ENEE 691, Problem Set 3 Solutions

Problem 3.7. Kaiser Problem 3.30

$a := 4.5\cdot 10^{-6}$ core radius

$n_1 := 1.48$ core index

$c_0 := 2.998\cdot 10^{8}$

$\Delta := \frac{0.22}{100}$

$\lambda_0 := 1.32\cdot 10^{-6}$ wavelength

$k_0 := \frac{2\cdot\pi}{\lambda_0}$

$\omega := c_0\cdot k_0$

$\Delta = \frac{n_1 - n_2}{n_1}$

$n_2 := -(\Delta - 1)\cdot n_1$

$n_2 = 1.476744$

$V := \frac{2\cdot\pi\cdot n_1\cdot a\cdot\sqrt{2\cdot\Delta}}{\lambda_0}$

$V = 2.102841$ V<2.405, so single mode

Solve Eq. (2.66) in Keiser, using normalized u and w

$u := 2$ Guess

Given

$$\frac{u\cdot J1(u)}{J0(u)} = \sqrt{V^2 - u^2}\cdot\frac{K1\left(\sqrt{V^2 - u^2}\right)}{K0\left(\sqrt{V^2 - u^2}\right)}$$

$uu := \text{Find}(u)$

$uu = 1.561714$

$w := \sqrt{V^2 - uu^2}$

$w = 1.408187$

$u := uu$

$k_1 := \frac{2\cdot\pi\cdot n_1}{\lambda_0}$ $k_2 := \frac{2\cdot\pi\cdot n_2}{\lambda_0}$

$$\beta := \sqrt{k_1^2 - \left(\frac{u}{a}\right)^2}$$

Calculation using cladding values

$$\beta 2 := \sqrt{k_2^2 - \left(\frac{w}{a}\right)^2}$$

$\beta = 7.03623 \times 10^6$

$$v_p := \frac{\omega}{\beta}$$

$\beta 2 = 7.022316 \times 10^6$

Note that β and β2 are not exactly equal. This is because the weakly-guiding approximation is not exact.

$v_p = 2.028138 \times 10^8$

From Keiser Eq. (3.21)

$$b := \frac{\frac{\beta^2}{k_0^2} - n_2^2}{n_1^2 - n_2^2}$$

$b = 0.447836$

$$b := \frac{\frac{\beta}{k_0} - n_2}{n_1 - n_2}$$

$\frac{\beta}{k_0} = 1.478203$

$1 - \frac{u^2}{V^2} = 0.448444$ Alternative formula for b

$b = 0.448109$

Waveguide dispersion effects can be calculated using the data in the graph on page 113 of Keiser. Alternatively, there is an approximation from Marcuse that allows the calculation of Vd2(Vb)/dV2, which is called DV below.

$DV := 0.08 + 0.549 \cdot (2.834 - V)^2$

Marcuse's formula, from A. Ghatak and K. Thyagarajan, "Introduction to Fiber Optics," Cambridge University Press, 1998

$DV = 0.373492$

$$D_{wg} := \frac{-n_2 \cdot \Delta}{c_0} \cdot \frac{1}{\lambda_0} \cdot DV$$

$D_{wg} = -3.066219 \times 10^{-6}$

Equivalent to -3.066 ps/nm.km

Other useful aproximations from Ghatak and Thyagarajan

$$u := \frac{\left(1 + \sqrt{2}\right) \cdot V}{\left[1 + \left(4 + V^4\right)^{0.25}\right]}$$

$$b := 1 - \frac{\left(1 + \sqrt{2}\right)^2}{\left[1 + \left(4 + V^4\right)^{0.25}\right]^2}$$

$u = 1.584986$

$b = 0.431883$

Problem 3.8

$a := 2\cdot 10^{-6}$ core radius

$n_1 := 1.5$ core index

$\Delta := 0.006$

$\lambda_0 := 1.55\cdot 10^{-6}$ wavelength

$$\Delta = \frac{n_1 - n_2}{n_1}$$

$n_2 := -(\Delta - 1)\cdot n_1$

$n_2 = 1.491$

$$V := \frac{2\cdot\pi\cdot n_1\cdot a\cdot\sqrt{2\cdot\Delta}}{\lambda_0}$$

$V = 1.332171$ V<2.405, so single mode

Solve Eq. (2.66) in Keiser, using normalized u and w

$u := 0.5$ Guess

Given

$$\frac{u\cdot J1(u)}{J0(u)} = \sqrt{V^2 - u^2}\cdot\frac{K1\left(\sqrt{V^2 - u^2}\right)}{K0\left(\sqrt{V^2 - u^2}\right)}$$

$uu := \text{Find}(u)$

$uu = 1.221284$

$w := \sqrt{V^2 - uu^2}$

$w = 0.532114$

$u := uu$

$r := 0, \frac{a}{100} .. 10\cdot a$

Poynting vector in core is

$$I_{core}(r) := J0\left(u\cdot\frac{r}{a}\right)^2$$

Poynting vector in cladding is

$$I_{clad} = \left(C\cdot K0\left(\frac{w_r}{a}\right)\right)^2$$

Constant C must be found by matching intensities at boundary between core and cladding

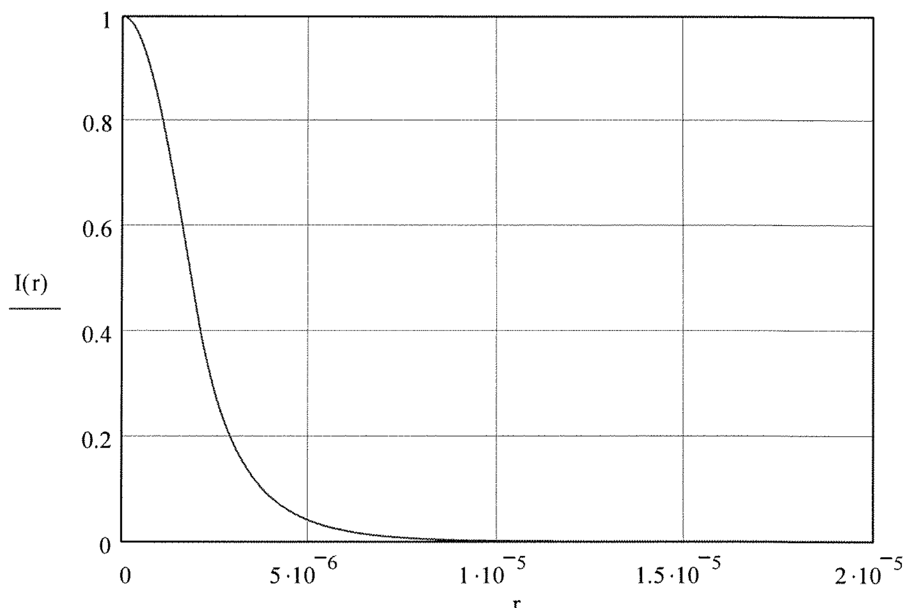
$$C := \frac{J0(u)^2}{K0(w)^2}$$

$C = 0.57195$

$I_{clad}(r) := C \cdot K0\left(w \cdot \frac{r}{a}\right)^2$ Cladding Poynting vector

$I(r) := \text{if}\left(r < a, I_{core}(r), I_{clad}(r)\right)$

Mode Profile



$$P_{core} := \int_0^a I_{core}(r)\,dr$$

$P_{core} = 1.578517 \times 10^{-6}$

$$P_{clad} := \int_a^{10 \cdot a} I_{clad}(r)\,dr$$

$P_{clad} = 5.366505 \times 10^{-7}$

$$\text{Fraction} := \frac{P_{clad}}{P_{core} + P_{clad}}$$

$\text{Fraction} = 0.253715$ ANSWER