ENEE 691 - Homework #4

① Keiser 4-3

(a) From Eqn (4-4) we have

$$1.540 = 1.424 + 1.266x + 0.266x^2$$

$$x^2 + 4.759x - 4.36 = 0.$$

Solve the quadratic equation taking the plus sign only:

$$x = \frac{1}{2}\left[-4.759 + \sqrt{(4.759)^2 + 4(.436)}\right]$$

$$\boxed{x = 0.090}$$

The emission wavelength is

$$\lambda = \frac{1.24}{1.540} = \boxed{805\ nm}$$

(b) $E_g = 1.424 + 1.266(0.015) + 0.266(0.015)^2$

$$\boxed{E_g = 1.443\ eV}$$

$$\lambda(\text{in }\mu m) = \frac{1.240}{E_g(\text{in eV})} = \frac{1.240}{1.443} = \boxed{0.859\ \mu m}$$

② Keiser 4-4

(a) $a(BC) = a(GaAs) = 5.6536$ Å

$a(BD) = a(GaP) = 5.4512$ Å

$a(AC) = a(InAs) = 6.0590$ Å

$a(AD) = a(InP) = 5.8696$ Å

$$a(x,y) = xy\,5.6536 + x(1-y)5.4512 + (1-x)y\,6.0590 + (1-x)(1-y)\,5.8696$$

$$\boxed{a(x,y) = 0.1894y - 0.4184x + 0.0130xy + 5.8696}$$

(2) (b) Substitute $a(x,y) = a(InP) = 5.8696\,Å$ into the expression for $a(x,y)$ in Part (a), then:

$$y = \frac{0.4184\,x}{0.1894 - 0.0130x} \approx \frac{0.4184x}{0.1894} = \boxed{2.20x}$$

(c) With $x = 0.26$ and $y = 0.56$,

$$E_g = 1.35 + 0.668(.26) - 1.17(0.56) + 0.758(0.26)^2 + 0.18(0.56)^2 - 0.069(.26)(.56) - .322(.26)^2(.56) + 0.33(.26)(.56)^2$$

$$\boxed{E_g = 0.981\text{ eV}}$$

(3) Keiser 4-9

(a) Using Eqn (4-28) with $\Gamma = 1$

$$g_{th} = \bar{\alpha} + \frac{1}{2L}\ln\left(\frac{1}{R_1R_2}\right)$$

$R_1 = R_2 = 0.32$ $\quad L = 500\,\mu m$ $\quad \bar{\alpha} = 10\,cm^{-1}$

$$g_{th} = 10\,cm^{-1} + \frac{1}{2(0.05\,cm)}\ln\left(\frac{1}{0.32}\right)^2$$

$$\boxed{g_{th} = 32.8\,cm^{-1}}$$

(b) With $R_1 = 0.9$ and $R_2 = 0.32$

$$g_{th} = 10\,cm^{-1} + \frac{1}{2(0.05\,cm)}\ln\left(\frac{1}{(0.9)(0.32)}\right)$$

$$\boxed{g_{th} = 22.5\,cm^{-1}}$$

(c) From Eqn (4-37) $\eta_{ext} = \eta_i(g_{th} - \bar{\alpha})/g_{th}$

For case (a): $\eta_{ext} = 0.65(32.8 - 10)/32.8 = \boxed{0.452}$

For case (b): $\eta_{ext} = 0.65(22.5 - 10)/22.5 = \boxed{0.361}$

④ Keiser 4-11

(a) $d = 0.1\mu m$ = thickness $L = 250\mu m$ = length.

$W = 1.0\mu m$ = width $n_1 = 3.55$ $n_2 = 3.20$ $R_1 = R_2 = 0.31$

$$D = \frac{2\pi d}{\lambda}(n_1^2 - n_2^2)^{1/2} = \frac{2\pi(0.1\mu m)}{(1.3\mu m)}(3.55^2 - 3.20^2)^{1/2} = \boxed{0.743}$$

$$\Gamma_T = \frac{D^2}{2 + D^2} = \boxed{0.216}$$

$$n_{eff}^2 = n_2^2 + \Gamma_T(n_1^2 - n_2^2) = 3.2^2 + (.216)(3.55^2 + 3.2^2) = \boxed{10.75}$$

$$W = \frac{2\pi w}{\lambda}(n_{eff}^2 - n_2^2)^{1/2} = \frac{2\pi(1.0\mu m)}{1.3\mu m}(10.75 - 3.2^2)^{1/2} = \boxed{3.45}$$

$$\Gamma_L = \frac{W^2}{2 + W^2} = \boxed{0.856}$$

(b) $\Gamma = \Gamma_T \Gamma_L = \boxed{0.185}$ = Total Confinement Factor

$$g_{th} = \frac{1}{\Gamma}\left[\bar{\alpha} + \frac{1}{2L}\ln\left(\frac{1}{R_1 R_2}\right)\right]$$

$$\boxed{g_{th} = 415.4\ cm^{-1}}$$

⑤ Keiser 4-18

From Eqn (4-48a) the effective refractive index is:

$$n_e = \frac{m\lambda_B}{2\Lambda} = \frac{2(1570nm)}{2(460nm)} = \boxed{3.4}$$

Use Eqn (4-48b):

For $m=0$: $\lambda = \lambda_B \pm \frac{\lambda_B^2}{2n_e L}\left(\frac{1}{2}\right) = \pm\frac{(1.57\mu m)(1570nm)}{4(3.4)(300\mu m)} + 1570nm$

$$\boxed{\lambda = 1570nm \pm 0.60nm}$$

For $m=1$: $\lambda = \lambda_B \pm \frac{\lambda_B^2}{2n_e L}\left(\frac{3}{2}\right) = \boxed{1570nm \pm 1.80nm}$

For $m=2$: $\lambda = \lambda_B \pm \frac{\lambda_B^2}{2n_e L}\left(\frac{5}{2}\right) = \boxed{1570nm \pm 3.0nm}$

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4-22. Since the dc component of $x(t)$ is 0.2, its range is $-2.36 < x(t) < 2.76$. The power has the form $P(t) = P_0[1 + mx(t)]$ where we need to find m and P_0. The average value is

$$< P(t)> \; = P_0[1 + 0.2m] = 1 \text{ mW}$$

The minimum value is

$P(t) = P_0[1 - 2.36m] \geq 0$ which implies $\boxed{m \leq \frac{1}{2.36} = 0.42}$

Therefore for the average value we have $< P(t)> \; = P_0[1 + 0.2(0.42)] \leq 1$ mW, which implies

$\boxed{P_0 = \frac{1}{1.084} = 0.92 \text{ mW}}$ so that $P(t) = 0.92[1 + 0.42x(t)]$ mW and

$i(t) = 10\, P(t) = 9.2[1 + 0.42x(t)]$ mA

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4-23. Substitute $x(t)$ into $y(t)$:

$$\begin{aligned}
y(t) &= a_1b_1 \cos \omega_1t + a_1b_2 \cos \omega_2t \\
&+ a_2(b_1^2\cos^2 \omega_1t + 2b_1b_2 \cos \omega_1t \cos \omega_2t + b_2^2\cos^2 \omega_2t) \\
&+ a_3(b_1^3\cos^3 \omega_1t + 3b_1^2b_2 \cos^2 \omega_1t \cos \omega_2t + 3b_1b_2^2\cos \omega_1t \cos^2 \omega_2t + b_2^3\cos^3 \omega_2t) \\
&+ a_4(b_1^4\cos^4 \omega_1t + 4b_1^3b_2 \cos^3 \omega_1t \cos \omega_2t + 6b_1^2b_2^2\cos^2 \omega_1t \cos^2 \omega_2t \\
&+ 4b_1b_2^3\cos \omega_1t \cos^3 \omega_2t + b_2^4\cos^4 \omega_2t)
\end{aligned}$$

Use the following trigonometric relationships:

i) $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

ii) $\cos^3 x = \frac{1}{4}(\cos 3x + 3\cos x)$

iii) $\cos^4 x = \frac{1}{8}(\cos 4x + 4\cos 2x + 3)$

iv) $2\cos x \cos y = \cos (x+y) + \cos (x-y)$

v) $\cos^2 x \cos y = \frac{1}{4}[\cos (2x+y) + 2\cos y + \cos (2x-y)]$

vi) $\cos^2 x \cos^2 y = \frac{1}{4}\ [1 + \cos 2x + \cos 2y + \frac{1}{2}\ \cos(2x+2y) + \frac{1}{2}\ \cos(2x-2y)]$

vii) $\cos^3 x \cos y = \frac{1}{8}\ [\cos(3x+y) + \cos(3x-y) + 3\cos(x+y) + 3\cos(x-y)]$

$k = 1$ or 2

then

$$y(t) = \frac{1}{2}\left[a_2b_1^2 + a_2b_2^2 + \frac{3}{4}a_4b_1^4 + 3a_4b_1^2b_2^2 + \frac{3}{4}a_4b_2^4\right] \quad \text{constant terms} = A_0$$

$$+\frac{3}{4}\left[a_3b_1^3 + 2a_3b_1b_2^2\right]\cos\omega_1 t + \frac{3}{4}\ a_3\left[b_2^3 + 2b_1^2b_2\right]\cos\omega_2 t \quad \text{fundamental terms} \Rightarrow A_1(\omega_k)$$

$$+\frac{b_1^2}{2}\left[a_2 + a_4b_1^2 + 3a_4b_2^2\right]\cos 2\omega_1 t + \frac{b_2^2}{2}\left[a_2 + a_4b_2^2 + 3a_4b_1^2\right]\cos 2\omega_2 t$$

2nd-order harmonic terms $\Rightarrow A_2(\omega_k)$

$$+\frac{1}{4}\ a_3b_1^3\ \cos 3\omega_1 t + \frac{1}{4}\ a_3b_2^3\ \cos 3\omega_2 t \quad \text{3rd-order harmonic terms} \Rightarrow A_3(\omega_k)$$

$$+\frac{1}{8}\ a_4b_1^4\ \cos 4\omega_1 t + \frac{1}{8}\ a_4b_2^4\ \cos 4\omega_2 t \quad \text{4th-order harmonic terms} \Rightarrow A_4(\omega_k)$$

$$+\left[a_2b_1b_2 + \frac{3}{2}a_4b_1^3b_2 + \frac{3}{2}a_4b_1b_2^3\right][\cos(\omega_1+\omega_2)t + \cos(\omega_1-\omega_2)t]$$

2nd-order intermodulation terms $\Rightarrow B_{1,\pm 1}$

$$+\frac{3}{4}\ a_3b_1^2\ b_2\,[\cos(2\omega_1+\omega_2)t + \cos(2\omega_1-\omega_2)t] \quad +\frac{3}{4}\ a_3b_1b_2^2\,[\cos(2\omega_2+\omega_1)t + \cos(2\omega_2-\omega_1)t]$$

3rd-order intermodulation terms $\Rightarrow B_{2,\pm 1}$ and $B_{\pm 1,2}$

$$+\frac{1}{2}\ a_4b_1^3\ b_2\,[\cos(3\omega_1+\omega_2)t + \cos(3\omega_1-\omega_2)t] \Rightarrow B_{3,\pm 1}$$

$$+\frac{3}{4}\ a_4b_1^2\ b_2^2\,[\cos(2\omega_1+2\omega_2)t + \cos(2\omega_1-2\omega_2)t] \Rightarrow B_{2,\pm 2}$$

$$+\frac{1}{2}\ a_4b_1b_2^3\,[\cos(3\omega_2+\omega_1)t + \cos(3\omega_2-\omega_1)t] \Rightarrow B_{\pm 1,3}$$

4th-order intermodulation terms

This output is of the form :

$$y(t) = A_0 + A_1(\omega_1)\cos\omega_1 t + A_2(\omega_1)\cos 2\omega_1 t + A_3(\omega_1)\cos 3\omega_1 t$$

$$+ A_4(\omega_1)\cos 4\omega_1 t + A_1(\omega_2)\cos\omega_2 t + A_2(\omega_2)\cos 2\omega_2 t$$

$$+ A_3(\omega_2)\cos 3\omega_2 t + A_4(\omega_2)\cos 4\omega_2 t + \sum_m \sum_n B_{mn}\cos(m\omega_1+n\omega_2)t$$

where $A_n(\omega_j)$ is the coefficient for the $\cos(n\omega_j)t$ term.

$= 1.$

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5-8. Using Eq. (5-10), the relfectivity at the gel-to-fiber interface is

$$R_{g-f} = \left(\frac{1.485 - 1.305}{1.485 + 1.305}\right)^2 = 4.16 \times 10^{-3}$$

The power loss is (see Example 5-3)

$$L = -10 \ \log \ (1 - R) = -10 \ \log \ (0.9958) = 0.018 \text{ dB}$$

When there is no index-matching gel, the joint loss is

$$R_{a-f} = \left(\frac{1.485 - 1.000}{1.485 + 1.000}\right)^2 = 0.038$$

The power loss is $L = -10 \ \log \ (1 - R) = -10 \ \log \ (0.962) = 0.17 \text{ dB}$

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5-16. The splice losses are found from the sum of Eqs. (5-35) through (5-37). First find NA(0) from Eq. (2-80b).

For fiber 1: $NA_1(0) = n_1\sqrt{2\Delta} = 1.46\sqrt{2(0.01)} = 0.206$

For fiber 2: $NA_2(0) = n_1\sqrt{2\Delta} = 1.48\sqrt{2(0.015)} = 0.256$

(a) The only loss is that from index-profile differences. From Eq. (5-37)

$$L_{1\to 2}(\alpha) = -10 \ \log \ \frac{1.80(2.00+2)}{2.00(1.80+2)} = 0.24 \text{ dB}$$

(b) The losses result from core-size differences and NA differences.

$$L_{2\to 1}(a) = -20 \ \log \ \left(\frac{50}{62.5}\right) = 1.94 \text{ dB}$$

$$L_{2\to 1}(NA) = -20 \ \log \ \left[\frac{.206}{.256}\right] = 1.89 \text{ dB}$$

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Problem 5-12

$n_1 := 1.48$ $\quad NA := 0.20$ $\quad n_2 := \sqrt{n_1{}^2 - NA^2}$ $\quad n_2 = 1.466$ $\quad \theta_c := \operatorname{asin}\left(\frac{n_2}{n_1}\right)$ $\quad \theta_c = 82.234 \cdot \deg$

We can use equation (5-30) to solve for the fiber loss due to lateral misalignment in equation (5-32). By defining dnorm as d/a, the relation becomes:

$$L_{Fd}(dnorm) := -10 \cdot \log\left[\frac{2}{\pi} \cdot \left[\operatorname{acos}(dnorm) - \left(1 - dnorm^2\right)^{\frac{1}{2}} \cdot \frac{1}{6} \cdot dnorm \cdot \left(5 - \frac{1}{2} \cdot dnorm^2\right)\right]\right]$$

We can solve for the loss due to longitudnal misalignment by rewriting equation (5-33) with s/a indicated by snorm, finding:

$$L_{Fs}(snorm) := -10 \cdot \log\left[\left(1 + snorm \cdot \tan\left(\theta_c\right)\right)^{-2}\right]$$

$i := 0 .. 100$

$dnorm_i := \frac{i}{100} \cdot 1.0$ $\qquad snorm_i := \frac{i}{100} \cdot 3.0$ $\qquad \theta_i := \frac{i}{100} \cdot 0.174533$ $\qquad \theta_{100} = 10 \cdot \deg$

With the equations for the variables p, q, and y, we can solve for the loss due to an angular displacement, θ, by using equation (5-34):

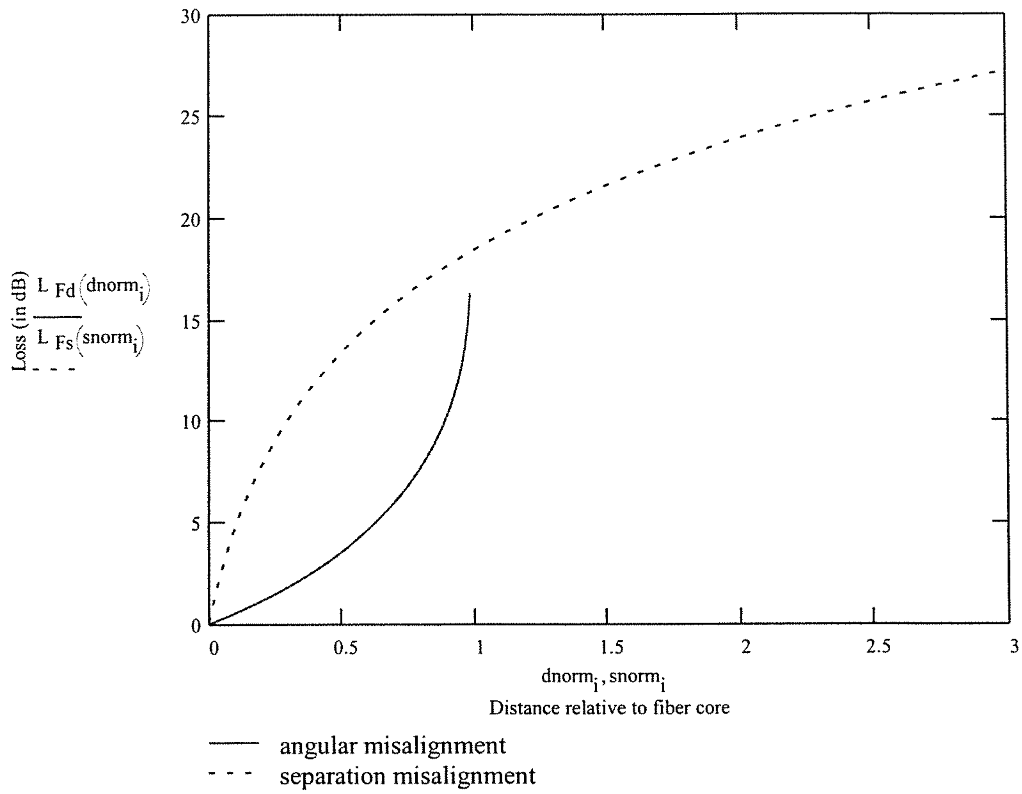
$$p_i := \frac{\cos\left(\theta_c\right) \cdot \left(1 - \cos\left(\theta_i\right)\right)}{\sin\left(\theta_c\right) \cdot \sin\left(\theta_i\right)} \qquad q_i := \frac{\cos\left(\theta_c\right)^3}{\left[\left(\cos\left(\theta_c\right)\right)^2 - \sin\left(\theta_i\right)^2\right]^{\frac{3}{2}}}$$

$$y_i := \frac{\cos\left(\theta_c\right)^2 \cdot \left(1 - \cos\left(\theta_i\right)\right) - \sin\left(\theta_i\right)^2}{\sin\left(\theta_c\right) \cdot \cos\left(\theta_c\right) \cdot \sin\left(\theta_i\right)}$$

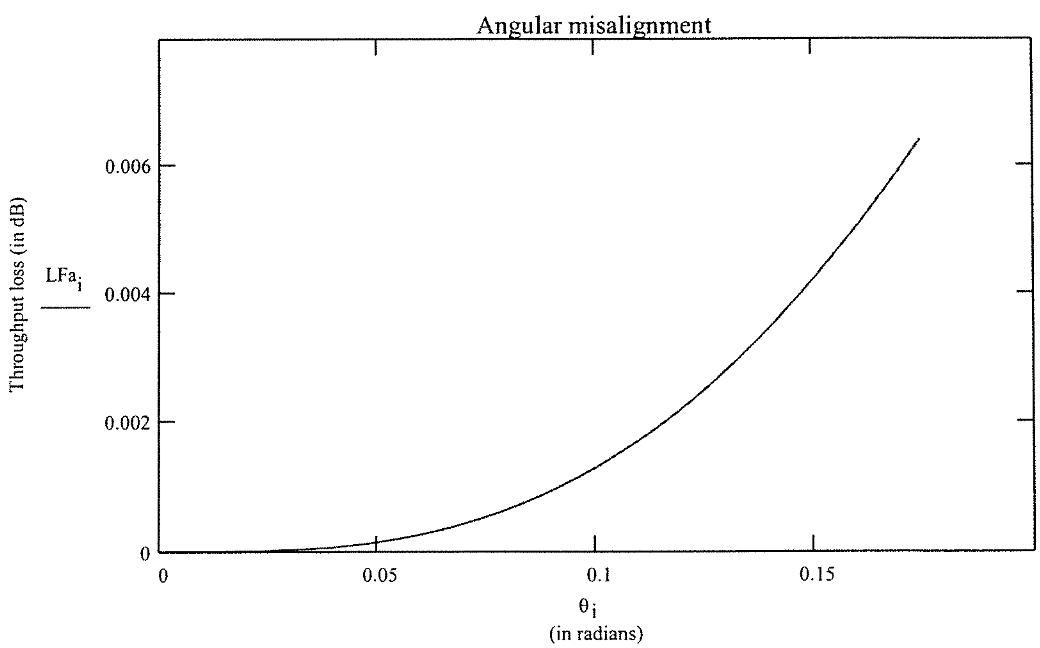
$$parg_i := \frac{1}{\pi} \cdot p_i \cdot \left[1 - \left(p_i\right)^2\right]^{\frac{1}{2}} + \frac{1}{\pi} \cdot \left(\operatorname{asin}\left(p_i\right)\right) \qquad yarg_i := \frac{1}{\pi} \cdot y_i \cdot \left[1 - \left(y_i\right)^2\right]^{\frac{1}{2}} + \left[\frac{1}{\pi} \cdot \left(\operatorname{asin}\left(y_i\right)\right)\right] + \frac{1}{2}$$

$$LFa_i := -10 \cdot \log\left[\cos\left[\theta_i \cdot \left(\frac{1}{2} - parg_i - q_i \cdot yarg_i\right)\right]\right]$$

5-12



30
25
20
15
10
5
0
0
0.5
1
1.5
2
2.5
3
Loss (in dB)
L Fd(dnorm_i)
L Fs(snorm_i)
dnorm_i, snorm_i
Distance relative to fiber core
angular misalignment
separation misalignment



Angular misalignment
0.006
0.004
0.002
0
0
0.05
0.1
0.15
Throughput loss (in dB)
LFa_i
θ_i
(in radians)

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Problem 5-20

$n_o := 1.470 \qquad n_{IM} := 1.470 \qquad n_{air} := 1.00$

$\theta_o := 8\ \mathrm{deg} \qquad d := 1 \cdot 10^{-6}$

$\theta_o = 0.1396\ \mathrm{rad}$

IM: index matching material gap

$$R_1 := \left(\frac{n_o - n_{IM}}{n_o + n_{IM}}\right)^2$$

$$\theta_1 := \mathrm{asin}\left(\frac{n_o}{n_{IM}} \cdot \sin(\theta_o)\right)$$

$$\theta_1 = 8\ \mathrm{deg}$$

$$\beta_1(\lambda) := \frac{4 \cdot \pi \cdot n_{IM} \cdot d \cdot \cos(\theta_1)}{\lambda}$$

$$T_1(\lambda, R) := \frac{(1-R)^2}{(1-R)^2 + 4 \cdot R \cdot \sin\left(\frac{\beta_1(\lambda)}{2}\right)}$$

Air gap

$$R_2 := \left(\frac{n_o - n_{air}}{n_o + n_{air}}\right)^2$$

$$\theta_2 := \mathrm{asin}\left(\frac{n_o}{n_{air}} \cdot \sin(\theta_o)\right)$$

$$\theta_2 = 11.805\ \mathrm{deg}$$

$$\beta_2(\lambda) := \frac{4 \cdot \pi \cdot n_{air} \cdot d \cdot \cos(\theta_2)}{\lambda}$$

$$T_2(\lambda, R) := \frac{(1-R)^2}{(1-R)^2 + 4 \cdot R \cdot \sin\left(\frac{\beta_2(\lambda)}{2}\right)}$$

5-20

$$L(\lambda) := 10 \cdot \log\left(\frac{T_1(\lambda, R_1)}{T_2(\lambda, R_2)}\right)$$

$$\lambda := 700 \cdot 10^{-9}, 701 \cdot 10^{-9} .. 1600 \cdot 10^{-9}$$