(1) $n_1 := 4$ $n_2 := 2$ refractive indices

$c_0 := 2.998 \cdot 10^8$

$\varepsilon_0 := 8.854 \cdot 10^{-12}$ fundamental constants

$\mu_0 := 4 \cdot \pi \cdot 10^{-7}$

$\varepsilon_1 := {n_1}^2 \cdot \varepsilon_0$

$\varepsilon_2 := {n_2}^2 \cdot \varepsilon_0$

Choose different wavelengths to get points on the dispersion curve

$j := 1, 2 .. 7$ $l := 1, 2 .. 7$ $q := 1, 2 .. 7$ $r := 1, 2 .. 7$

$i := 1$

$\lambda_0 := i \cdot 10^{-6} \cdot 0.1$

$\omega := \dfrac{2 \cdot \pi \cdot c_0}{\lambda_0}$ $k_1 := \dfrac{2 \cdot \pi \cdot n_1}{\lambda_0}$

$d := 0.6 \cdot 10^{-6}$ $k_2 := \dfrac{2 \cdot \pi \cdot n_2}{\lambda_0}$

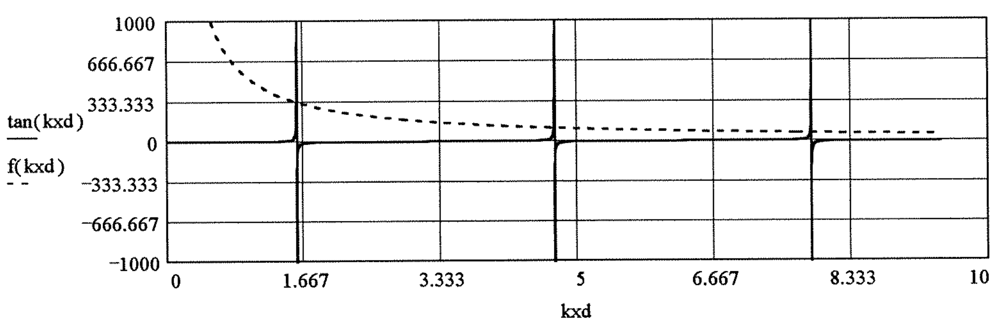
Plot a figure like (17.8)

For the TE modes

$$f(kxd) := \sqrt{\frac{4 \cdot \pi^2 \cdot \left({n_1}^2 - {n_2}^2\right) \cdot d^2}{\left(\lambda_0\right)^2 \cdot kxd^2} - 1} \cdot \frac{\varepsilon_1}{\varepsilon_2}$$

Generally $\mu_1 = \mu_2$

$kxd := 0.001, 0.002 .. 3 \cdot \pi$



Solve Eq.(17.121)

$kxd := 1.56$ Guess for m=0 mode

Given

$f(kxd)=\tan(kxd)$

$a := Find(kxd)$

$a = 1.56779$ $\quad$ $kxd := a$

$\gamma := \sqrt{\frac{kxd^2}{d^2} - k_1^{\,2}}$ $\quad$ $\beta := \sqrt{k_1^{\,2} - \frac{kxd^2}{d^2}}$

$\gamma = 2.51314 \cdot 10^8 i$

$\beta = 2.51314 \cdot 10^8$

$\frac{2 \cdot \pi}{\lambda_0} = 6.28319 \cdot 10^7$

$k_1 = 2.51327 \cdot 10^8$

$k_2 = 1.25664 \cdot 10^8$

This is a point on the dispersion curve for m=0

$\omega = 1.8837 \cdot 10^{16}$

Points on the light lines

Solutions found for m=0 by changing frequency

$\omega\omega_{1,0} := 1.8837 \cdot 10^{15}$ $\quad$ $kk_{1_1} := 2.51327 \cdot 10^7$ $\quad$ $kk_{2_1} := 1.25664 \cdot 10^7$ $\quad$ $\beta\beta_{1,0} := 2.50012 \cdot 10^7$

$\omega\omega_{2,0} := 9.41848 \cdot 10^{14}$ $\quad$ $kk_{1_2} := 1.25664 \cdot 10^7$ $\quad$ $kk_{2_2} := 6.28319 \cdot 10^6$ $\quad$ $\beta\beta_{2,0} := 1.23113 \cdot 10^7$

$\omega\omega_{3,0} := 6.279 \cdot 10^{14}$ $\quad$ $kk_{1_3} := 8.37758 \cdot 10^6$ $\quad$ $kk_{2_3} := 4.18879 \cdot 10^6$ $\quad$ $\beta\beta_{3,0} := 8.00589 \cdot 10^6$

$\omega\omega_{4,0} := 4.70925 \cdot 10^{14}$ $\quad$ $kk_{1_4} := 6.28319 \cdot 10^6$ $\quad$ $kk_{2_4} := 3.14159 \cdot 10^6$ $\quad$ $\beta\beta_{4,0} := 5.80141 \cdot 10^6$

$\omega\omega_{5,0} := 3.7674 \cdot 10^{14}$ $\quad$ $kk_{1_5} := 5.02655 \cdot 10^6$ $\quad$ $kk_{2_5} := 2.51327 \cdot 10^6$ $\quad$ $\beta\beta_{5,0} := 4.442 \cdot 10^6$

$\omega\omega_{6,0} := 3.1395 \cdot 10^{14}$ $\quad$ $kk_{1_6} := 4.18878 \cdot 10^6$ $\quad$ $kk_{2_6} := 2.0944 \cdot 10^6$ $\quad$ $\beta\beta_{6,0} := 3.51087 \cdot 10^6$

$\omega\omega_{7,0} := 2.691 \cdot 10^{14}$ $\quad$ $kk_{1_7} := 3.59039 \cdot 10^6$ $\quad$ $kk_{2_7} := 1.7952 \cdot 10^6$ $\quad$ $\beta\beta_{7,0} := 2.83214 \cdot 10^6$

Solve Eq.(17.121)

$kxd := 4.67$ $\quad$ **Guess for m=2 mode**

Given

$f(kxd)=\tan(kxd)$

$a := Find(kxd)$

$a = 4.70338$ $\qquad$ $kxd := a$

$$\gamma := \sqrt{\frac{kxd^2}{d^2} - k_1^{\,2}} \qquad \beta := \sqrt{k_1^{\,2} - \frac{kxd^2}{d^2}}$$

$\gamma = 2.51205 \cdot 10^8\, i$ $\qquad$ $\omega = 1.8837 \cdot 10^{16}$

$\beta = 2.51205 \cdot 10^8$

$\frac{2\cdot\pi}{\lambda_0} = 6.28319 \cdot 10^7$

This is a point on the dispersion curve for m=2

$k_1 = 2.51327 \cdot 10^8$

$k_2 = 1.25664 \cdot 10^8$

Points on the light lines

Solutions found by changng frequency

$\omega\omega_{1,2} := 1.8837 \cdot 10^{16}$ $\qquad$ $\beta\beta_{1,2} := 2.51205 \cdot 10^8$

$\omega\omega_{2,2} := 6.279 \cdot 10^{15}$ $\qquad$ $\beta\beta_{2,2} := 8.3411 \cdot 10^7$

$\omega\omega_{3,2} := 3.7674 \cdot 10^{15}$ $\qquad$ $\beta\beta_{3,2} := 4.966 \cdot 10^7$

$\omega\omega_{4,2} := 1.8837 \cdot 10^{15}$ $\qquad$ $\beta\beta_{4,2} := 2.3925 \cdot 10^7$

$\omega\omega_{5,2} := 9.41848 \cdot 10^{14}$ $\qquad$ $\beta\beta_{5,2} := 1.01066 \cdot 10^7$

$\omega\omega_{6,2} := 6.279 \cdot 10^{14}$ $\qquad$ $\beta\beta_{6,2} := 4.84522 \cdot 10^6$

$\omega\omega_{7,2} := 4.70925 \cdot 10^{14}$ $\qquad$ $\beta\beta_{7,2} := 3.14599 \cdot 10^6$

Points on the dispersion curves for other modes are obtained in a similar way.
For the even symmetry P-wave modes Eq.(17.101) must be solved.

Solve Eq.(17.121)

$kxd := 6$ $\qquad$ **Guess for m=4 mode**

Given

$f(kxd) = \tan(kxd)$

$a := \mathrm{Find}(kxd)$

$a = 29.78663$ $\qquad$ $kxd := a$

$$\gamma := \sqrt{\frac{kxd^2}{d^2} - k_1^{\,2}} \qquad \beta := \sqrt{k_1^{\,2} - \frac{kxd^2}{d^2}}$$

$\gamma = 2.46376 \cdot 10^8\, i$

$\beta = 2.46376 \cdot 10^8$ This is a point on the dispersion curve for m=4

$\frac{2 \cdot \pi}{\lambda_0} = 6.28319 \cdot 10^7$

$k_1 = 2.51327 \cdot 10^8$ Points on the light lines

$k_2 = 1.25664 \cdot 10^8$

Solve Eq.(17.121)

$kxd := 14$ Guess for m=6 mode

Given

$f(kxd) = \tan(kxd)$

$a := \text{Find}(kxd)$

$a = 14.11$ $kxd := a$

$\gamma := \sqrt{\frac{kxd^2}{d^2} - k_1^{\,2}}$ $\beta := \sqrt{k_1^{\,2} - \frac{kxd^2}{d^2}}$

$\gamma = 2.50225 \cdot 10^8 \, i$

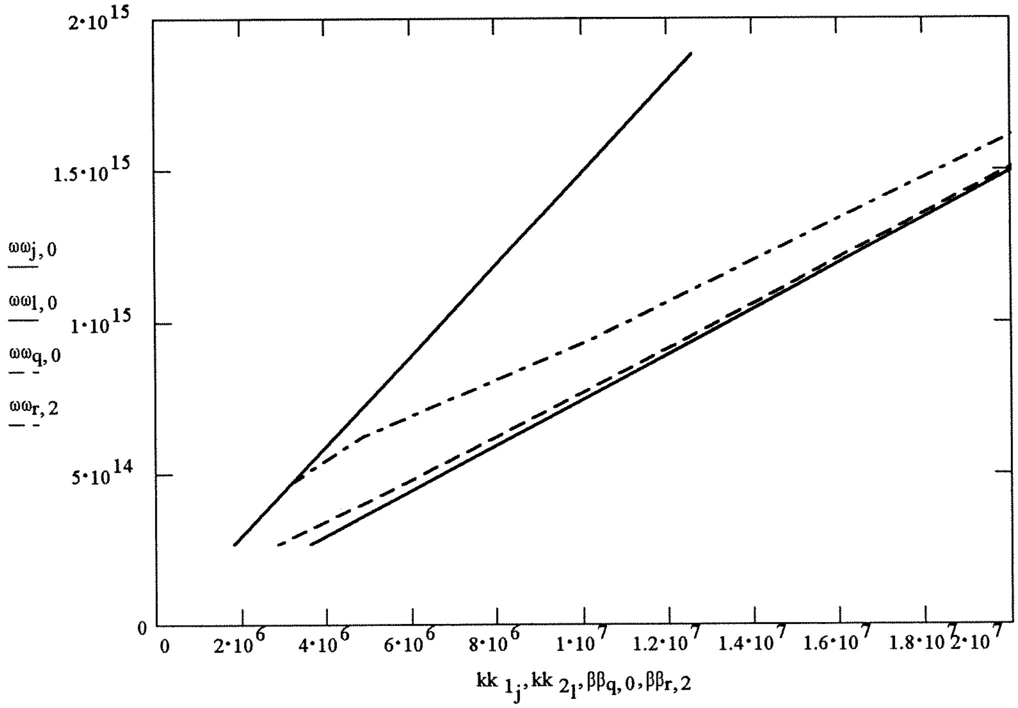
$\beta = 2.50225 \cdot 10^8$ This is a point on the dispersion curve for m=6

$\frac{2 \cdot \pi}{\lambda_0} = 6.28319 \cdot 10^7$

$k_1 = 2.51327 \cdot 10^8$ Points on the light lines

$k_2 = 1.25664 \cdot 10^8$

DISPERSION CURVES FOR m=0,2



(2)

$n_1 := 1.5$ $n_2 := 1.485$ **refractive indices**

$c_0 := 2.998 \cdot 10^{8}$

$\varepsilon_0 := 8.854 \cdot 10^{-12}$ **fundamental constants**

$\mu_0 := 4 \cdot \pi \cdot 10^{-7}$

$\varepsilon_1 := {n_1}^2 \cdot \varepsilon_0$

$\varepsilon_2 := {n_2}^2 \cdot \varepsilon_0$

$a := 2 \cdot 10^{-6}$

$$\Delta := \frac{n_1 - n_2}{n_1}$$

$$\lambda_c := \frac{2 \cdot \pi \cdot n_1 \cdot a \cdot \sqrt{2 \cdot \Delta}}{2.405}$$

$\lambda_c = 1.1084115 \cdot 10^{-6}$ **Cutoff wavelength for second mode to appear**

$\lambda_0 := 1.5 \cdot \lambda_c$

$$k_1 := \frac{2 \cdot \pi \cdot n_1}{\lambda_0}$$

$$w(u) := \sqrt{2 \cdot {k_1}^2 \cdot a^2 \cdot \Delta - u^2}$$

$u := 1$ **Guess**

Given

$$\frac{u \cdot J1(u)}{J0(u)} = \frac{w(u) \cdot K1(w(u))}{K0(w(u))}$$

$uu := \mathrm{Find}(u)$

$uu = 1.3685721$ **Solution for u**

$i := 1, 2 .. 100$

$$ww := \sqrt{2 \cdot k_1^{\,2} \cdot a^2 \cdot \Delta - uu^2}$$

$j := 1, 2 .. 100$

$ww = 0.8352772$

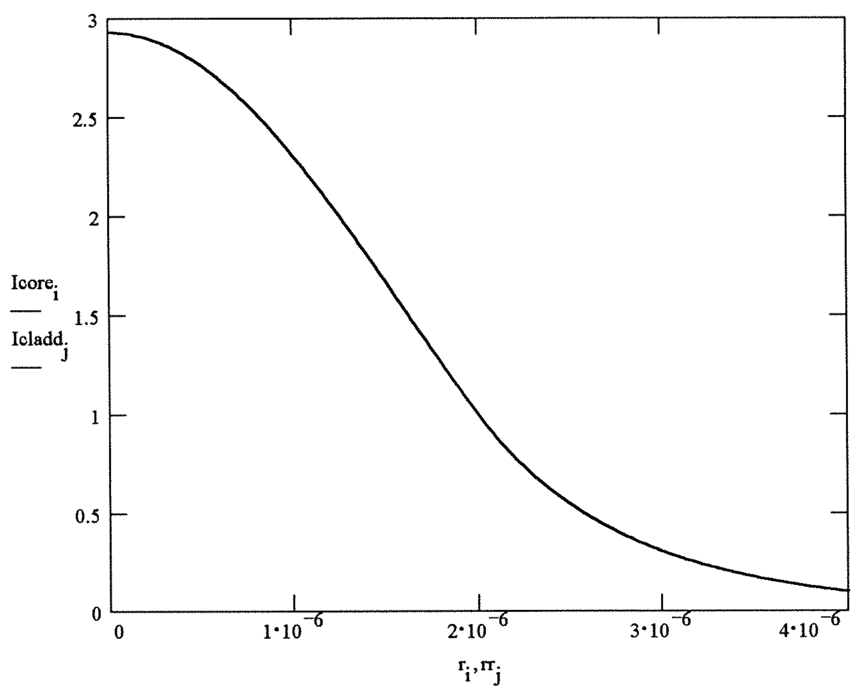
$$r_i := a \cdot \frac{i-1}{99}$$

$$Icore_i := \left(\frac{J0\left(uu \cdot \frac{r_i}{a} \right)}{J0(uu)} \right)^2$$

$$rr_j := a + (j-1) \cdot \frac{a}{99}$$

$$Icladd_j := \left(\frac{K0\left(ww \cdot \frac{rr_j}{a} \right)}{K0(ww)} \right)^2$$

H E11 (LP01) Mode Profile



(3) Normalize intensity curves from problem (2)

$j := 1, 2 .. 200$

$$I_j := \mathrm{if}\left(j \le 100, \frac{Icore_j}{Icore_1}, \frac{Icladd_{j-100}}{Icore_1}\right) \qquad r_j := (j-1)\cdot\frac{a}{199}$$

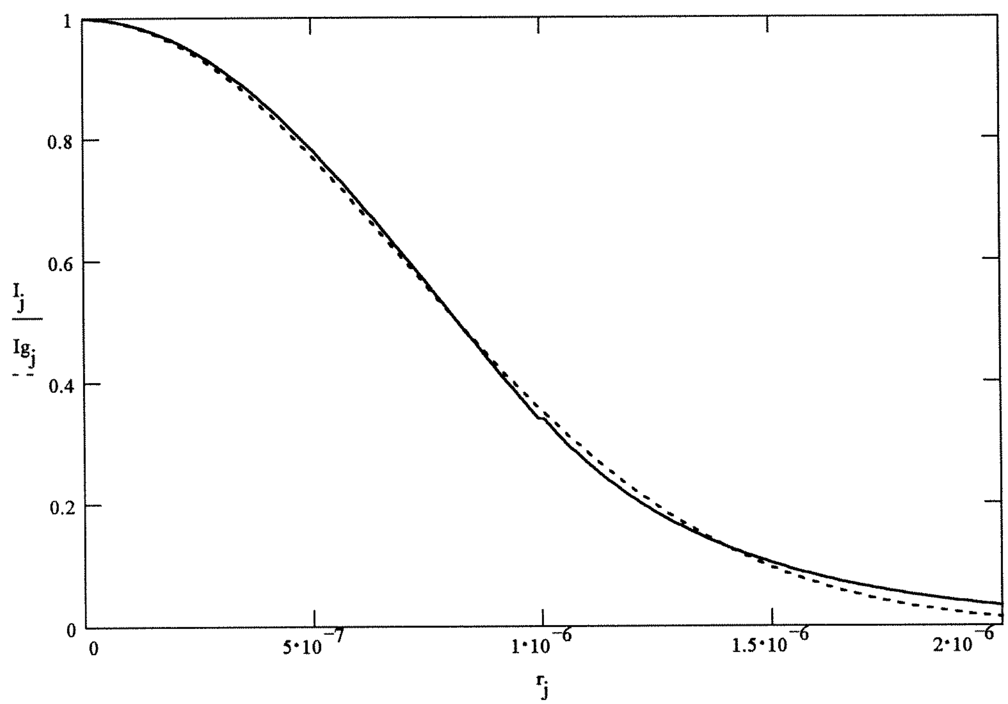
Gaussian function

$\sigma := 0.9833\cdot 10^{-6}$ Best σ computed by trial and error

$$Ig_j := e^{-\frac{(r_j)^2}{\sigma^2}} \qquad S := \sum_j \left(I_j - Ig_j\right)^2$$

$S = 0.0272905$

HE11 Mode and Gaussian Fit



(4) $\Delta = \frac{n_1 - n_2}{n_1} = \frac{1.465 - 1.46}{1.465} = 0.003413$

For $a = 5\,\mu m$ $\lambda = 1.55\,\mu m$ $V^2 = \frac{8\pi^2}{(1.55)^2}(1.465)^2\, 25 \times 0.003413$

$V = 2.4532$

) For $a = 5\,\mu m$ $\lambda = 0.85\,\mu m$ $V^2 = \frac{8\pi^2 (1.465)^2\, 25 \times 0.003413}{(0.85)^2}$

$V = 4.4735$

or $a = 25\,\mu m$ $\lambda = 1.55\,\mu m$ $V = 12.266$

) For $a = 25\,\mu m$ $\lambda = 0.85\,\mu m$ $V = 22.368$

) For $V = 2.4532$ only LP_{01} and LP_{11} modes will propagate

$LP_{01} = HE_{11}$, 2 degenerate modes

$LP_{11} = TE_{01}, TM_{01}, HE_{21}$ (2 degenerate modes) = 4 modes

ie 6 modes propagate

(ii) For $V = 4.4735$, the following Bessel function zeros are less than V: J_{01}, J_{11} $J_{01} = 2.405$, $J_{11} = 3.832$

so $TM_{01}, TE_{01}, EH_{11}, HE_{11}, HE_{31}, HE_{21}, HE_{12}$ can propagate

ie LP_{01}, LP_{11} & LP_{21}, LP_{02} = 12 modes

(iii) For $V = 12.266$, LP_{01} propagates (has no cut-off)

For J_0 the following zeros are smaller:

2.405	LP_{11}
5.52	LP_{12}
8.654	LP_{13}
11.792	LP_{14}

For J_1 & J_{-1} the following zeros are smaller

3.832	LP_{21}, LP_0
7.016	LP_{22}, LP_0
10.173	LP_{23}, LP_0

For J_2 & J_{-2} the following zeros are smaller

5.136	LP_{31}
8.417	LP_{32}
11.62	LP_{33}

For J_3 & J_{-3} the following zeros are smaller

6.38	LP_{41}
9.76	LP_{42}

For J_4 & J_{-4} the following zeros are smaller

7.588	LP_{51}
11.065	LP_{52}

For J_5 & J_{-5} the following are smaller

8.7715	LP_6

For J_6 & J_{-6} :	9.936	LP_{71}
For J_7 & J_{-7}	11.086	LP_{81}
For J_8 & J_{-8}	12.225	LP_{91}

Total number of modes
$= 18 \times 4 + 4 \times 2$
$= 80$ modes

For the LP_{ml} mode to propagate

$$V < J_{(m-1)l}$$

Each LP_{0l} mode is 2-fold degenerate

Each LP_{ml} $(m \neq 0)$ is 4-fold degenerate

An approximation for the number of modes is obtained from $Q = \frac{2V}{\pi}$

and the total number of modes is $M \sim \frac{\pi^2 Q^2}{8}$

For $V = 12.266$ $Q = 7.808$

$M \sim$ 75 modes (Exact answer is 80)

(4) For $V = 22.368$ $Q = 14.24$

$M \sim$ 250 modes

(5)

$n_1 := 1.465$ $\quad n_2 := 1.46$ **refractive indices**

$c_0 := 2.998 \cdot 10^8$

$\varepsilon_0 := 8.854 \cdot 10^{-12}$ **fundamental constants**

$\mu_0 := 4 \cdot \pi \cdot 10^{-7}$

$\varepsilon_1 := n_1^{\,2} \cdot \varepsilon_0$

$\varepsilon_2 := n_2^{\,2} \cdot \varepsilon_0$

$a := 50 \cdot 10^{-6}$ **Fiber radius**

$$\Delta := \frac{n_1 - n_2}{n_1}$$

$$\lambda_c := \frac{2 \cdot \pi \cdot n_1 \cdot a \cdot \sqrt{2 \cdot \Delta}}{2.405}$$

$\lambda_c = 1.5810791 \cdot 10^{-5}$ **Cutoff wavelength for second mode to appear**

$\lambda_0 := 1.55 \cdot 10^{-6}$

$k_1 := \dfrac{2 \cdot \pi \cdot n_1}{\lambda_0}$ $\qquad k_1 = 5.9386235 \cdot 10^6$

$$w(u) := \sqrt{2 \cdot k_1^{\,2} \cdot a^2 \cdot \Delta - u^2}$$

$u := 2$ **Guess**

Given

$\dfrac{u \cdot J1(u)}{J0(u)} = \dfrac{w(u) \cdot K1(w(u))}{K0(w(u))}$ **LP01 mode**

$uu := \mathrm{Find}(u)$ **Solution for u for LP01 (HE11mode)**

$uu = 2.3104976$

$$\beta := \sqrt{k_1^{\,2} - \left(\frac{uu}{a}\right)^2}$$

$\beta = 5.9384437 \cdot 10^6$

For LP01 10 μm fiber, uu=1.9872448, β=5.9352976*10^6

For LP01 50μm fiber, uu=2.3104976, β=5.9384437*10^6

$u := 2.5$

Given

$$\frac{u \cdot J0(u)}{J1(u)} = \frac{-w(u) \cdot K0(w(u))}{K1(w(u))}$$

$uu := Find(u)$ **Solution for LP11 mode (includes HE21 and TE01**

$uu = 3.6811143$

$uu = 3.6811143$

$$\beta := \sqrt{k_1^{\,2} - \left(\frac{uu}{a}\right)^2}$$

$\beta = 5.9381672 \cdot 10^6$

For LP11 10μm fiber, uu=3.1405702, β=5.9303135*10^6

For LP11 50μm fiber, uu=3.6811143, β=5.9381672*10^6