

**ENEE 496 Spring 2004**  
**Solutions to First Examination Questions**

(1) Homogeneous broadening affects all particles equally -- all particles are identical.

Examples: natural broadening, pressure broadening, Stark broadening.

Inhomogeneous broadening results because nonequivalent particles are being observed, which have different center frequencies.

Examples: Doppler broadening, different environments in a real crystal or glass.

Lorentzian line shape is derived by carrying out Fourier transform on

$$e(t) = E_0 \cdot \cos(\omega t) e^{-\frac{t}{\tau_c}}$$

$$\text{If } \tau = \frac{1}{A_{21}} \quad \Delta\nu = \frac{A_{21}}{2 \cdot \pi}$$

$$\text{If } \tau = \frac{1}{A_2} \quad \Delta\nu = \frac{A_2}{2 \cdot \pi}$$

$$\text{Including lower level } \Delta\nu = \frac{A_2 + A_1}{2 \cdot \pi}$$

(2)

$$I = I_0 e^{-\frac{\gamma_0(\nu) I}{I_s}}$$

$$g(\nu_0, \nu) = \frac{\frac{2}{(\pi \cdot \Delta\nu)}}{1 + \left[ \frac{2 \cdot (\nu - \nu_0)}{\Delta\nu} \right]^2}$$

$$\gamma(\nu) = \frac{\gamma(\nu_0)}{\left[ 1 + \left[ \frac{2 \cdot (\nu - \nu_0)}{\Delta\nu} \right]^2 \right]}$$

$I_{0S} := 10$  saturation intensity at line center

$\gamma_0 := 5 \cdot 1000$  At  $\Delta\nu$  from line center line shape is 1/5 of peak value

$I_0 := 10$   $l := 10^{-2}$

$\Delta\nu := 10^{10}$

$g := \frac{1}{1 + \left(\frac{2 \cdot 5 \cdot 10^8}{\Delta\nu}\right)^2}$  relative line shape function 500 MHz from line center

$g = 0.99$

$I_S := \frac{I_{0S}}{g}$

$I_S = 10.1$  saturation intensity for problem

$I := 10.1$  Guess

Given

$$I = I_0 \cdot e^{\gamma_0 \cdot l \cdot \frac{(I - I_0)}{I_S}}$$

$I_{out} := \text{Find}(I)$

$I_{out} = 475.986$

Without gain saturation

$$I_{out} := I_0 \cdot e^{\gamma_0 \cdot l}$$

$I_{out} = 5.185 \times 10^{22}$

$$(4) \quad \lambda_0 := 10^{-6} \quad h := 6.626 \cdot 10^{-34} \quad k := 1.38 \cdot 10^{-23}$$

$$c := 2.998 \cdot 10^8 \quad T := 300$$

Ratio required is

$$R = \frac{B \cdot \rho(\nu)}{A} = \frac{c^3}{8 \cdot \pi \cdot h \cdot \nu^3} \cdot \rho$$

$$\nu := \frac{c}{\lambda_0}$$

$$\rho := \frac{8 \cdot \pi \cdot h \cdot \nu^3}{c^3} \cdot \frac{1}{e^{\frac{h \cdot \nu}{k \cdot T}} - 1}$$

$$\rho \cdot 10^{35} = 2.415$$

$$R := \frac{c^3}{8 \cdot \pi \cdot h \cdot \nu^3} \cdot \rho$$

$$R \cdot 10^{21} = 1.45$$

Ratio is  $1.45 \times 10^{-21}$

g

ss.