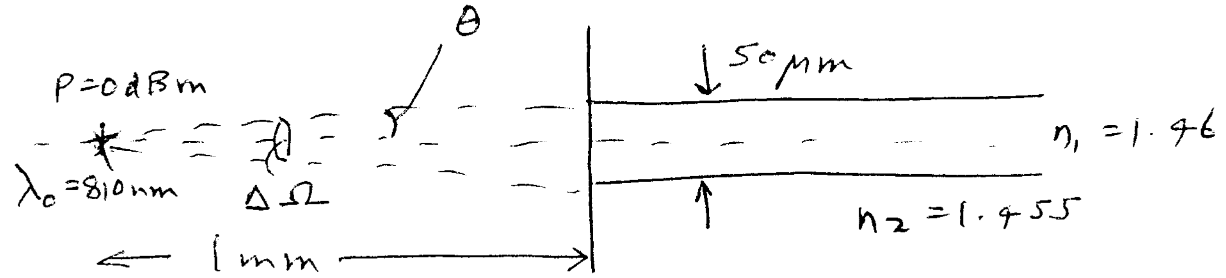
ENEE 691
Problem Set #1 SOLUTIONS

(1)



$\theta = \arctan\left(\frac{25}{1000}\right)$ $\theta = 0.02499$ rad.
1.432°

The NA of the fiber is $NA = \sqrt{n_1^2 - n_2^2} = \sqrt{0.0145} = 0.121$
$\theta_0 = 6.939^\circ$

All light that reaches core will be guided

$\Delta\Omega = \frac{A}{R^2}$ where A is subtended area
R > 1 mm

$\Delta\Omega \simeq \frac{\pi(25\times10^{-6})^2}{10^{-6}} = 1.963\times10^{-3}$ sr

Fraction of power into core is $\frac{1.963\times10^{-3}}{4\pi} = 1.5625\times10^{-4}$

power in is 10^{-3} W $\times 1.5625\times10^{-4} = 1.5625\times10^{-7}$ W
1.5625×10^{-7} W $\equiv -38.06$ dBm

(2) Maximum number of modes is found from

$$\frac{d}{\lambda_0} = \frac{m}{4\sqrt{n_1^2 - n_2^2}}$$

where $m = \text{int}\left(\frac{4d\sqrt{n_1^2 - n_2^2}}{\lambda_0}\right) = 14$

Therefore 15 modes can propagate (approximate model)

The V-number of the fiber is

$$V = \frac{2\pi}{\lambda_0} n_1 a \sqrt{2\Delta} = \frac{2\pi a}{\lambda_0} (NA) = 23.46$$

$$\frac{V^2}{2} = 275 \text{ modes}$$

(3) The critical angle $\theta_c = \arcsin \frac{1.455}{1.46} = 1.488$ rad

$$\theta_a = \frac{\pi}{2} - \theta_c = 0.08278 \text{ rad}$$

For $m = 1$ $\quad \sin\theta_z = \frac{\lambda_0}{4 n_1 d}$ $\quad \theta_z = 5.5479 \times 10^{-3}$ rad

The Goos-Haenchen shift is

$$z_s \simeq \frac{\lambda}{\pi n_1 \theta_z} \frac{1}{\sqrt{\theta_a^2 - \theta_z^2}} = 3.854 \times 10^{-4} \text{ m}$$

(4) The intensity distribution $I(\theta) = \cos^3\theta$ for $0 \le \theta \le \pi/2$ must be normalized to take into account the whole output power.

Intensity has units W/m^2 so at radius r the total power emitted over a hemisphere is

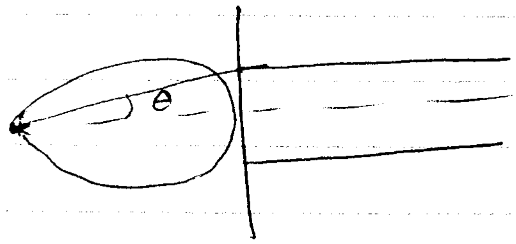
$$P = P_o \int_0^{\pi/2} I(\theta)\, 2\pi r^2 \sin\theta \, d\theta$$

$$P = P_o \int_0^{\pi/2} 2\pi r^2 \sin\theta \cos^3\theta \, d\theta = 2\pi r^2 P_o \left[\frac{-\cos^4\theta}{4} \right]_0^{\pi/2}$$

$$P = \frac{\pi r^2 P_o}{2} \Rightarrow P_o = \frac{2P}{\pi r^2}$$

So with $P = 0.1\,dBm \equiv 1.0233\,mW$

$$P_o = \frac{6.515 \times 10^{-4}}{r^2}\ W/m^2$$



The max θ into the fiber is

$$\theta = \arctan \frac{25 \times 10^{-6}}{100 \times 10^{-6}} = 14.04°$$

Only light within the NA will be guided so
$\theta_{max} = \theta_0 = 6.934° = 0.121\ rad$
The power into the fiber is

$$P_{in} = \int_0^{\theta_0} P_o\, 2\pi r^2 \sin\theta \, d\theta = 6.515 \times 10^{-4} \times 7.2 \times 10^{-3} \times 2\pi$$

$$= 29.47\,\mu W$$

(6) The maximum time delay is between the $m=0$ and $m=14$ modes

For $m=14$ $\quad \sin\theta_z = \frac{14\lambda_0}{4n_1 d} \Rightarrow \theta_z = 4.45^\circ$

For a fiber of length L

$$\Delta T = \frac{n_1 L}{c_0}\left[\frac{1}{\cos\theta_z} - 1\right] = 1.476\times10^{-11} L \text{ seconds}$$

for 100 Mb/s RZ the pulses are 5 ns long with a clock period of 10 ns.

If the delay reaches > 2.5 ns we have a problem.

Therefore $L_{max} = \frac{2.5\times10^{-9}}{1.476\times10^{-11}} = 169\,m$