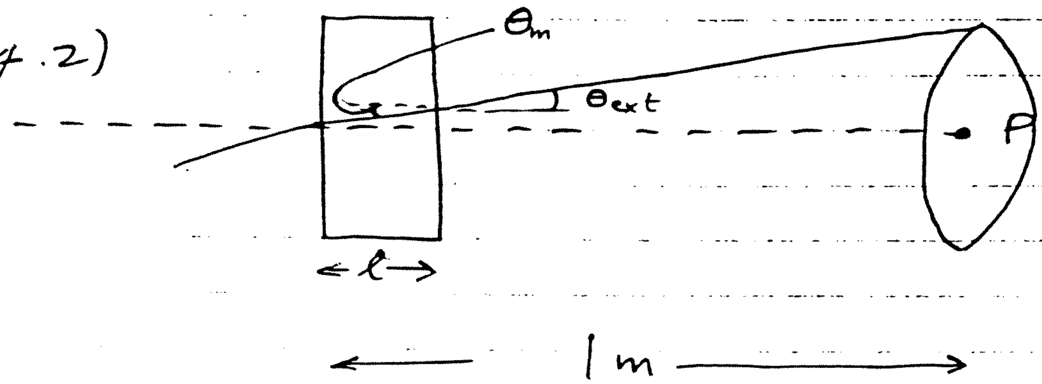
(4.2)



Internal angles for rings satisfy $\cos\theta_m = \dfrac{m c_0}{2n\ell\nu}$

For center of pattern $\dfrac{\text{"}m\text{"} c_0}{2n\ell\nu} = 1$

$\therefore \text{"}m\text{"} = \dfrac{2n\ell\nu}{c_0} = \dfrac{2n\ell}{\lambda_0} = \dfrac{3\times 3\times 10^{-3}}{510.6\times 10^{-9}}$

$\text{"}m\text{"} = 17626.32197$

$\therefore$ integers for 1st 3 rings are $\underset{(m)}{17626}, \underset{(m-1)}{17625}, \underset{(m-2)}{17625}$

$\cos\theta_m = \dfrac{17626\times 510.6\times 10^{-9}}{9\times 10^{-3}}$ $\theta_m = 0.00604$ rad $= 0.346^\circ$

$\cos\theta_{m-1} = \dfrac{17625\times 510.6\times 10^{-9}}{9\times 10^{-3}}$ $\theta_{m-1} = 0.01225$ rad $= 0.702^\circ$

$\cos\theta_{m-2} = \dfrac{17624\times 510.6\times 10^{-9}}{9\times 10^{-3}}$ $\theta_{m-2} = 0.0162$ rad $= 0.93^\circ$

Ring radii are $\tan\theta_{ext} \times 1$; $\theta_{ext} = n\,\theta_m$

$\therefore$ Ring radii are 9.06 mm, 18.38 mm, 24.3 mm

to get a bright spot at the center adjust thickness so "m" = 17626 exactly

$\ell = \dfrac{m\lambda_0}{2n\ell}$ $\Delta\ell = \dfrac{\Delta m\,\lambda_0}{2n} = \dfrac{-0.32197\times 510.6\times 10^{-9}}{3}$

$= -54.8\text{ nm}$

(4.3) (i) For a bright spot at the center of the pattern $\frac{m\lambda_0}{2n\ell} = 1$ with m an integer

In this case

$$\text{"}m\text{"} = \frac{2n\ell}{\lambda_0} = \frac{2 \times 1.55 \times 7.4 \times 10^{-3}}{488.79 \times 10^{-9}} = 46932.22038$$

change thickness so m is 46932 exactly (you could also move to m = 46933, but this would be a larger thickness change)

for m = 46932

$$\Delta \ell = -\frac{0.22038 \times 488.79 \times 10^{-9}}{2 \times 1.55} = \underline{34.75\,\text{nm}}$$

THINNER

Therefore cool the etalon by ΔT where

$$\frac{\Delta \ell}{\ell} = \alpha \, \Delta T \quad \text{with } \alpha = 3 \times 10^{-6}\ K^{-1}$$

$$\Delta T = \frac{34.75 \times 10^{-9}}{3 \times 10^{-6} \times 7.4 \times 10^{-3}} = \underline{1.565 \text{ degrees } K}$$

COOL BY THIS AMOUNT

(ii) If there is only one ring then there is not a 2nd ring. To get a 2nd ring use m = 46931 and $\cos\theta_m = \frac{m\lambda_0}{2n\ell}$

gives $\cos\theta_m = \frac{46931 \times 488.79 \times 10^{-9}}{2 \times 1.55 \times 7.4 \times 10^{-3}} \Rightarrow \theta_m = 0.00721$ rad $= 0.413$ degrees

This is the internal angle in the etalon outside the etalon the beam divergence angle θ_{ext} must satisfy

$$\theta_{ext} < n \sin \theta_{m=46931}$$

$$\underline{\theta_{ext} < 0.64^{\circ}}$$

(iii) $\lambda_0 = 489.32$ nm is also present

For this wavelength

$$\text{"}m\text{"} = \frac{2 \times 1.55 \times 7.4 \times 10^{-3}}{489.32 \times 10^{-9}} = 46881.38641$$

This integer is not within one of the "m" for $\lambda_0 = 488.79$ nm, so the rings do not appear within the same order

Ring 46932 of $\lambda_0 = 488.79$nm is adjacent to ring 46881 of $\lambda_0 = 489.32$nm

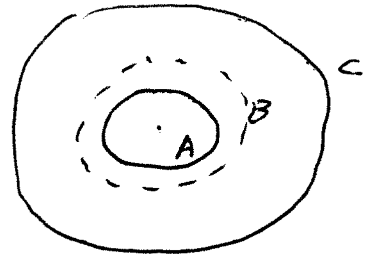
The first ring A of $\lambda_0 = 488.79$nm has $\theta_m = 0.003065$ rad

second ring C of $\lambda_0 = 488.79$nm has $\theta_{m-1} = 0.00721$ rad

First ring B of $\lambda_0 = 489.32$nm has $\theta_m = 0.00406$ rad

The rings at $\theta_m = 0.003065$ and $\theta_m = 0.00406$ are closest and must be distinguished

Pattern looks like



To distinguish between these 2 rings their spacing must be equivalent to $\Delta\nu_{1/2} = \dfrac{\Delta\nu_{FSR}}{F}$

$$= \frac{c/2l}{\pi\sqrt{R}/(1-R)}$$

For $\lambda_0 = 489.32\text{nm}$ to overlap $\lambda_0 = 488.79\text{m}$ we would need an effective "m" of

$$\text{"}m\text{"} = \frac{2nl\cos\theta_m}{\lambda_0} = \frac{2\times1.55\times7.4\times10^{-3}\times\cos(0.00406)}{488.79}$$

$$= 46931.83358$$

The effective "Δm" $= 46932 - 46931.83358$
$= 0.1664$

$\Delta m = 1$ is equivalent to $\Delta\nu_{FSR}$

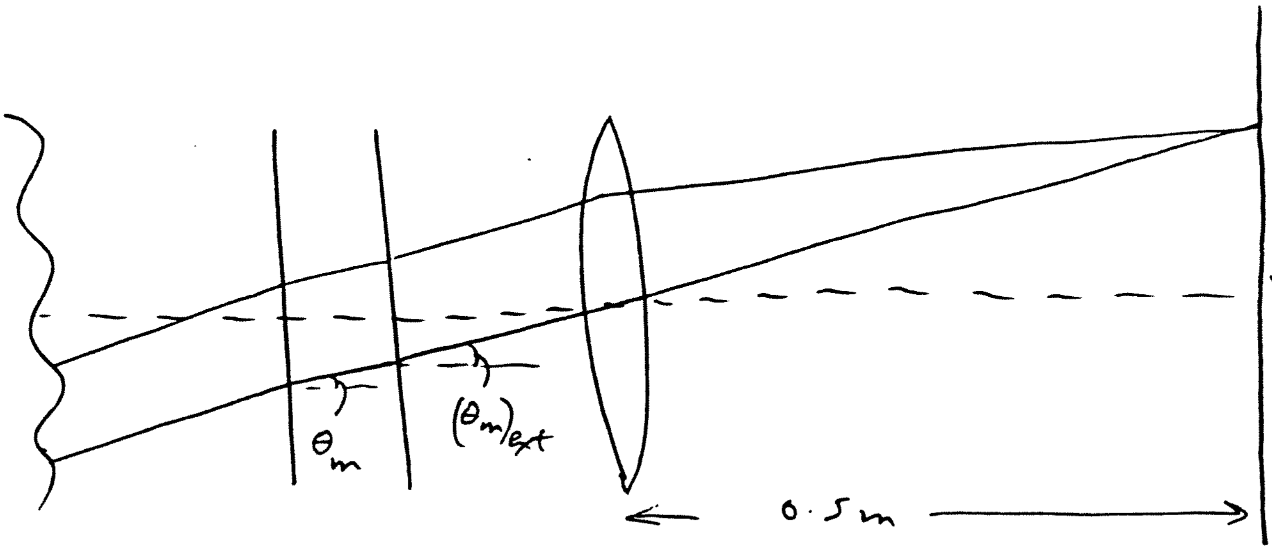
Therefore $\Delta m = 0.1664$ is equivalent to $0.1664\,\Delta\nu_{FSR}$

Therefore $\dfrac{1}{F} = 1\Big/\dfrac{\pi\sqrt{R}}{1-R} = 0.1664$

$R = 0.596$

59.6% reflectance

(4.4)



find lowest ring radius number

$$m = \frac{2nl}{\lambda_0} = \frac{3 \times 2 \times 10^{-3}}{510.554 \times 10^{-9}} = 11751.94$$

$m = 11751$

to get rings through aperture on screen

$$(0.5\,m) \times \tan(\theta_m')_{ext} \leq 30\,mm$$

$$(\theta_m')_{ext} < 3.4336^\circ$$

$$\theta_m' < 2.289^\circ \quad \left(\text{small angles}\right)$$

find integer corresponding to 2.289°

$\cos\theta_m' = \dfrac{m'\lambda_0}{2nl}$ gives $m' = 11742(.562)$

ie ring 11743 goes through, 11742 does not

Therefore 9 rings go through

To get a bright spot at center move to $m = 11752$

$$\ell = \frac{m\lambda_0}{2n\ell} \qquad n = \frac{m\lambda_0}{2\ell} = \frac{11752 \times 510.559 \times 10^{-9}}{\times 10^{-3}}$$

$$= 1.500007652$$

$$\underline{\Delta n = 7.652 \times 10^{-6}}$$

(5.1) $R := 10^{24} \cdot 0.5$ Pumping rate of upper laser level is equal to pump rate of pump band times transfer efficiency

$\tau_2 := 7 \cdot 10^{-4}$ Lifetime of upper laser level

$N_2 := R \cdot \tau_2$ Steady state population of upper laser level

$A_{21} := 10^3$ $\lambda_0 := 10^{-6}$ $\Delta\nu := 10^9$

$n := 1.6$ refractive index

$\lambda := \frac{\lambda_0}{n}$

The gain at line center of the homogeously broadened line is

$$\gamma_0 := \frac{N_2 \cdot \lambda^2 \cdot A_{21}}{8 \cdot \pi} \cdot \frac{2}{\pi \cdot \Delta\nu}$$

$\gamma_0 = 3.463$ gain at line center in units of m^-1

$\gamma_t = \alpha - \frac{1}{l} \cdot \ln(r_1 \cdot r_2)$ threshold gain

$R_1 := 1$ $l := 20 \cdot 10^{-3}$

$r_1 := 1$

$\alpha := 0$

$\gamma_t = -\frac{1}{l} \cdot \ln\left(\sqrt{R_2}\right)$ $\gamma_t := \gamma_0$

$$R_2 := \exp\left(-\gamma_t \cdot l\right)^2$$

$R_2 = 0.871$

Minimum mirror reflectance is 87.1%

(5.2) The inhomogeneous gain lineshape is

$$\gamma(\nu) = \gamma_0 \cdot e^{-4\cdot(\nu - \nu_0)^2 \cdot \ln(2)} \qquad \text{Eq.(a)}$$

where γ_0 is the gain at line center

$\alpha := 0.001$ $\quad l := 1$ $\quad R_1 := 0.99$ $\quad R_2 := 0.99$ $\quad \gamma_0 := 1$

$r_1 := \sqrt{R_1}$ $\quad r_2 := \sqrt{R_2}$ $\quad \Delta\nu_D := 3\cdot 10^9$ $\quad c_0 := 2.998\cdot 10^8$

The threshold gain is

$$\gamma_t := \alpha - \frac{1}{l}\cdot \ln(r_1 \cdot r_2)$$

$$\gamma_t = 0.011$$

Re-write Eq.(a) as $\quad \gamma_t = \gamma_0 \cdot e^{-4\cdot\Delta\nu_t^2 \cdot \ln(2)}$

where $\Delta\nu_t$ is the frequency spacing from line center in units of $\Delta\nu_D$ at which the gain has dropped down to γ_t

$$\Delta\nu_t := \left[\frac{\ln\left(\frac{\gamma_0}{\gamma_t}\right)}{4\cdot\ln(2)}\right]^{\frac{1}{2}}$$

$$\Delta\nu_t = 1.275$$

$$\Delta\nu_t := \Delta\nu_t \cdot \Delta\nu_D$$

The mode spacing in the laser resonator is

$$\Delta\nu_{FSR} := \frac{c_0}{2\cdot l}$$

$\Delta\nu_{FSR} = 1.499\cdot 10^8$ $\qquad$ 150MHz mode spacing

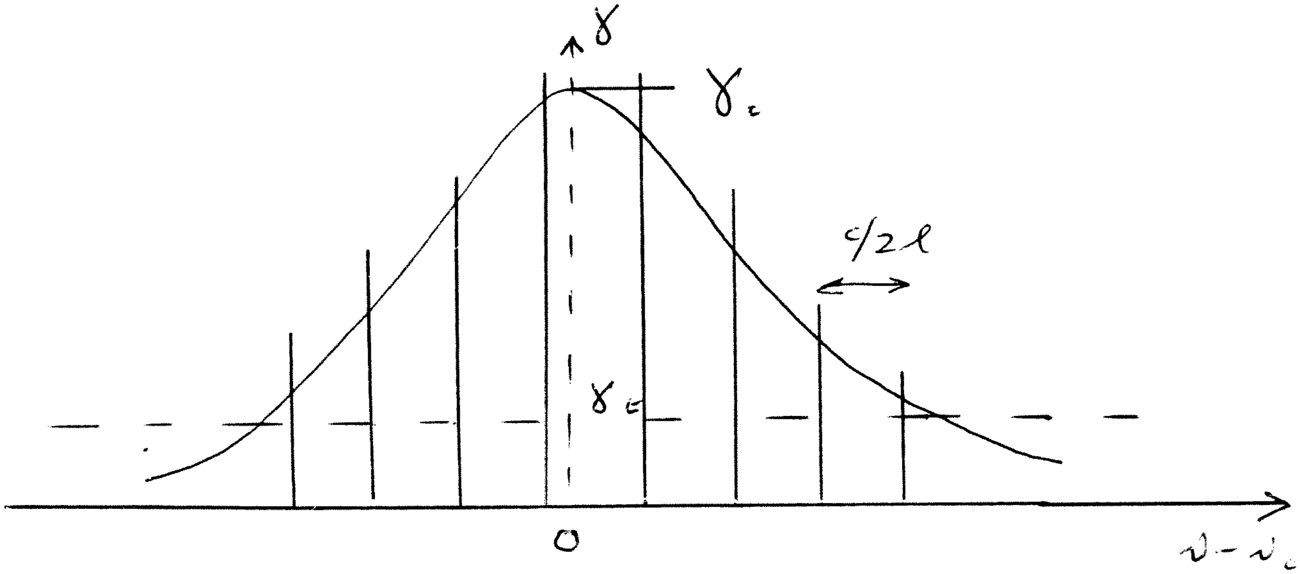
Number of mode spacings that fit under gain curve for gains greater than γ_t is

$$N := \frac{\Delta\nu_t}{\Delta\nu_{FSR}}$$

$$N = 25.512$$

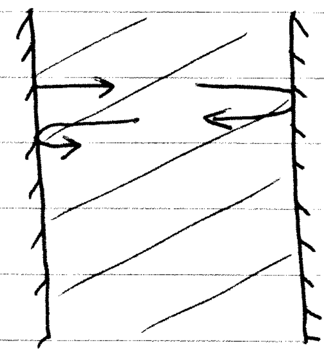
N = 25.512

26 modes should oscillate



In this example 7 mode spacings fit for $\gamma > \gamma_t$ $\rightarrow$ 8 modes oscillate

(5.4)



Intensity goes from $I_0 \rightarrow A I_0$ one one round trip

Since Electric Field amplitude $\propto \sqrt{\text{Intensity}}$ the electric field amplitude changes by a factor of $\sqrt{A}$ per round trip, excluding the additional loss through each mirror. Therefore, per pass, the electric field amplitude changes by a factor of $A^{1/4}$

Eq. (4.20) is modified in this case to be

$$E_t = E_0 e^{-i\delta_0} \tau\tau' A^{1/4} \left(1 + \rho'^2 \sqrt{A} e^{-i\delta} + \rho'^4 A e^{-2i\delta} + \dots\right)$$

it is as if the reflection coefficient is changed from ρ' to $\rho' A^{1/4}$

Eq. (4.22) becomes $E_t = \dfrac{E_0 e^{-i\delta_0} \tau\tau' A^{1/4}}{1 - R\sqrt{A}\, e^{-i\delta}}$

$$\frac{I_t}{I_i} = \frac{T^2 \sqrt{A}}{(1-R\sqrt{A})^2 + 4R\sqrt{A} \sin^2\left(\frac{\delta}{2}\right)}$$

The peak transmission is $\dfrac{(1-R)^2 \sqrt{A}}{(1-R\sqrt{A})^2}$

Note that Eq. (4.49) is a different result because for this result the intensity loss <u>per single pass</u> was $I_0 \rightarrow A I_0$

(5.6) Define various quantities

$l := 1$ $c_0 := 2.997 \cdot 10^8$ $\lambda_0 := 632.8 \cdot 10^{-9}$ $n := 1.0001$ $\Delta v := 10^8$

$R := 0.99$

$c := \frac{c_0}{n}$

$\Delta v_{FSR} := \frac{c}{2 \cdot l}$ Cavity mode spacing

$F := \pi \cdot \frac{R}{(1 - R)}$ Finesse of cavity

$F = 311.018$

$\Delta v_{half} := \frac{\Delta v_{FSR}}{F}$

$\Delta v_{half} = 4.818 \cdot 10^5$ $\Delta v_{FSR} = 1.498 \cdot 10^8$

The spacing between two oscillating modes is found from Eq.(5.29)

$$\nu_1 = \nu_m - (\nu_m - \nu_c) \frac{\Delta \nu_{1/2}}{\Delta \nu}$$

$$\nu_2 = \nu_{m-1} - (\nu_{m-1} - \nu_c) \frac{\Delta \nu_{1/2}}{\Delta \nu}$$

$$\nu_1 - \nu_2 = (\nu_m - \nu_{m-1}) - (\nu_m - \nu_{m-1}) \frac{\Delta \nu_{1/2}}{\Delta \nu}$$

$$= \Delta \nu_{FSR} - \Delta \nu_{FSR} \frac{\Delta \nu_{1/2}}{\Delta \nu}$$

$\Delta v_{modes} := \Delta v_{FSR} - \Delta v_{FSR} \cdot \frac{\Delta v_{half}}{\Delta v}$

$\Delta v_{modes} = 1.491 \cdot 10^8$

The beat frequency observed if this frequency is mixed with a 150MHz oscillator is

$\Delta v_{beat} := 150 \cdot 10^6 - \Delta v_{modes}$

$\Delta v_{beat} = 8.868 \cdot 10^5$

The observed beat frequency will be 886.8kHz

(5.7) $\lambda_0 := 325\cdot 10^{-9}$ laser wavelength (actually a He-Cd laser)

$c_0 := 2.998\cdot 10^8$ velocity of light

$n := 1$

$\gamma_0 := 0.1$ gain at line center

$\Delta\nu_D := 3\cdot 10^9$ Doppler FWHM

$l := 500\cdot 10^{-3}$ cavity length

$\alpha := 0.01$ distributed loss

The spacing between cavity modes is:

$$\Delta\nu := \frac{c_0}{2\cdot n\cdot l}$$

$\Delta\nu = 2.998\cdot 10^8$ cavity mode spacing is 300MHz

For 10 longitudinal modes to oscillate the loss line must cross the Doppler broadened gain profile at approximately $5\Delta\nu$ from the line center. A more accurate calculation proceeds a follows:

$$m := \frac{\frac{c_0}{\lambda_0}}{\Delta\nu}$$

$m = 3.076923077\cdot 10^6$

The cavity mode nearest to line center has m=3076923

$m - 3076923 = 0.076923077$

The cavity mode 5 cavity modes away has m=3076928. This mode is separated from line center by:

$\Delta\nu_1 := (3076928 - m)\cdot\Delta\nu$

$\Delta\nu_1 = 1.475938462\cdot 10^9$

The other extreme cavity mode of the ten that must oscillate has m=3076919, which is displaced from line center by:

$\Delta\nu_2 := (m - 3076919)\cdot\Delta\nu$

$\Delta\nu_2 = 1.222261538\cdot 10^9$

If the mode spaced by $\Delta\nu_1$ oscillates, then ten modes will oscillate.

We must solve

We must solve

$$\alpha - \frac{1}{l}\cdot \ln(R) = \gamma_0\cdot \exp\left[-\left(\frac{2\cdot \Delta\nu_1}{\Delta\nu_D}\right)^2\cdot \ln(2)\right]$$

$R := 0.9$ guess

Given

$$\alpha - \frac{1}{l}\cdot \ln(R) = \gamma_0\cdot \exp\left[-\left(\frac{2\cdot \Delta\nu_1}{\Delta\nu_D}\right)^2\cdot \ln(2)\right]$$

$a := \mathrm{Find}(R)$

$a = 0.979652259$

$R := a$

The mirror reflectances needed are 97.97%

Check the calculation

The loss line is:

$$\alpha - \frac{1}{l}\cdot \ln(R) = 0.051115217$$

$$\gamma_t := \gamma_0\cdot \exp\left[-\left(\frac{2\cdot \Delta\nu_1}{\Delta\nu_D}\right)^2\cdot \ln(2)\right]$$

$\gamma_t = 0.051115217$

The threshold gain agrees with the loss line