ENEE 496 Spring 2004

Problem Set 3. Due Tuesday March 30, 2004

(1) Davis Problem 4.1

(2) Davis Problem 4.2

(3) Davis Problem 4.3

(4) Davis Problem 4.4

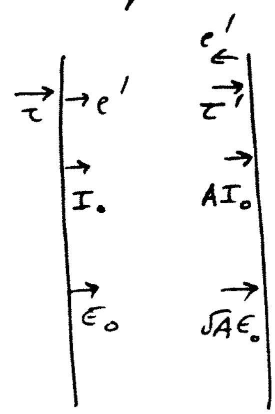
(5) Davis Problem 4.6

(6) Plot the transmittance of a Fabry-Perot device with R=0.985 as a function of phase shift δ. Show three peaks. Fit one of the peaks to a Lorentzian function and thereby show that the FWHM of the transmittance peaks obeys $\Delta\delta = 2\pi/F$ or equivalently in frequency $\Delta\nu_{1/2} = \Delta\nu/F$, where $F = \pi\sqrt{R}/(1-R)$.

(4.1) Proof of Eq. (4.49)

Loss per pass (of Intensity) is a factor A

e.g



The electric field change on one pass by a factor $\sqrt{A}$.

With the same notation as in the notes the maximum transmitted electric field is

$$E_t = E_0 \tau\tau'\sqrt{A} + E_0\tau\tau'\rho'^2 (A)^{3/2} + E_0\tau\tau'(\rho')^4 A^{5/2} + \dots$$

We can treat all the phase factors as $= 1$, since for maximum transmission they are all in phase

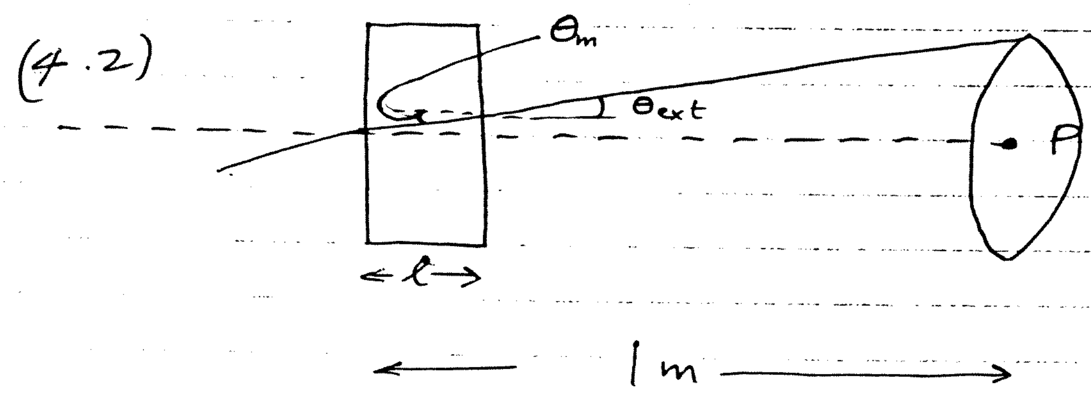
$$E_t = E_0\tau\tau'\sqrt{A}\left[1 + \rho'^2 A + \rho'^4 A^2 + \dots\right]$$

$$= \frac{E_0\tau\tau'\sqrt{A}}{1-\rho'^2 A} = \frac{E_0 T\sqrt{A}}{1-RA}$$

$$\frac{|E_t|^2}{|E_0|^2} = \frac{T^2 A}{(1-RA)^2} = \frac{(1-R)^2 A}{(1-RA)^2}$$

Q.E.D.

(4.2)



Internal angles for rings satisfy $\cos\theta_m = \dfrac{m c_0}{2 n l \omega}$

For center of pattern $\dfrac{\text{"}m\text{"} c_0}{2 n l \omega} = 1$

$\therefore$ "m" $= \dfrac{2 n l \omega}{c_0} = \dfrac{2 n l}{\lambda_0} = \dfrac{3 \times 3 \times 10^{-3}}{510.6 \times 10^{-9}}$

"m" $= 17626.32197$

$\therefore$ integers for 1st 3 rings are 17626, 17625, 17625
(m) (m − 1) (m − 2)

$\cos\theta_m = \dfrac{17626 \times 510.6 \times 10^{-9}}{9 \times 10^{-3}}$ $\theta_m = 0.00604$ rad $= 0.346°$

$\cos\theta_{m-1} = \dfrac{17625 \times 510.6 \times 10^{-9}}{9 \times 10^{-3}}$ $\theta_{m-1} = 0.01225$ rad $= 0.702°$

$\cos\theta_{m-2} = \dfrac{17624 \times 510.6 \times 10^{-9}}{9 \times 10^{-3}}$ $\theta_{m-2} = 0.0162$ rad $= 0.93°$

Ring radii are $\tan\theta_{ext} \times 1$; $\theta_{ext} = n\,\theta_m$

$\therefore$ Ring radii are 9.06 mm, 18.38 mm, 24.3 mm
to get a bright spot at the center adjust thickness so "m" = 17626 exactly

$l = \dfrac{m \lambda_0}{2 n l}$ $\Delta l = \dfrac{\Delta m\, \lambda_0}{2 n} = \dfrac{-0.32197 \times 510.6 \times 10^{-9}}{3}$

$= -54.8$ nm

(4.3) (i) For a bright spot at the center of the pattern $\frac{m\lambda_0}{2nl} = 1$ with m an integer

In this case

$$\text{"}m\text{"} = \frac{2nl}{\lambda_0} = \frac{2 \times 1.55 \times 7.4 \times 10^{-3}}{488.79 \times 10^{-9}} = 46932.22038$$

change thickness so m is 46932 exactly
(you could also move to m = 46933, but this would be a larger thickness change)

for m = 46932

$$\Delta l = -\frac{0.22038 \times 488.79 \times 10^{-9}}{2 \times 1.55} = \underline{34.75\,\text{nm}}$$

THINNER

Therefore cool the etalon by ΔT where

$$\frac{\Delta l}{l} = \alpha \Delta T \quad \text{with } \alpha = 3 \times 10^{-6}\ K^{-1}$$

$$\Delta T = \frac{34.75 \times 10^{-9}}{3 \times 10^{-6} \times 7.4 \times 10^{-3}} = \underline{1.565 \text{ degrees K}}$$

COOL BY THIS AMOUNT

(ii) If there is only one ring then there is not a 2nd ring. To get a 2nd ring use $m = 46931$ and $\cos\theta_m = \frac{m\lambda_0}{2nl}$

gives $\cos\theta_m = \frac{46931 \times 488.79 \times 10^{-9}}{2 \times 1.55 \times 7.4 \times 10^{-3}} \Rightarrow \theta_m = 0.00721$ rad $= 0.413$ degrees

This is the internal angle in the etalon outside the etalon the beam divergence angle θ_{ext} must satisfy

$$\theta_{ext} < n \sin\theta_{m=46931}$$

$$\underline{\theta_{ext} < 0.64^{\circ}}$$

(iii) $\lambda_0 = 489.32$ nm is also present

For this wavelength

$$"m" = \frac{2 \times 1.55 \times 7.4 \times 10^{-3}}{489.32 \times 10^{-9}} = 46881.38641$$

This integer is not within one of the "m" for $\lambda_0 = 488.79$ nm, so the rings do not appear within the same order

Ring 46932 of $\lambda_0 = 488.79$nm is adjacent to ring 46881 of $\lambda_0 = 489.32$nm

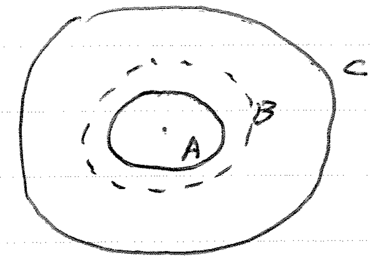
The first ring A of $\lambda_0 = 488.79$ nm has $\theta_m = 0.003065$ rad

second ring C of $\lambda_0 = 488.79$ nm has $\theta_{m-1} = 0.00721$ rad

First ring B of $\lambda_0 = 489.32$nm has $\theta_m = 0.00406$ rad

The rings at $\theta_m = 0.003065$ and $\theta_m = 0.00406$ are closest and must be distinguished

Pattern looks like



To distinguish between these 2 rings their spacing must be equivalent to $\Delta\nu_{1/2} = \frac{\Delta\nu_{FSR}}{F}$

$$= \frac{c/2l}{\pi\sqrt{R}/(1-R)}$$

For $\lambda_o = 489.32\text{nm}$ to overlap $\lambda_o = 488.79\text{m}$ we would need an effective "m" of

$$\text{"m"} = \frac{2nl\cos\theta_m}{\lambda_o} = \frac{2\times 1.55\times 7.4\times 10^{-3}\times\cos(0.00406)}{488.79}$$

$$= 46931.83358$$

The effective "Δm" $= 46932 - 46931.83358$

$$= 0.1664$$

$\Delta m = 1$ is equivalent to $\Delta\nu_{FSR}$

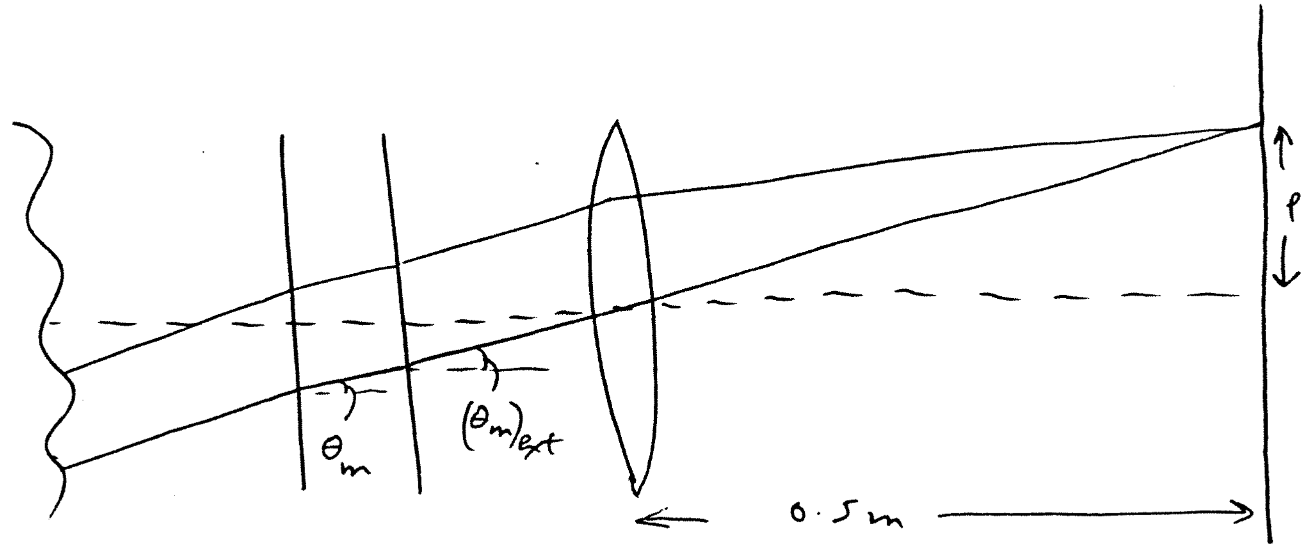
Therefore $\Delta m = 0.1664$ is equivalent to $0.1664\,\Delta\nu_{FSR}$

Therefore $\frac{1}{F} = 1\Big/\frac{\pi\sqrt{R}}{1-R} = 0.1664$

$R = 0.596$

$\underline{59.6\%\ \text{reflectance}}$

(4.4)



Find lowest ring radius number

$$m = \frac{2nl}{\lambda_0} = \frac{3 \times 2 \times 10^{-3}}{510.554 \times 10^{-9}} = 11751.94$$

$m = 11751$

to get rings through aperture on screen

$$(0.5m) \times \tan(\theta_{m'})_{ext} \leq 30\,mm$$

$$(\theta_{m'})_{ext} < 3.4336^\circ$$

$$\theta_{m'} < 2.289^\circ \quad \left(\begin{array}{c}\text{small} \\ \text{angles}\end{array}\right)$$

Find integer corresponding to 2.289°

$\cos\theta_{m'} = \frac{m'\lambda_0}{2nl}$ gives $m' = 11742(.562)$

ie ring 11743 goes through, 11742 does not

Therefore 9 rings go through

To get a bright spot at center move to $m = 11752$

$$\ell = \frac{m\lambda_0}{2n\ell} \qquad n = \frac{m\lambda_0}{2\ell} = \frac{11752 \times 510.559 \times 10^{-9}}{\times 10^{-3}}$$

$$= 1.500007652$$

$$\underline{\Delta n = 7.652 \times 10^{-6}}$$

(4.6) The transmittance of the Fabry-Perot interferometer is

$$T_{FP} = \frac{I_t}{I_i} = \frac{1}{1 + \frac{4R}{(1-R)^2} \sin^2 \delta/2}$$

with $F = \frac{\pi \sqrt{R}}{1-R}$ $\quad T_{FP} = \frac{1}{1 + \frac{4F^2}{\pi^2} \sin^2 \delta/2}$

$(T_{FP})_{max} = 1$ $\qquad (T_{FP})_{min} = \frac{1}{1 + 4F^2/\pi^2}$

$$\frac{(T_{FP})_{max}}{(T_{FP})_{min}} = C = 1 + \frac{4F^2}{\pi^2}$$ Q.E.D.

(6) $R := 0.985$ mirror reflectance

Plot one transmission peak. Choose the center phase angle δ to be 0

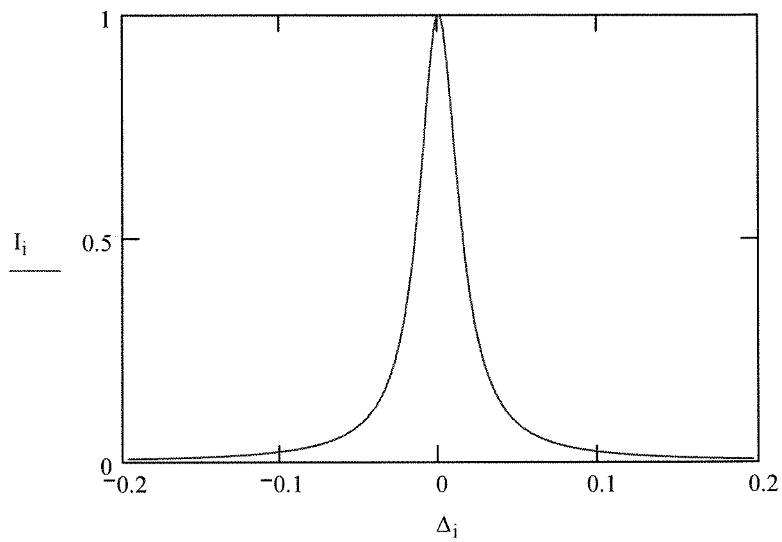
Since all the peaks are the same shape we only need the relative shape

Use phase angles from $-\pi/2$ to $\pi/2$

$i := 1, 2 .. 1000$

$$\Delta_i := \frac{-\pi}{16} + (i-1)\cdot\frac{\pi}{8\cdot 999}$$ A range of phase differences from the center

$$I_i := \frac{1}{1 + \frac{4\cdot R}{(1-R)^2}\cdot\sin\left(\frac{\Delta_i}{2}\right)^2}$$ Various normalized transmitted intensities of the system



Let's fit this to a Lorentzian function of the form

$$g_i = \frac{1}{1 + \left(\frac{2\,\Delta_i}{\Delta\delta}\right)^2}$$ where $\Delta\delta$ is the FWHM of the line

$vy_i := I_i$ $\qquad vx_i := \Delta_i$

$$F(zf,u) := \begin{bmatrix} \dfrac{u_1}{1+\left(\dfrac{2\cdot zf}{u_2}\right)^2} \\ \dfrac{1}{1+\left(\dfrac{2\cdot zf}{u_2}\right)^2} \\ \dfrac{\dfrac{u_1\cdot 8\cdot zf^2}{(u_2)^3}}{\left[1+\left(\dfrac{2\cdot zf}{u_2}\right)^2\right]^2} \end{bmatrix}$$

This is a curve fitting procedure in Mathcad

$vg_1 := 1$ $\qquad vg_2 := 0.02$

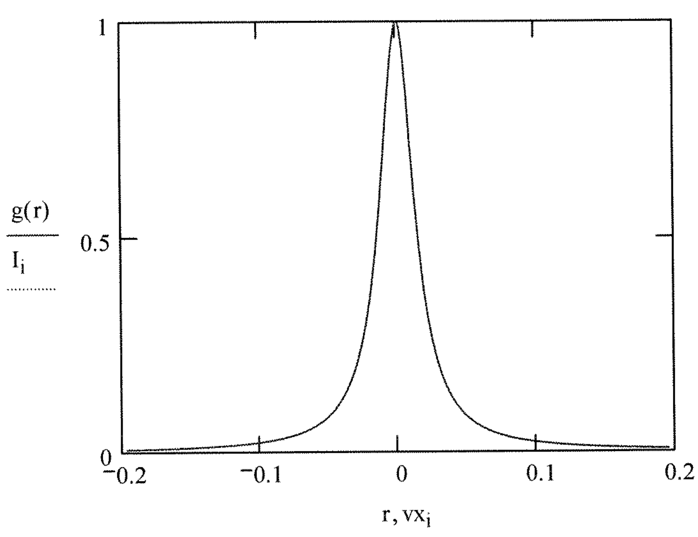
initial guesses for fitting parameters

$$P := \text{genfit}(vx, vy, vg, F)$$

$P = \begin{pmatrix} 1 \\ 0.03 \end{pmatrix}$ FWHM is 0.03

$$r := \frac{-\pi}{16}, \frac{-\pi}{16} + 0.001 .. \frac{\pi}{16}$$

$g(r) := F(r,P)_1$ fitted curve



)ata and fitted curve

'WHM determined is 0.248

$$\text{Finesse} := \frac{\pi\cdot\sqrt{R}}{1-R} \qquad \text{Finesse} = 207.863$$

$\dfrac{2\cdot\pi}{\text{Finesse}} = 0.03$ This is the FWHM and is the result that was to be proved

The way the problem was setup the spacing between transmission peaks is 2π