SOLUTIONS TO HW # 4

4-3. (a) From Eq. (4-4) we have $1.540 = 1.424 + 1.266x + 0.266x^2$ or

$x^2 + 4.759x - 0.436 = 0$. Solving this quadratic equation yields (taking the plus sign only)

$$x = \frac{1}{2}\left[-4.759 + \sqrt{(4.759)^2 + 4(.436)}\right] = \underline{0.090}$$

The emission wavelength is $\lambda = \frac{1.240}{1.540} = 805$ nm.

(b) $E_g = 1.424 + 1.266(0.15) + 0.266(0.15)^2 = 1.620$ eV, so that

$$\lambda = \frac{1.240}{1.620} = 766 \text{ nm}$$

4-4. (a) The lattice spacings are as follows:

a(BC) = a(GaAs) = 5.6536 Å

a(BD) = a(GaP) = 5.4512 Å

a(AC) = a(InAs) = 6.0590 Å

a(AD) = a(InP) = 5.8696 Å

$$a(x,y) = xy\,5.6536 + x(1-y)\,5.4512 + (1-x)y\,6.0590 + (1-x)(1-y)5.8696$$

$$= 0.1894y - 0.4184x + 0.0130xy + 5.8696$$

(b) Substituting a(xy) = a(InP) = 5.8696 Å into the expression for a(xy) in (a), we have

$$y = \frac{0.4184x}{0.1894 - 0.0130x} \approx \frac{0.4184x}{0.1894} = 2.20x$$

(c) With x = 0.26 and y = 0.56, we have

$$E_g = 1.35 + 0.668(.26) - 1.17(.56) + 0.758(.26)^2 + 0.18(.56)^2$$

$$- .069(.26)(.56) - .322(.26)^2(.56) + 0.03(.26)(.56)^2 = 0.956 \text{ eV}$$

4-9. (a) Using Eq. (4-28) with $\Gamma = 1$

$$g_{th} = \frac{1}{0.05 \text{ cm}} \ln\left(\frac{1}{0.32}\right)^2 + 10 \text{ cm}^{-1} = 55.6 \text{ cm}^{-1}$$

(b) With $R_1 = 0.9$ and $R_2 = 0.32$,

$$g_{th} = \frac{1}{0.05 \text{ cm}} \ln\left[\frac{1}{0.9(0.32)}\right] + 10 \text{ cm}^{-1} = 34.9 \text{ cm}^{-1}$$

(c) From Eq. (4-37) $\eta_{ext} = \eta_i (g_{th} - \bar{\alpha})/g_{th}$;

thus for case (a): $\eta_{ext} = 0.65(55.6 - 10)/55.6 = 0.53$

For case (b): $\eta_{ext} = 0.65(34.9 - 10)/34.9 = 0.46$

4-10. Using Eq. (4-4) to find E_g and Eq. (4-3) to find λ, we have for x = 0.03,

$$\lambda = \frac{1.24}{E_g} = \frac{1.24}{1.424 + 1.266(0.3) + 0.266(0.3)^2} = 1.462\ \mu\text{m}$$

From Eq. (4-38)

$$\eta_{ext} = 0.8065\ \lambda(\mu\text{m}) \frac{dP(\text{mW})}{dI(\text{mA})}$$

Taking dI/dP = 0.5 mW/mA, we have $\eta_{ext} = 0.8065\ (1.462)(0.5) = 0.590$

4-11. (a) From the given values, D = 0.74, so that $\Gamma_T = 0.216$

Then $n_{eff}^2 = 10.75$ and W = 3.45, yielding $\Gamma_L = 0.856$

(b) The total confinement factor then is $\Gamma = 0.185$

4-18. From Eq. (4-48a) the effective refractive index is

$$n_e = \frac{m\lambda_B}{2\Lambda} = \frac{2(1570 \text{ nm})}{2(460 \text{ nm})} = 3.4$$

Then, from Eq. (4-48b), for m = 0

$$\lambda = \lambda_B \pm \frac{\lambda_B^2}{2n_eL}\left(\frac{1}{2}\right) = 1570 \text{ nm} \pm \frac{(1.57\ \mu\text{m})(1570 \text{ nm})}{4(3.4)(300\ \mu\text{m})} = 1570 \text{ nm} \pm 1.20 \text{ nm}$$

Therefore for m = 1, $\lambda = \lambda_B \pm 3(1.20 \text{ nm}) = 1570 \text{ nm} \pm 3.60 \text{ nm}$

For m = 2, $\lambda = \lambda_B \pm 5(1.20 \text{ nm}) = 1570 \text{ nm} \pm 6.0 \text{ nm}$

4-22. Since the dc component of $x(t)$ is 0.2, its range is $-2.36 < x(t) < 2.76$. The power has the form $P(t) = P_0[1 + mx(t)]$ where we need to find m and P_0. The average value is

$$< P(t)> = P_0[1 + 0.2m] = 1 \text{ mW}$$

The minimum value is

$$P(t) = P_0[1 - 2.36m] \geq 0 \qquad \text{which implies } m \leq \frac{1}{2.36} = 0.42$$

Therefore for the average value we have $< P(t)> = P_0[1 + 0.2(0.42)] \leq 1$ mW, which implies

$$P_0 = \frac{1}{1.084} = 0.92 \text{ mW} \qquad \text{so that } P(t) = 0.92[1 + 0.42x(t)] \text{ mW and}$$

$$i(t) = 10\ P(t) = 9.2[1 + 0.42x(t)] \text{ mA}$$

4-23. Substitute $x(t)$ into $y(t)$:

$$y(t) = a_1b_1 \cos \omega_1 t + a_1b_2 \cos \omega_2 t$$
$$+ a_2(b_1^2\cos^2 \omega_1 t + 2b_1b_2 \cos \omega_1 t \cos \omega_2 t + b_2^2\cos^2 \omega_2 t)$$
$$+ a_3(b_1^3\cos^3 \omega_1 t + 3b_1^2 b_2 \cos^2 \omega_1 t \cos \omega_2 t + 3b_1 b_2^2\cos \omega_1 t \cos^2 \omega_2 t + b_2^3\cos^3 \omega_2 t)$$
$$+a_4(b_1^4\cos^4 \omega_1 t + 4b_1^3 b_2 \cos^3 \omega_1 t \cos \omega_2 t + 6b_1^2 b_2^2\cos^2 \omega_1 t \cos^2 \omega_2 t$$
$$+ 4b_1 b_2^3\cos \omega_1 t \cos^3 \omega_2 t + b_2^4\cos^4 \omega_2 t)$$

Use the following trigonometric relationships:

i) $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

ii) $\cos^3 x = \frac{1}{4}(\cos 3x + 3\cos x)$

iii) $\cos^4 x = \frac{1}{8}(\cos 4x + 4\cos 2x + 3)$

iv) $2\cos x \cos y = \cos (x+y) + \cos (x-y)$

v) $\cos^2 x \cos y = \frac{1}{4}\ [\cos (2x+y) + 2\cos y + \cos (2x-y)]$

vi) $\cos^2 x \cos^2 y = \frac{1}{4}\ [1 + \cos 2x + \cos 2y + \frac{1}{2}\ \cos(2x+2y) + \frac{1}{2}\ \cos(2x-2y)]$

vii) $\cos^3 x \cos y = \frac{1}{8}\ [\cos(3x+y) + \cos(3x-y) + 3\cos(x+y) + 3\cos(x-y)]$

then

$$y(t) = \frac{1}{2}\left[a_2 b_1^2 + a_2 b_2^2 + \frac{3}{4} a_4 b_1^4 + 3 a_4 b_1^2 b_2^2 + \frac{3}{4} a_4 b_2^4\right] \qquad \text{constant terms}$$

$$+ \frac{3}{4}\left[a_3 b_1^3 + 2 a_3 b_1 b_2^2\right] \cos \omega_1 t + \frac{3}{4}\ a_3\left[b_2^3 + 2 b_1^2 b_2\right] \cos \omega_2 t \qquad \text{fundamental terms}$$

$$+ \frac{b_1^2}{2}\left[a_2 + a_4 b_1^2 + 3 a_4 b_2^2\right] \cos 2\omega_1 t + \frac{b_2^2}{2}\left[a_2 + a_4 b_2^2 + 3 a_4 b_1^2\right] \cos 2\omega_2 t$$

2nd-order harmonic terms

$$+ \frac{1}{4}\ a_3 b_1^3 \cos 3\omega_1 t + \frac{1}{4}\ a_3 b_2^3 \cos 3\omega_2 t \qquad \text{3rd-order harmonic terms}$$

$$+ \frac{1}{8}\ a_4 b_1^4 \cos 4\omega_1 t + \frac{1}{8}\ a_4 b_2^4 \cos 4\omega_2 t \qquad \text{4th-order harmonic terms}$$

$$+ \left[a_2 b_1 b_2 + \frac{3}{2} a_4 b_1^3 b_2 + \frac{3}{2} a_4 b_1 b_2^3\right][\cos(\omega_1+\omega_2)t + \cos(\omega_1-\omega_2)t]$$

2nd-order intermodulation terms

$$+ \frac{3}{4}\ a_3 b_1^2\ b_2\ [\cos(2\omega_1+\omega_2)t + \cos(2\omega_1-\omega_2)t] \quad + \frac{3}{4}\ a_3 b_1 b_2^2\ [\cos(2\omega_2+\omega_1)t + \cos(2\omega_2-\omega_1)t]$$

3rd-order intermodulation terms

$$+ \frac{1}{2}\ a_4 b_1^3\ b_2\ [\cos(3\omega_1+\omega_2)t + \cos(3\omega_1-\omega_2)t]$$
$$+ \frac{3}{4}\ a_4 b_1^2\ b_2^2\ [\cos(2\omega_1+2\omega_2)t + \cos(2\omega_1-2\omega_2)t]$$
$$+ \frac{1}{2}\ a_4 b_1 b_2^3\ [\cos(3\omega_2+\omega_1)t + \cos(3\omega_2-\omega_1)t] \qquad \text{4th-order intermodulation}$$

terms

This output is of the form

$$y(t) = A_0 + A_1(\omega_1) \cos \omega_1 t + A_2(\omega_1) \cos 2\omega_1 t + A_3(\omega_1) \cos 3\omega_1 t$$
$$+ A_4(\omega_1) \cos 4\omega_1 t + A_1(\omega_2) \cos \omega_2 t + A_2(\omega_2) \cos 2\omega_2 t$$
$$+ A_3(\omega_2) \cos 3\omega_2 t + A_4(\omega_2) \cos 4\omega_2 t + \sum_m \sum_n B_{mn} \cos(m\omega_1+n\omega_2)t$$

where $A_n(\omega_j)$ is the coefficient for the $\cos(n\omega_j)t$ term.

5-8. Using Eq. (5-10), the relfectivity at the gel-to-fiber interface is

$$R_{g-f} = \left(\frac{1.485-1.305}{1.485+1.305}\right)^2 = 4.16\times 10^{-3}$$

The power loss is (see Example 5-3)

$$L = -10 \ \log \ (1-R) = -10 \ \log \ (0.9958) = 0.018 \ \text{dB}$$

When there is no index-matching gel, the joint loss is

$$R_{a-f} = \left(\frac{1.485-1.000}{1.485+1.000}\right)^2 = 0.038$$

The power loss is $L = -10 \ \log \ (1-R) = -10 \ \log \ (0.962) = 0.17 \ \text{dB}$

5-16. The splice losses are found from the sum of Eqs. (5-35) through (5-37). First find NA(0) from Eq. (2-80b).

For fiber 1: $NA_1(0) = n_1\sqrt{2\Delta} = 1.46\sqrt{2(0.01)} = 0.206$

For fiber 2: $NA_2(0) = n_1\sqrt{2\Delta} = 1.48\sqrt{2(0.015)} = 0.256$

(a) The only loss is that from index-profile differences. From Eq. (5-37)

$$L_{1\to 2}(\alpha) = -10 \ \log \ \frac{1.80(2.00+2)}{2.00(1.80+2)} = 0.24 \ \text{dB}$$

(b) The losses result from core-size differences and NA differences.

$$L_{2\to 1}(a) = -20 \ \log \left(\frac{50}{62.5}\right) = 1.94 \ \text{dB}$$

$$L_{2\to 1}(NA) = -20 \ \log \left[\frac{.206}{.256}\right] = 1.89 \ \text{dB}$$