ENEE 496 LASERS AND ELECTRO-OPTIC DEVICES

First Examination, Thursday, March 21, 2002. 3:30 - 4:45pm

ANSWER THREE (3) QUESTIONS – IF MORE THAN THREE ARE ANSWERED, BEST THREE WILL COUNT

(1) Explain carefully the difference between homogeneous and inhomogeneous broadening and give two mechanisms for each. (4pts.)

An excited state decays only to one other state with Einstein coefficient A_{21}. By treating the decay as an exponentially damped wave prove that the lineshape that results from this decay is Lorentzian and derive an expression for its FWHM in terms of A_{21}. (5pts.)

What is the Lorentzian FWHM of a state with radiative lifetime 10ns that is also being destroyed by collisions at a rate 3×10^8 s^{-1}? (1pt.)

(2) Using the definition for gain:

$$\gamma = \frac{1}{I}\frac{dI}{dz}$$

derive an expression that allows the output from a saturated amplifier to be determined (3pts.).

A homogeneously broadened amplifier 2m long has a small signal gain at line center of 2m^{-1}. An input signal $\Delta\nu$ from line center has intensity of 1W m^{-2} and the saturation intensity at line center is 0.5W m^{-2}. What is the ouput intensity from the amplifier? (7 pts.)

Hint: $\Delta\nu$ is the FWHM of the line (homogeneously broadened).

(3) Discuss three of the following:

(a) Why the black-body radiation spectrum does not depend on the shape of a large cavity for which it is calculated.

(b) The use of Brewster windows on a laser.

(c) The lineshape function.

(d) Hole burning.

(4) Starting with the following basic definitions derive expressions for the relations between gain and complex susceptibility, and phase velocity and complex susceptibility. (8pts.)

$$\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P},$$

$$\mathbf{P} = \epsilon_0\chi\mathbf{E},$$

$$\mathbf{D} = \epsilon_r\epsilon_0\mathbf{E},$$

$$k = \omega\sqrt{\mu\epsilon}.$$

Hints: a plane wave propagates with time and space dependence $\sim e^{j(\omega t - kz)}$, the susceptibility that results from interaction with a transition adds to the background value of the refractive index.

Sketch the behavior of the real and imaginary parts of the complex susceptibility as the frequency passses through an atomic transition. (2pts.)

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SOLUTIONS

(1) Homogeneous broadening is identical for all particles. Each particle contributes at the same center frequency, and has the same homogeneous FWHM. Examples include natural broadening and pressure broadening.

Inhomogeneous broadeneing results when the particles contributing to the overall lineshape are not equivalent, generally because different particles have different center frequencies. Exaples include Doppler broadening in a gas, and inhomogeneous broadening in a crystal or glass where different local enviroments shift center frequencies around in a random way.

For derivation of natural broadening using a damped cosine wave see book.

The FWHM that results is:

$$\Delta\nu = \frac{A_{21}}{2\cdot\pi}$$

For natural decay plus collisions:

$\tau_{natural} := 10\cdot 10^{-9}$ $\qquad$ $X_2 := 3\cdot 10^{8}$

$$A_2 := \frac{1}{\tau_{natural}}$$

$\tau_{overall} := \dfrac{1}{A_2 + X_2}$ $\qquad$ See Eq. (2.43)

$\dfrac{1}{2\cdot\pi\cdot\tau_{overall}} = 6.366\times 10^{7}$ $\qquad$ ANSWER

(2) For derivation see book:

$$I = I_0\cdot e^{\gamma_0(\nu)\cdot l - \frac{(I-I_0)}{I_s(\nu)}}$$

$\gamma_0(\nu_0) := 2$ $\qquad$ small signal gain at line center

$\gamma_0 := 2$

At $\Delta\nu$ from line center, using the Lorentzian lineshape function

$$\gamma := \frac{\gamma_0}{5}$$

$I_s(\nu_0) := 0.5$

At $\Delta\nu$ from line center: $\quad I_s := 5\cdot 0.5$ $\quad$ From $\quad I_s(\nu)$ $\quad$ proportional to $\quad \dfrac{1}{g(\nu_0,\nu)}$

$I_0 := 1$ $\qquad$ input intensity

$l := 2$ $\qquad$ length of amplifier

$I_{max} := I_0\cdot e^{\gamma\cdot l}$ $\qquad$ Maximum possible output w/o saturation

$I := I_{max}$ guess $I_{max} = 2.226$ $I_s = 2.5$

Given

$$1 = I_0 \cdot \frac{e^{\gamma \cdot l - \frac{(I - I_0)}{I_s}}}{I}$$ solve iteratively

$I_{out} := \text{find}(I)$

$I_{out} = 1.689$ ANSWER

(3) Short answers:

(a) If blackbody radiation distribution depended on cavity shape we could connect cavities of different shape via appropriate bandpass filters and make a heat engine that would violate the second law of thermodynamics.

(b) Wave polarized in plane if incidence suffers no reflection at Brewster's angle. This avoids spurious additional reflections in resonator formed from two mirrors.

(c) Lineshape function is normalized:

$$\int_{-\infty}^{\infty} g(\nu_o, \nu)\, d\nu = 1$$

(d) Hole burning is the local depression of curve that results from saturation effects in an inhomogeneously broadened amplifier.

(4) See book. Key results are:

$\varepsilon_r^{1} = n^2 + \chi^{1} - j \cdot \chi^{11}$ χ^{1} is real part of susceptibility from transition

$k^{1} = k_0 \cdot \sqrt{\varepsilon_r}$ χ^{11} is imaginary part of susceptibility from transition

$\gamma = \frac{-k \cdot \chi^{11}}{n^2}$ gain $k = k_0 \cdot n$ $n^2 = \varepsilon_r$

$c^{1} = \frac{\omega}{k + \Delta k}$ new phase velocity

$\Delta k = k \cdot \frac{\chi^{1}}{2 \cdot n^2}$