ENEE 691 Spring 2002

Solutions to Questions on First Examination

(1) Short answers:
(a) Modal dispersion - different ray paths.
(b) Material dispersion - n varies with λ.
(c) Waveguide dispersion - β varies with ω.
(d) Polarization Mode Dispersion - fiber is not perfectly cylindrically symmetric, two polarization eigenmodes. Pulse energy wanders between these two modes and broadens pulse.

$$v_g = \frac{d\beta}{d\omega}$$

$$\beta = \frac{\omega n}{c_0}$$

$$\frac{d\beta}{d\omega} = \frac{n}{c_0} + \frac{\omega}{c_0}\frac{dn}{d\omega} = \frac{n}{c_0} + \frac{\omega}{c_0}\frac{dn}{d\lambda}\frac{d\lambda}{d\omega}$$

$$\lambda = \frac{2\pi c_0}{n\omega} \quad \rightarrow \quad \frac{d\lambda}{d\omega} = -\frac{2\pi c_o}{n\omega^2}$$

$$\frac{d\beta}{d\omega} = \frac{1}{c_0}\left[n - \lambda\frac{dn}{d\lambda}\right].$$

This leads to the desired result.

Calculate $dv_g/d\lambda$ and use $\tau = L/v_g$.

$$\frac{d\tau}{d\lambda} = -\frac{L}{c_0}\lambda\frac{d^2 n}{d\lambda^2}.$$

Desired result follows.

For l=1km, $\Delta\lambda$=20nm, pulse spreading is clearly 120×20 = 2400ps/km.

(2) Solution to $-\partial^2\Phi/\partial\phi^2$ is $\Phi = A\cos\nu\phi + B\sin\nu\phi$. For a single valued solution $\cos(\phi + 2m\pi) = cos(\phi)$, for example. So ν must be a positive or negative integer, or zero.

Solutions to Bessel's equation that satisfy boundary conditions must be of form $J_v(ur/a)$ in core and $K_\nu(wr/a)$ in cladding. More explanations in book.

From universal b.c. for $m = 0$, cutoff results from $uJ_1(u)/uJ_0(u) = 0$. This equation does not blow up as $u \rightarrow 0$, so there is no cutoff for the lowest LP mode, namely HE_{11}.

$$v = \frac{2\pi a}{\lambda_0}\sqrt{n_1^2 - n_2^2} = \frac{2\pi a n_1}{\lambda_0}\sqrt{2\Delta}.$$

Maximum V for single mode operation is 2.405, which gives $2a$=4.82μm.

(3) Short answers:

(a) NA is sine of maximum entrance half angle.

$$NA = \sqrt{n_1^2 - n_2^2},$$

which gives in this case NA=0.113.

(b) Goos Hänchen shift results from phase shift on total internal reflection.

(c) Bent fiber are always "leaky". There are no perfecly guided modes in a bent fiber. Leakage increases with reduction in bend radius.

(d) Beam must be focused to a beam waist on the cleaved end face of the fiber. Focused spot size must match mode field radius in fiber.

(4) The phase condition in the lateral direction in a meridional section gives

$$4a\frac{2\pi n_1}{\lambda_0}\cos\theta = 2m\pi,$$

which gives

$$m = \frac{4an_1}{\lambda_0}\sqrt{n_1^2 - n_2^2},$$

which gives the number of modes as $N = m + 1$, where

$$N = 1 + 2\pi/V.$$

Six LP modes implies LP_{01}, LP_{11}, LP_{21}, LP_{02}, LP_{31}, and LP_{12}. The next mode would be LP_{41}. So $V < 6.38$, which gives the minimum wavelength as 1.438μm.

These six LP modes contain 20 TE, TM, HE and EH modes, including the cos and sin two-fold degeneracy for all but the TE and TM modes. $V^2/2$=20.35. This agrees well with the exact calculation.