt.) A ray of light enters this system 0.01m from the axis at an angle of 2° . What are its parameters when it leaves? (3 pts.) With what parameters must the ray enter the system to leave 0.005m from the axis and parallel to the axis? (3pts.) What type of optical system could this be? (3pts.)

(2) A plane wave in a semiconductor of refractive index 3 strikes the plane boundary with a second semiconductor of refractive index 4 at an angle of incidence of 50° . The wave is circularly polarized. What fraction of the intensity is transmitted? (6 pts.) What is the polarization state of the reflected light? (2 pts.) At what angle would the wave need to be incident for the reflected light to be linearly polarized? (2 pts.)

(3) Discuss three of the following:

- (a) Material dispersion in an optical fiber
- (b) How to minimize modal dispersion in a graded-index optical fiber
- (c) Photoconductive detectors
- (d) Anti-reflection layers

(4) A Gaussian beam leaves a small beam waist of size w_0 and expands towards a thin lens of focal length f . The emerging beam is found to have a plane phase front immediately after the lens. What is the spot size at this point (3 pts.) and how far from the lens was the original beam waist? (3 pts.)

What fraction of the beam will pass through the lens if $w_0 = 10\mu\text{m}$, and the beam waist is 100mm from a lens of aperture diameter 5mm? (4 pts.)

The following equation may be useful:

$$w^2(z) = w_0^2 \left[1 + \left(\frac{\lambda z}{\pi w_0^2} \right)^2 \right]$$

(5) Derive an expression for the numerical aperture (N.A.) of a step-index optical fiber with core index n_1 and cladding index n_2 . (3 pts.)

For a step-index fiber to be single mode, its core size must be sufficiently small. A higher order mode can exist in the fiber if its zig-zag path satisfies a phase condition. This phase condition is that the effective thickness perpendicular to the axis must be such that a round

trip in the lateral direction is a multiple of 2π . Derive an expression for the maximum thickness $2d$ of a slab waveguide that will be single mode. (3 pts.)

A point source of light of power 1mW is placed right against the end of a single-mode fiber on axis. The fiber has $n_1 = 1.6$, $n_2 = 1.5$, and the core diameter is $10\mu\text{m}$. What power is guided into the fiber? (4 pts.)

(6) Explain what happens when unpolarized light enters a uniaxial crystal travelling at angle θ to the optic axis. (3 pts.)

The index surfaces in a uniaxial crystal describe the index of ordinary and extraordinary waves *in their direction of propagation*. Derive expressions for these surfaces in terms of n_o and n_e . (4 pts.)

A uniaxial crystal slab with parallel faces is $10\mu\text{m}$ thick *exactly*. It is cut so that its faces are parallel to the optic axis. In the slab $n_o=2$, $n_e=3$. Light of wavelength 633nm enters the slab travelling perpendicular to the slab faces. What retardation is produced by the slab? (2 pts.) How much thicker would the slab need to be to act as a $\lambda/2$ plate? (1 pt.)

ENEE 488K Final Examination

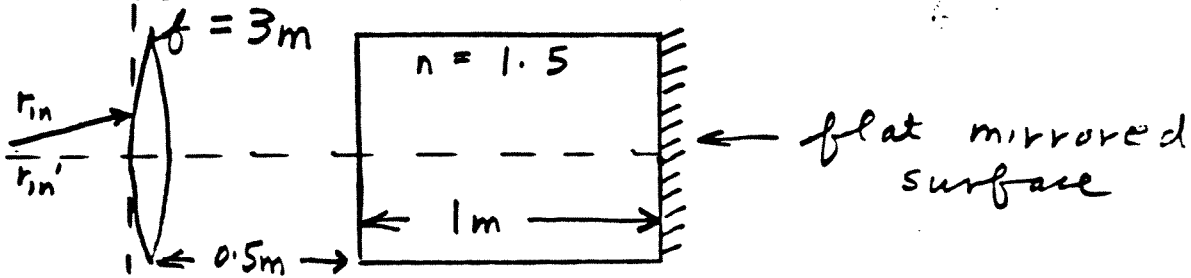
Tuesday, December 15, 1992, 1:30 - 3:30 pm

ANSWER 4 QUESTIONS

(if more than 4 are answered best 4 will count)

- (1) Derive the ray transfer matrices for
 - (a) An interface between air and glass (ref. index n)
 - (b) A length d of uniform medium
 - (c) A thin lens [Hint: $\det(M) = 1$]

In the arrangement below an input ray enters with $r_{in} = 0.1m$, $r'_{in} = 0.01$ rad. What are its ray parameters when it returns to the input plane?

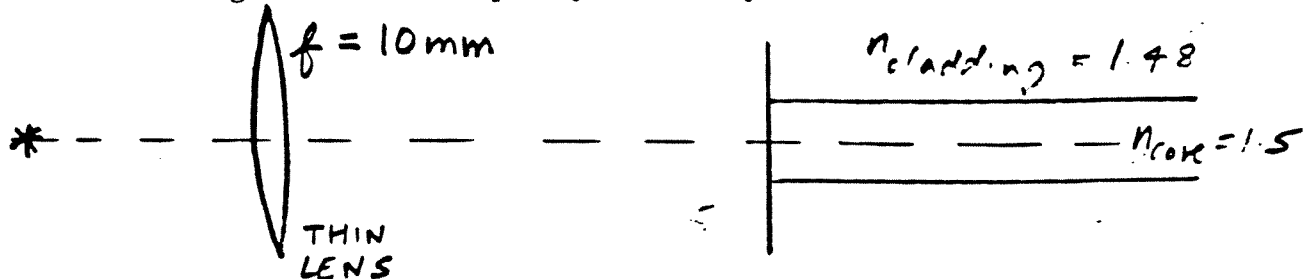


- (2) Discuss three of the following:
 - (a) The Fraunhofer diffraction pattern of a small circular aperture
 - (b) Material dispersion
 - (c) LEDs
 - (d) Spatial filters

[Substantial mathematics is not necessary to obtain full marks, but answers must contain significant relevant information.]

- (3) A circular cross-section, step-index optical fiber has: $n_{core} = 1.5$, $n_{cladding} = 1.48$, $r_{core} = 25\mu m$. Calculate the numerical aperture of the fiber. 5

In the arrangement below what is the maximum fraction of the light that can be injected into the fiber and be guided? Use a simple ray model for your answer.



[Hint: The relative placement of lens, fiber and source can be adjusted.]

- (4) In a uniaxial crystal the permitted polarization directions and refractive indices for waves traveling in the crystal can be described with the aid of the indicatrix. Explain how this is done. 3

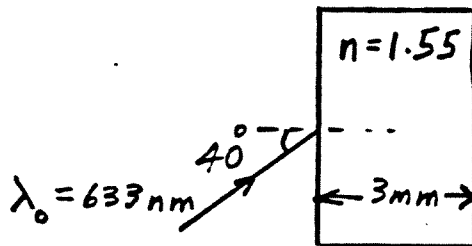
In a crystal with $n_e = 1.5$, $n_o = 2$, a ray is traveling at an angle of 45° to the optic axis. What is the direction of the wave vector and what value of $n_e(\theta)$ does this wave see?

(5) A Gaussian beam is being focused to a point d away from a thin lens of focal length f . The incident Gaussian beam has a spotsize w_1 and radius of curvature R_1 at the lens, which can be assumed to be far from the beam waist. Calculate the value of d and the spotsize w_f of the focused spot.

What are the values of d and w_f if $R_1 = 10\text{m}$, $w_1 = 10\text{mm}$, and $\lambda_0 = 1.06\mu\text{m}$? $f = 100\text{mm}$

$$\frac{1}{q(z)} = \frac{1}{R(z)} - i\left(\frac{\lambda}{\pi w^2(z)}\right).$$

(6) What is the overall transmittance through the slab arrangement shown below if the incident light is coherent and is an equal mixture of S and P polarizations.



What is the polarization state of the reflected light?

The transformed impedance formula is

$$Z_3'' = Z_2' \left(\frac{Z_3' \cos k_2 d' + i Z_2' \sin k_2 d'}{Z_2' \cos k_2 d' + i Z_3' \sin k_2 d'} \right).$$