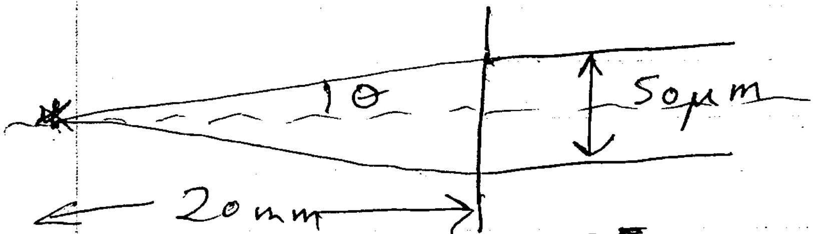
EXAM # 1 ENEE 691 Spring 2001

(1) $NA \equiv \sin\theta_0 = \sqrt{n_1^2 - n_2^2}$

$$NA = \sqrt{(n_1 - n_2)(n_1 + n_2)}$$
$$\cong \sqrt{\left(\frac{n_1 - n_2}{n_1}\right) 2 n_1^2}$$
$$= n_1 \sqrt{2\Delta}$$
$$= 1.46 \sqrt{0.004}$$
$$= 0.0923^\circ = 0.09243 \text{ rad}$$
$$= 5.3^\circ$$



$\theta = \arctan \frac{25}{20{,}000}$ $\quad \theta = 0.0716^\circ$
$\cong 0.00125 \text{ rad}$,

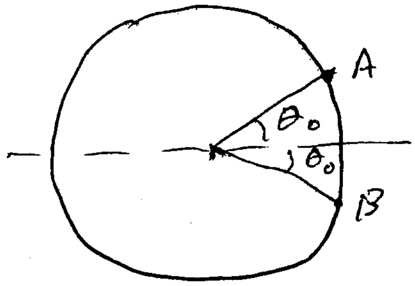
since $\theta < NA'$ all light reaching core will be guided

solid angle is $\frac{\pi \times 25^2 \times 10^{-12}}{400 \times 10^{-6}} = 4.909 \times 10^{-6}$ rad

power in is $\frac{10^{-2} \times 4.909 \times 10^{-6}}{4\pi}$

$= 3.906$ nW

When point source is against end of fiber the fraction of light entering the fiber is determined by the solid angle associated with θ_0



The area of the sphere part of which AB is limiting segment is

$$\int_0^{2\pi}\int_0^{\theta_0} r^2 \sin\theta \, d\theta \, d\phi$$

$$= -2\pi r^2 \cos\theta \Big|_0^{\theta_0}$$

$$= 2\pi r^2 (1-\cos\theta_0)$$

The power fraction is

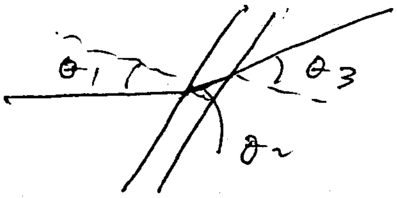
$$P \, \frac{2\pi r^2 (1-\cos\theta_0)}{4\pi r^2} = \frac{P}{2}(1-\cos\theta_0)$$

$$= \frac{10^{-2}}{2}(1-\cos\theta_0) = 21.34\,\mu W$$

(2)

$$\rho = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{\frac{1}{n_1} - 1}{\frac{1}{n_1} + 1} = \frac{1-n_1}{1+n_1}$$

$$R = |\rho|^2 = \left(\frac{1-n_1}{1+n_1}\right)^2 = 0.0343$$



$\theta_1 = 8^\circ$

$1.455 \sin 8^\circ = 1.3 \sin\theta_2$

$\theta_2 = 8.96^\circ$

$1.3 \sin\theta_2 = \sin\theta_3 \qquad \theta_3 = 11.68^\circ$

P wave impedances are of the form

$$Z = \frac{Z_0 \cos\theta}{n}$$

Ignore the Z_0, it cancels.

$Z_1' \propto 0.6806$

$Z_2' \propto 0.7598$

$Z_3' \propto 0.9793$

$$k_2 d' = k_2 d_2 \cos\theta_2 = \frac{2\pi}{\lambda_2}\frac{\lambda_2}{4}\cos\theta_2$$

$$= \frac{\pi}{2}\cos\theta_2 = 1.552$$

$$Z_3'' \propto 0.7598\left(\frac{0.9793\cos(1.552) + j\,0.7598\sin(1.552)}{0.7598\cos(1.552) + j\,0.9793\sin(1.552)}\right)$$

$$\propto \frac{0.01399 + j\,0.5772}{0.01428 + j\,0.9791} = \underline{0.5894 - 0.00561}$$

$$\rho = \frac{0.5894 - j\,0.00569 - 0.6806}{0.5894 - j\,0.00569 + 0.6806}$$

$$= \frac{-0.0912 - j\,0.00569}{1.27 - j\,0.00569}$$

$$R = |\rho|^2 = 0.00518$$

The power at port 5 is determined by R, the reflectance of a cleaved, uncoated facet
$R = 0.0343$

$$P_2 = \underbrace{\frac{P}{2}\left(\frac{1}{2}\right)R\left(\frac{1}{2}\right)(1-R)}_{P_{51}} + \frac{P_{51}}{(1-R)}(R)\left(\frac{1}{2}\right)R\left(\frac{1}{2}\right)(1-R) + \cdots$$

$$= (1-R)\left[\frac{R}{8} + \frac{R^3}{32} + \frac{R^5}{128} + \cdots\right]P$$

additional contribution

$$P(1-R)\left[\frac{R^3}{128} + \frac{R^5}{512} + \cdots\right] + \text{smaller terms}$$

$$= \underline{0.00415\ \text{mW}}$$

(4) V number is $\dfrac{2\pi \times 1.46 \times 12\times10^{-6}\sqrt{0.01}}{1.55\times10^{-6}}$ $= 7.102$ (5)

The following Bessel function zeros apply: J_{01}, J_{02}

J_{11} J_{12}

J_{21}

J_{31}

The cutoff of a LP_{ml} mode is determined by the condition

$$\frac{u\, J_{m-1}(u)}{J_m(u)} = 0$$

as then $u = V$ and $w = 0$. The mode spreads into the cladding and is no longer guided

For $m=0$ $J_0(0) = 1$ so $u=0$ is a solution this is LP_{01}

LP_{01} LP_{02} LP_{03} propagate

for $m=1$ Zeros of J_0 apply

LP_{11} LP_{12} propagate

for $m=0$ zeros of J_1 apply

For $m=2$ zeros of J_1 apply

LP_{21} LP_{22} propagate

for $m=3$ zeros of J_2 apply

LP_{31} propagates

For $m=9$ zeros of J_3 apply

LP_{41} propagates

LP modes	modes	degeneracy
LP_{01}	HE_{11}	2
LP_{02}	HE_{12}	2
LP_{03}	HE_{13}	2
LP_{11}	TE_{01}, TM_{01} HE_{21}	4
LP_{12}	TE_{02}, TM_{02}, HE_{22}	4
LP_{21}	HE_{31}, EH_{11}	4
LP_{22}	HE_{32}, EH_{12}	4
LP_{31}	HE_{41}, EH_{21}	4
LP_{41}	HE_{51}, EH_{31}	4
9 LP modes	17 modes	30 modes including degeneracy

$\frac{V^2}{2}$ predicts 25 modes