ENEE 496 LASERS AND ELECTRO-OPTIC DEVICES

Second Examination, Tuesday, May 7, 2002, 3:30 - 4:45 pm

ANSWER THREE (3) QUESTIONS - IF MORE THAN THREE ARE ANSWERED, BEST THREE WILL COUNT

(1)Prove that the relative upper level populations needed to achieve inversion in a 3-level relative to a 4-level laser are

$$\frac{(N_2)_{3-level}}{(N_2)_{4-level}} \simeq \frac{(g_2/g_1)N + N_t}{[(g_2/g_1)+1]N_t},$$

where N_t is the threshold inversion (4 pts.).

A gas laser has an upper laser level 3000cm^{-1} above the ground state and a lower laser level 1000cm^{-1} above the ground state. If you can assume that the upper laser level population contains 25% of all the atoms, and does not change with temperature, how much lower is the population inversion at 1000K than at 300K (6 pts.). You may assume that atoms are only in the three states upper laser level, lower laser level, and ground state. $g_2 = 2$; $g_1 = 4$; $g_{ground\ state} = 2$. The pumping process of the upper laser level does not give any significant population of the lower laser level.

Hint: at 300K kT=208.6cm^{-1}.

(2)A laser reaches the threshold for oscillation when a plane wave makes a round trip within the resonator and returns to its starting point with the same amplitude and phase as it began with. Use this idea to prove that the threshold phase condition gives the oscillating laser frequencies as:

$$\nu = \nu_m - (\nu_m - \nu_0)\frac{\Delta\nu_{1/2}}{\Delta\nu}. \qquad \text{(4pts.)}$$

ν_m is a passive resonance of the cavity The following information may be useful:

$$\Delta k = \frac{k\chi'}{2n^2}$$

$$\Delta\nu_{1/2} = c/(2lF)$$

$$\gamma(\nu) = -\frac{k\chi''}{n^2}$$

$$\chi' = \frac{2(\nu_0 - \nu)}{\Delta\nu}\chi''$$

$$F = \pi\sqrt{R}/(1-R).$$

The amplitude condition for laser oscillation is

$$\gamma_t = \alpha - \frac{1}{\ell}\ln r_1 r_2,$$

where ℓ is the cavity length, which can be assumed to be completely filled with amplifying medium, and r_1, r_2, are the mirror reflection coefficients, respectively .

The peak small-signal gain of an inhomogeneously broadened gas laser operating at 488.0000 nm is 5 m^{-1} at line center. The distributed loss coefficient $\alpha = 0$. The cavity length is almost exactly 500mm, the Doppler linewidth (FWHM) is 2GHz. The laser has a homogeneous contribution to its linewidth of 10MHz. 10 modes are found to be oscillating, with one precisely at the line center. What are the mirror reflectances, if both can be assumed to be identical.(4 pts.). What is the precise length of the laser cavity, if the index of the gas is n=1.0001. (2 pts.)

Hint: for a Doppler broadened line

$$\gamma(\nu) = \gamma(\nu_0)\exp\{-[2(\nu-\nu_0)/\Delta\nu_D)^2]\ln 2\}.$$

(3) Discuss three of the following:

(a) The excitation processes in the low pressure carbon dioxide laser

(b) Mode locking

(c) Pumping arrangements for solid-state glass or crystalline insulating lasers

(d) The DF chemical laser

(4)The transmittance condition for a Fabry-Perot etalon is

$$\cos\theta_m = \frac{mc_0}{2n\ell\nu_m}.$$

Explain what will be seen if such a device is illuminated with a bright, point, monochromatic source. Explain also what will be seen if the device is illuminated with an extended source emitting a mixture of monochromatic blue and monochromatic red light if a lens is used to focus the output light onto a screen. (4 pts.).

A glass etalon with n=1.46 and of thickness 3mm is illuminated with a bright monochromatic point source placed 3mm from the entrance face of the etalon. The wavelength of the source is 501.7nm. Calculate the radii of the first three rings observed on a screen placed 1m behind the etalon (4 pts.).

The etalon changes thickness with temperature according to

$$\Delta\ell = \alpha\ell\Delta T,$$

where α is the thermal expansion coefficinet , which in this case is $10^{-6}\ \mathrm{K}^{-1}$.What is the minimum temperature change of the etalon needed to get a bright spot at the center of the ring pattern? (2 pts.)

ENEE 496 SECOND EXAMINATION 2002

SOLUTIONS

(1) In a 3-level laser, lower laser level is the ground state

$$N_1 + N_2 = N$$

$$N_2 = N - N_1$$

Population inversion needed is (3-level)

$$N_2 - \frac{g_2}{g_1} N_1 = N_t$$

$$N_2 - \frac{g_2}{g_1}(N - N_2) = N_t$$

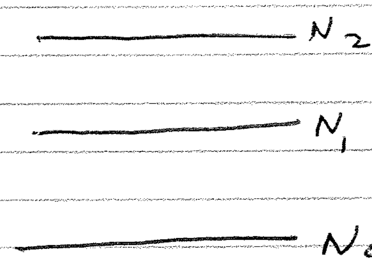
$$N_2\left(1 + \frac{g_2}{g_1}\right) = \frac{g_2}{g_1} N + N_t$$

In a 4-level laser $N_2 \simeq N_t$

Therefore

$$\frac{(N_2)_{3\text{-level}}}{(N_2)_{4\text{-level}}} = \frac{\frac{g_2}{g_1} N + N_t}{\left(\frac{g_2}{g_1} + 1\right) N_t}$$

In the 3-state system



$$N_0 + N_1 + N_2 = N,$$

$$\frac{N_1}{N_0} = \frac{g_1}{g_0} e^{-\frac{\Delta E_{10}}{kT}}, \quad N_2 = \frac{N}{4}$$

Clearly $N_1 + N_0 = \frac{3N}{4}$

At 300 K

$\frac{N_1}{N_0} = 2e^{-1000/208.6} = 0.0166$

At 1000K $\frac{N_1}{N_0} = 2e^{-\frac{1000 \times 300}{1000 \times 208.6}} = 0.237 \times 2 = 0.475$

The population inversion is $N_2 - \frac{g_2}{g_1} N_1 = N_2 - \frac{1}{2} N_1$

At 300K $N_1 = 0.0166 N_0$ so $N_0 = 0.7334 N$

$N_1 = 0.0122 N$

Inversion is $0.25N - \frac{1}{2}(0.0122N) = 0.244N$

At 1000K $N_1 = 0.475N$ so $N_0 = 0.508N$

$N_1 = 0.242N$

Inversion is $0.25N - \frac{1}{2}(0.242N) = 0.129N$

Ratio is $\frac{0.129}{0.244} = 0.529$

(2) For derivation of $\nu = \nu_m - (\nu_m - \nu_0)\frac{\Delta\nu_{1/2}}{\Delta\nu}$

see the back

For the 500 mm cavity $\Delta\nu = \frac{c}{2\ell} = \frac{2.998 \times 10^8}{1 \times 1.0001}$

$\Delta\nu = 2.9977 \times 10^8$ Hz

10 modes implies 4.5 mode spacings on each side of line center, so loss line cuts through lineshape at $(\nu - \nu_0) = \pm 1.349$ GHz

The gain at these point is

$$\gamma_t = 5 \exp\left\{-\left[\frac{2 \times 1.349}{2}\right]^2 \ln 2\right\} = 0.283 \times 5 = 1.416$$

so using $\gamma_t = -\frac{1}{\ell} \ln R$ in this case

$$\cancel{1.895}\ 1.416 = -2 \ln R \qquad R = 0.493$$

$$\frac{mc}{2\ell} = \frac{c}{\lambda} \qquad m = \frac{2\ell}{\lambda} = \frac{1.0001}{488 \times 10^{-9}}$$

$m = 2049385.2$

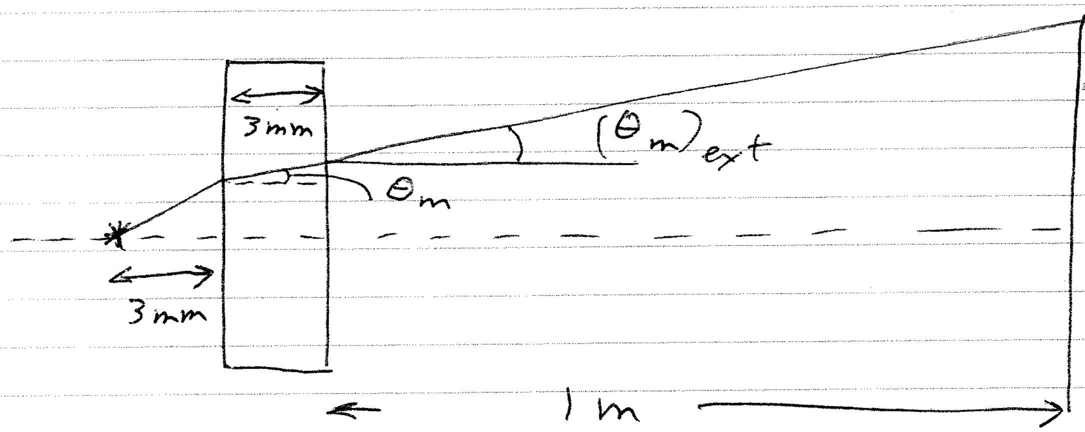
nearest integer is $m = 2049385$

$$\ell = \frac{2049385}{2}\lambda = 499.99994 \text{ mm}$$

Difference from 500 mm is 60 nm

(4) With a bright, point, monochromatic source a series of rings of the appropriate color will be seen

With an extended source, when a lens is used, rings will again appear. For 2 wavelengths there will interleaved rings of alternate red and blue.



For center of pattern $\frac{m c_0}{2 n \ell \nu_m} = 1$

$$m = \frac{2 n \ell}{\lambda_0} = \frac{2 \times 1.46 \times 3 \times 10^{-3}}{501.7 \times 10^{-9}} = 17460.634$$

First ring is $m = 17460$

$\cos\theta_m = \frac{m \lambda_0}{2 n \ell}$ $\quad 10\,\theta_m = 4.882^\circ$

$\theta_m = 0.4882^\circ \quad (\theta_m)_{ext} = n\theta_m = 0.7128^\circ$

$\theta_{m-1} = 0.7838^\circ \quad (\theta_{m-1})_{ext} = 1.1444^\circ$

$\theta_{m-2} = 0.9952^\circ \quad (\theta_{m-2})_{ext} = 1.453^\circ$

In each case the ring radius 1m away is

$r_m = (3 \times 10^{-3})(\theta_m)_{ext} + (3 \times 10^{-3})\theta_m + (\theta_m)_{ext} = 1.4674\,\theta_m$

gives $r_m = 12.5\text{ mm}$, $r_{m-1} = 20.07\text{mm}$, $r_{m-2} = 25.49\text{m}$

To get a bright spot at the center closest integer is 17461

Now $l = 3 \times 10^{-3}$ mm $= 17460.634 \times \frac{\lambda_0}{2n}$

So $\Delta l = 0.366 \frac{\lambda_0}{2n} = \frac{125.77 \text{ nm}}{2} = 62.885$ nm

So $\Delta T = \frac{125.77 \times 10^{-9}}{2 \times 10^{-6} \times 3 \times 10^{-3}} = \frac{41.92^\circ}{2}$

$= \underline{\underline{20.96^\circ}}$