ENEE 496 Spring 2004

Problem Set 4. Due Tuesday April 15, 2004

(1) Davis Problem 5.1

(2) Davis Problem 5.2

(3) Davis Problem 5.3

(4) Davis Problem 5.5

(5) Davis Problem 5.6

(6) Davis Problem 5.7

(5.1) $R := 10^{24} \cdot 0.5$ Pumping rate of upper laser level is equal to pump rate of pump band times transfer efficiency

$\tau_2 := 7 \cdot 10^{-4}$ Lifetime of upper laser level

$N_2 := R \cdot \tau_2$ Steady state population of upper laser level

$A_{21} := 10^3$ $\lambda_0 := 10^{-6}$ $\Delta\nu := 10^9$

$n := 1.6$ refractive index

$$\lambda := \frac{\lambda_0}{n}$$

The gain at line center of the homogeously broadened line is

$$\gamma_0 := \frac{N_2 \cdot \lambda^2 \cdot A_{21}}{8 \cdot \pi} \cdot \frac{2}{\pi \cdot \Delta\nu}$$

$\gamma_0 = 3.463$ gain at line center in units of m^-1

$$\gamma_t = \alpha - \frac{1}{l} \cdot \ln(r_1 \cdot r_2)$$ threshold gain

$R_1 := 1$ $l := 20 \cdot 10^{-3}$

$r_1 := 1$

$\alpha := 0$

$$\gamma_t = -\frac{1}{l} \cdot \ln\left(\sqrt{R_2}\right)$$ $\gamma_t := \gamma_0$

$$R_2 := \exp\left(-\gamma_t \cdot l\right)^2$$

$R_2 = 0.871$

Minimum mirror reflectance is 87.1%

(5.2) The inhomogeneous gain lineshape is

$$\gamma(\nu) = \gamma_0 \cdot e^{-4\cdot(\nu - \nu_0)^2 \cdot \ln(2)} \qquad \text{Eq.(a)}$$

where γ_0 is the gain at line center

$\alpha := 0.001$ $\quad l := 1$ $\quad R_1 := 0.99$ $\quad R_2 := 0.99$ $\quad \gamma_0 := 1$

$r_1 := \sqrt{R_1}$ $\quad r_2 := \sqrt{R_2}$ $\quad \Delta\nu_D := 3\cdot 10^9$ $\quad c_0 := 2.998\cdot 10^8$

The threshold gain is

$$\gamma_t := \alpha - \frac{1}{l}\cdot\ln(r_1\cdot r_2)$$

$$\gamma_t = 0.011$$

Re-write Eq.(a) as $\quad \gamma_t = \gamma_0 \cdot e^{-4\cdot\Delta\nu_t^2\cdot\ln(2)}$

where $\Delta\nu_t$ is the frequency spacing from line center in units of $\Delta\nu_D$ at which the gain has dropped down to γ_t

$$\Delta\nu_t := \left[\frac{\ln\left(\frac{\gamma_0}{\gamma_t}\right)}{4\cdot\ln(2)}\right]^{\frac{1}{2}}$$

$$\Delta\nu_t = 1.275$$

$$\Delta\nu_t := \Delta\nu_t\cdot\Delta\nu_D$$

The mode spacing in the laser resonator is

$$\Delta\nu_{FSR} := \frac{c_0}{2\cdot l}$$

$\Delta\nu_{FSR} = 1.499\cdot 10^8$ $\quad$ 150MHz mode spacing

Number of mode spacings that fit under gain curve for gains greater than γ_t is

$$N := \frac{\Delta\nu_t}{\Delta\nu_{FSR}}$$

$$N = 25.512$$

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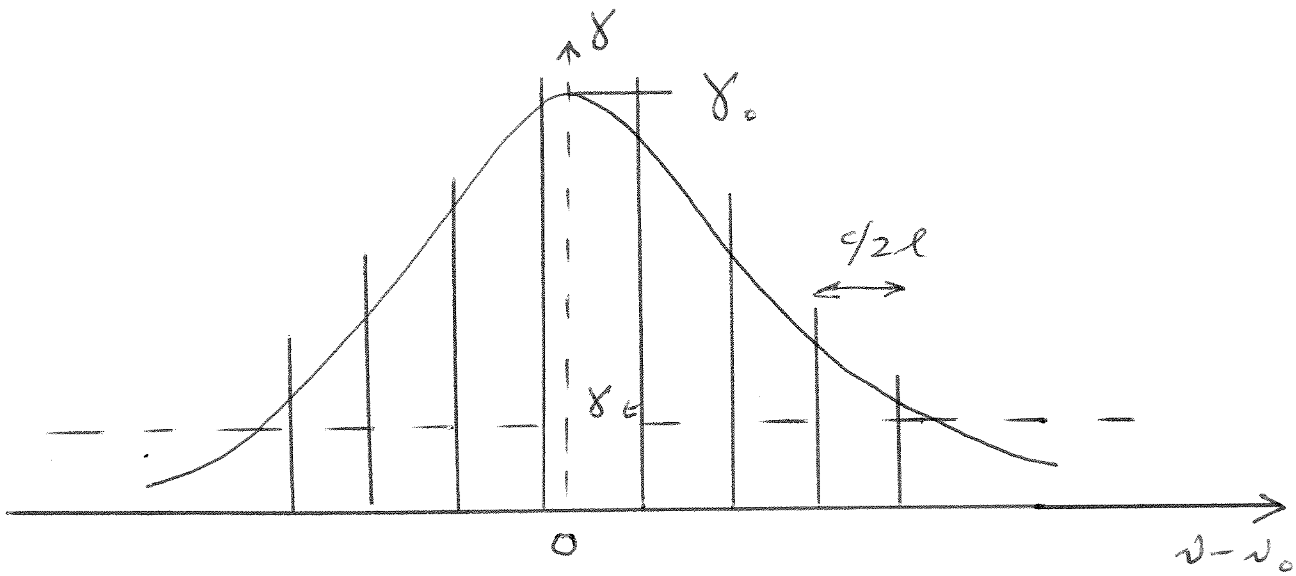
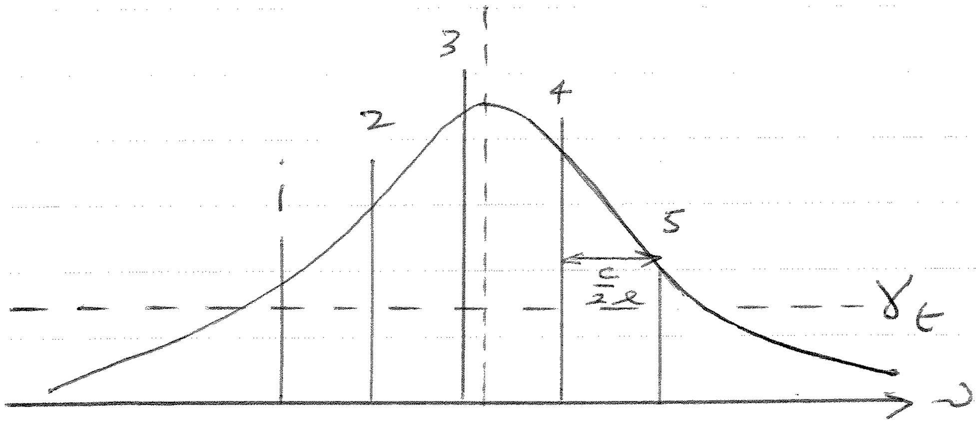
26 modes should oscillate

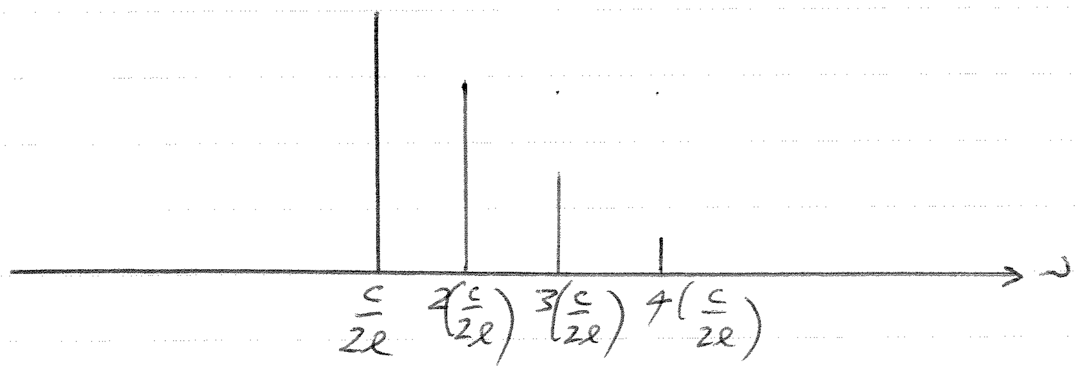


In this example 7 mode spacings fit for $\gamma > \gamma_t$ $\rightarrow$ 8 modes oscillate

(5.3) The gain curve and oscillating modes will be



The beat spectrum, to first order, will be



The frequencies of adjacent oscillating modes satisfy Eq. (5.29)

$$\nu_i = \nu_m - (\nu_m - \nu_0)\,\frac{\Delta\nu_{1/2}}{\Delta\nu}$$

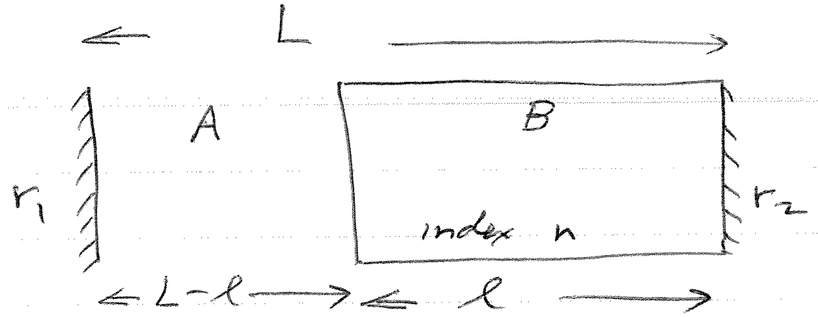
$$\nu_{i-1} = \nu_{m-1} - (\nu_{m-1} - \nu_0)\,\frac{\Delta\nu_{1/2}}{\Delta\nu}$$

$$\nu_i - \nu_{i-1} = (\nu_m - \nu_{m-1}) - (\nu_m - \nu_{m-1})\,\frac{\Delta\nu_{1/2}}{\Delta\nu}$$

$$= \frac{c}{2}\left(1 - \frac{\Delta\nu_{1/2}}{\Delta\nu}\right)$$

independent of m, so beat at c/2l is not split (to this order of approximation)

(5.5)



The propagation constant in region A is k_1,
The propagation constant in region B is k_2'

The amplitude condition is that the wave can make a round trip between the two mirrors and retain its original amplitude

i.e. $r_1 r_2 e^{(\gamma - \alpha)\ell} = 1$

which gives $\gamma = \alpha - \frac{1}{\ell} \ln r_1 r_2$

where α is the distributed loss in region B.
we assume that there is no distributed loss in region A

The phase condition is

$$2k_1(L-\ell) + 2k_2'\ell = 2m\pi$$

$k_1 = \frac{2\pi}{\lambda_0}$ (region A assumed to be air)

$k_2' = nk_1 + \frac{k_2 \chi'(\nu)}{2n^2} = nk_1 + \Delta k \qquad k_2 = nk_1$

$\chi'(\nu) = \frac{2(\nu_0 - \nu)}{\Delta\nu} \chi''(\nu) = -\frac{2(\nu_0-\nu)n\gamma}{k_2} \qquad \gamma = -\frac{k_2 \chi''}{n^2}$

Therefore $\Delta k = -\frac{k_2}{2n^2}\left(\frac{2(\nu_0-\nu)}{\Delta\nu}\right)\frac{n^2\gamma}{k_2} = -\frac{(\nu_0-\nu)\gamma}{\Delta\nu}$

Therefore, the phase condition becomes

$$k_1 (L-\ell) + n k_1 \ell - (\nu_0 - \nu) \frac{\gamma \ell}{\Delta\nu} = m\pi$$

$$k_1 = \frac{2\pi\nu}{c_0}$$

$$k_1 \left[(L-\ell) + n\ell - \frac{(\nu_0-\nu)}{\Delta\nu} \frac{\gamma \ell}{k_1} \right] = m\pi$$

which gives $\nu \left[1 - \frac{(\nu_0-\nu)}{\Delta\nu} \frac{\gamma}{k_1} \frac{\ell}{(L-\ell+n\ell)} \right] = \frac{m c_0}{2(L-\ell+n\ell)} = \nu_m$

where ν_m is the mth resonance of the passive cavity of "optical length" $(L-\ell+n\ell)$

Therefore $\nu = \nu_m + \frac{(\nu_0-\nu)}{\Delta\nu} \frac{\gamma c_0}{2\pi} \frac{\ell}{(L-\ell+n\ell)}$

We expect ν to be near ν_m, Therefore

$$\nu = \nu_m - (\nu_m - \nu_0) \frac{\gamma(\nu_m) c_0}{2\pi \Delta\nu} \frac{\ell}{(L-\ell+n\ell)}$$

near threshold $\gamma(\nu_m) \simeq \frac{1-R}{\ell}$

Therefore $\nu = \nu_m - (\nu_m - \nu_0) \frac{c_0 (1-R)}{2\pi \Delta\nu (L-\ell+n\ell)}$

and comparing with Eq (5.28) in this case

$\Delta\nu_{1/2} = \frac{c_0 (1-R)}{2\pi (L-\ell+n\ell)}$ (for $R \simeq 1$)

and finally

$$\nu = \nu_m - (\nu_m - \nu_0) \frac{\Delta\nu_{1/2}}{\Delta\nu}$$

(5.6) Define various quantities

$l := 1$ $c_0 := 2.997 \cdot 10^8$ $\lambda_0 := 632.8 \cdot 10^{-9}$ $n := 1.0001$ $\Delta\nu := 10^8$

$R := 0.99$

$$c := \frac{c_0}{n}$$

$$\Delta\nu_{FSR} := \frac{c}{2 \cdot l}$$ Cavity mode spacing

$$F := \pi \cdot \frac{R}{(1 - R)}$$ Finesse of cavity

$F = 311.018$

$$\Delta\nu_{half} := \frac{\Delta\nu_{FSR}}{F}$$

$\Delta\nu_{half} = 4.818 \cdot 10^5$ $\Delta\nu_{FSR} = 1.498 \cdot 10^8$

The spacing between two oscillating modes is found from Eq.(5.29)

$$\nu_1 = \nu_m - (\nu_m - \nu_c) \frac{\Delta\nu_{1/2}}{\Delta\nu}$$

$$\nu_2 = \nu_{m-1} - (\nu_{m-1} - \nu_c) \frac{\Delta\nu_{1/2}}{\Delta\nu}$$

$$\nu_1 - \nu_2 = (\nu_m - \nu_{m-1}) - (\nu_m - \nu_{m-1}) \frac{\Delta\nu_{1/2}}{\Delta\nu}$$

$$= \Delta\nu_{FSR} - \Delta\nu_{FSR} \frac{\Delta\nu_{1/2}}{\Delta\nu}$$

$$\Delta\nu_{modes} := \Delta\nu_{FSR} - \Delta\nu_{FSR} \cdot \frac{\Delta\nu_{half}}{\Delta\nu}$$

$\Delta\nu_{modes} = 1.491 \cdot 10^8$

The beat frequency observed if this frequency is mixed with a 150MHz oscillator is

$$\Delta\nu_{beat} := 150 \cdot 10^6 - \Delta\nu_{modes}$$

$\Delta\nu_{beat} = 8.868 \cdot 10^5$

The observed beat frequency will be 886.8kHz

(5.7) $\lambda_0 := 325\cdot 10^{-9}$ laser wavelength (actually a He-Cd laser)

$c_0 := 2.998\cdot 10^{8}$ velocity of light

$n := 1$

$\gamma_0 := 0.1$ gain at line center

$\Delta\nu_D := 3\cdot 10^{9}$ Doppler FWHM

$l := 500\cdot 10^{-3}$ cavity length

$\alpha := 0.01$ distributed loss

The spacing between cavity modes is:

$$\Delta\nu := \frac{c_0}{2\cdot n\cdot l}$$

$\Delta\nu = 2.998\cdot 10^{8}$ cavity mode spacing is 300MHz

For 10 longitudinal modes to oscillate the loss line must cross the Doppler broadened gain profile at approximately 5$\Delta\nu$ from the line center. A more accurate calculation proceeds a follows:

$$m := \frac{\frac{c_0}{\lambda_0}}{\Delta\nu}$$

$m = 3.076923077\cdot 10^{6}$

The cavity mode nearest to line center has m=3076923

$m - 3076923 = 0.076923077$

The cavity mode 5 cavity modes away has m=3076928. This mode is separated from line center by:

$\Delta\nu_1 := (3076928 - m)\cdot\Delta\nu$

$\Delta\nu_1 = 1.475938462\cdot 10^{9}$

The other extreme cavity mode of the ten that must oscillate has m=3076919, which is displaced from line center by:

$\Delta\nu_2 := (m - 3076919)\cdot\Delta\nu$

$\Delta\nu_2 = 1.222261538\cdot 10^{9}$

If the mode spaced by $\Delta\nu_1$ oscillates, then ten modes will oscillate.

We must solve

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$$\alpha - \frac{1}{l}\cdot\ln(R) = \gamma_0\cdot\exp\left[-\left(\frac{2\cdot\Delta\nu_1}{\Delta\nu_D}\right)^2\cdot\ln(2)\right]$$

$R := 0.9$ guess

Given

$$\alpha - \frac{1}{l}\cdot\ln(R) = \gamma_0\cdot\exp\left[-\left(\frac{2\cdot\Delta\nu_1}{\Delta\nu_D}\right)^2\cdot\ln(2)\right]$$

$a := \text{Find}(R)$

$a = 0.979652259$

$R := a$

The mirror reflectances needed are 97.97%

Check the calculation

The loss line is:

$$\alpha - \frac{1}{l}\cdot\ln(R) = 0.051115217$$

$$\gamma_t := \gamma_0\cdot\exp\left[-\left(\frac{2\cdot\Delta\nu_1}{\Delta\nu_D}\right)^2\cdot\ln(2)\right]$$

$\gamma_t = 0.051115217$

The threshold gain agrees with the loss line