ENEE 380 QUIZ SOLUTIONS

(1) $|\mathbf{A}| = \sqrt{3^2+4^2+2^2} = \sqrt{29}$

unit vector is $\frac{\mathbf{A}}{|\mathbf{A}|} = \hat{\mathbf{A}} = \frac{1}{\sqrt{29}}(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$

$= 0.557\hat{\mathbf{i}} + 0.743\hat{\mathbf{j}} - 0.377\hat{\mathbf{k}}$

(2) $|\mathbf{A}| = \sqrt{3^2-4^2+2^2} = \sqrt{29}$

unit vector at the point in spherical coordinates is

$\hat{\mathbf{A}} = \frac{1}{\sqrt{29}}(3\hat{\mathbf{r}} - 4\hat{\theta} + 2\hat{\phi}) = 0.557\hat{\mathbf{r}} - 0.743\hat{\theta} + 0.337\hat{\phi}$

(3) $\mathbf{A}\cdot\mathbf{B} = 2+2-15 = -11$

(4) $\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 2 & -5 \\ 1 & 2 & 3 \end{vmatrix} = 13\hat{\mathbf{i}} - 11\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

(5) vector required is $\frac{\mathbf{A}\times\mathbf{B}}{|\mathbf{A}\times\mathbf{B}|} = \frac{13\hat{\mathbf{i}}-11\hat{\mathbf{j}}+3\hat{\mathbf{k}}}{\sqrt{13^2+11^2+3^2}}$

$= \frac{13\hat{\mathbf{i}}-11\hat{\mathbf{j}}+3\hat{\mathbf{k}}}{\sqrt{299}} = \frac{13\hat{\mathbf{i}}-11\hat{\mathbf{j}}+3\hat{\mathbf{k}}}{11.292}$

$= 0.752\hat{\mathbf{i}} - 0.636\hat{\mathbf{j}} + 0.173\hat{\mathbf{k}}$

(6) $(\mathbf{u}\cdot\mathbf{v})^2 = (|\mathbf{u}||\mathbf{v}|\cos\theta)^2 + (|\mathbf{u}||\mathbf{v}|\sin\theta)^2 = |u|^2|v|^2$

(7) $\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C} - \mathbf{C}(\mathbf{A}\cdot\mathbf{B})$

the "back cab" rule

$= (\mathbf{i} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})(4-4+30) - (2\hat{i} - 4\hat{j} - 6\hat{\mathbf{k}})(2+2-15)$

$= (30\hat{\mathbf{i}} + 60\hat{\mathbf{j}} + 90\mathbf{k}) - (-22\hat{\mathbf{i}} + 44\hat{\mathbf{j}} + 66\hat{\mathbf{k}})$

$= 52\hat{\mathbf{i}} + 16\hat{\mathbf{j}} + 24\hat{\mathbf{k}}$

$\mathbf{B}\times\mathbf{C} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 2 & 3 \\ 2 & -4 & -6 \end{vmatrix} = 0\hat{\mathbf{i}} + 12\hat{\mathbf{j}} - 8\hat{\mathbf{k}}$

$\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = 0+12+40 = 52$

(8) A vector perpendicular to $\mathbf{m}\times\mathbf{n}$ is $\ell' = q(\mathbf{m}\times\mathbf{n})$ where q is a scalar. If $\ell\ell' = 1$ then clearly $\ell' = \frac{(\mathbf{m}\times\mathbf{n})}{\ell\cdot(\mathbf{m}\times\mathbf{n})}$ (plug it in and see)

(9) $\frac{|\vec{AC}|}{|\vec{CB}|} = \lambda \qquad |\vec{AC}| = \lambda x$

$|\vec{BC}| = x$

$\vec{BA} = \mathbf{a} - \mathbf{b}$

$\vec{BC} = \vec{BA}\left(\frac{1}{1+\lambda}\right)$

$\mathbf{c} = \mathbf{b} + \vec{BC} = \mathbf{b} + (\mathbf{a} - \mathbf{b})(\frac{1}{1+\lambda})$

(10) In cylindrical coordinates

$\mathbf{A} = 2\cos\phi\hat{\rho} - 2\rho\hat{\phi} + 3\hat{z}$

$x = \rho\cos\phi \quad y = \rho\sin\phi \quad z = z$

$\cos\phi = \frac{x}{\rho} = \frac{x}{\sqrt{x^2+y^2}} \qquad \sin\phi = \frac{y}{\rho} = \frac{y}{\sqrt{x^2+y^2}}$

$\mathbf{A} = \frac{2x}{\sqrt{x^2+y}^2}\hat{\rho} - 2\sqrt{x^2+y^2}\hat{\phi} + 3\hat{\mathbf{z}}$

$A_x = \mathbf{A}\cdot\hat{\mathbf{i}} = \frac{2x}{\sqrt{x^2+y^2}}\hat{\rho}\ \cdot\hat{\mathbf{i}} - 2\sqrt{x^2+y^2}\hat{\phi}\cdot\hat{\mathbf{i}}$

$= \frac{2x}{\sqrt{x^2+y^2}}\cos\phi + 2\sqrt{x^2+y^2}\sin\phi$

$= \frac{2x^2}{x^2+y^2} + \frac{2y\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}} = \frac{2x^2}{x^2+y^2} + 2y$

$A_z = A_z$

$A_y = \mathbf{A}\cdot\hat{\mathbf{j}} = \frac{2x}{\sqrt{x^2y^2}}\hat{\rho}\cdot\hat{\mathbf{j}} - 2\sqrt{x^2+y^2}\hat{\phi}\cdot\hat{\mathbf{j}}$

$= \frac{2x}{\sqrt{x^2+y^2}}\sin\phi - 2\sqrt{x^2+y^2}\cos\phi$

$= \frac{2xy}{x^2+y^2} - 2\sqrt{x^2+y^2}\frac{x}{\sqrt{x^2+y^2}}$

$= \frac{2x_y}{x^2+y^2} - 2x$

(11) $(r,\theta,\phi) = (6, 60°, 120°)$

$x = r\sin\theta\cos\phi = -2.598$

$y = r\sin\theta\sin\phi = 4.5$

$z = r\cos\theta = -3$

In cylindrical coordinates

$\rho = \sqrt{x^2+y^2} = 5.196$

$\phi = \tan^{-1}\frac{y}{x} = 120°$

$z = z = -3$

(12) $V = 3x^2 \cos y - \ln y y^2 + 3xz^2$

$\frac{\partial V}{\partial x} = 6x \cos y + 3z^2$

$\frac{\partial V}{\partial y} = -3x^2 \sin y - y - 2y \ln y$

$\frac{\partial V}{\partial z} = 6xz$

$\text{grad}V = (6x \cos y + 3z^2)\hat{\mathbf{i}} - (3x^2 \sin y - y - 2y \ln y)\hat{\mathbf{j}} + 6xz\hat{\mathbf{k}}$

(13) $\mathbf{B} = \sin(xy)\hat{\mathbf{i}} + 2z^3\hat{\mathbf{j}} + 3\tan y\hat{\mathbf{k}}$

$\frac{\partial B_x}{\partial x} = y \cos(xy)$

$\frac{\partial B_y}{\partial y} = 0 \qquad \frac{\partial B_z}{\partial x} = 0$

$\text{div}\mathbf{B} = y \cos(xy)$

(14) $\mathbf{B} = \sin(xy)\hat{\mathbf{i}} + 2z^3\hat{\mathbf{j}} + 3\tan y\hat{\mathbf{k}}$

$$\text{curl}\mathbf{B} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \\ \sin(xy) & 2z^3 & 3\tan y \end{vmatrix} = \hat{\mathbf{i}}(3\text{sec}^2 y - 6z^2) + \hat{\mathbf{j}}(0) + \hat{\mathbf{k}}(-x \cos xy)$$

(15) 0

(16) 0

(17) $\text{divgrad}\phi = \nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}$

$\phi = x^2 - \frac{3}{y} + 6 \sin z$

$\nabla^2\phi = 2 - \frac{6}{y^3} - 6 \sin z$

(18) $\mathbf{F} = (3y - c_1 z)\hat{\mathbf{i}} + (c_2 x - 2z)\hat{\mathbf{j}} - (c_3 y + \mathbf{z})\hat{\mathbf{k}}$

$$\text{curl}\mathbf{F} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial j} & \frac{\partial}{\partial z} \\ 3y - c_1 z & c_2 x - 2z & c_3 y + z \end{vmatrix} = 0$$

$-c_3 + 2 = 0 \qquad c_3 = 2$

$-c_1 = 0 \qquad c_1 = 0$

$c_2 - 3 = 0 \qquad c_2 = 3$

(19) $\mathbf{E} = \left(\frac{25}{r^2}\right)\hat{\mathbf{r}}$

At $P = (-3, 4, -5) \quad r^2 = 9 + 16 + 25 = 50$

$\hat{\mathbf{r}} = \frac{-3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}}}{\sqrt{50}}$

$|\mathbf{E}| = \frac{25}{50} = 0.5$

$E_x = -0.5 \times \frac{3\hat{\mathbf{i}}}{\sqrt{50}} = \frac{-1/5\hat{\mathbf{i}}}{\sqrt{50}} = -0.212\hat{\mathbf{i}}$

For $\mathbf{B} = 2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}} \quad |\mathbf{B}| = \sqrt{4+4+1} = 3$

$\hat{\mathbf{B}} = \frac{2\hat{\mathbf{i}}-2\hat{\mathbf{j}}+\hat{\mathbf{k}}}{3}$

$\mathbf{E} = \frac{0.5}{\sqrt{50}}\left(-3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}}\right)$

$\mathbf{r} \cdot \hat{\mathbf{B}} = \cos\theta = \frac{(-3\hat{\mathbf{i}}+4\hat{\mathbf{j}}-5\hat{\mathbf{k}})}{\sqrt{50}} \cdot \frac{(2\hat{\mathbf{i}}-2\hat{\mathbf{j}}+\hat{\mathbf{k}})}{3}$

$\cos\theta = \frac{-6-8-5}{3\times\sqrt{50}} = \frac{-19}{3\times\sqrt{50}} \quad \theta = -26.41°$

(20) $\mathbf{E} = y\hat{\mathbf{i}} + x\hat{\mathbf{j}}$

P_1P_2 lie on the parabola $x = 2y^2$

$\mathbf{d}\ell = \hat{\mathbf{i}}dx + \hat{\mathbf{j}}dy$

$\mathbf{E} \cdot d\ell = ydx + xdy$

On the parabola this gives $\mathbf{E}d\ell = \sqrt{x/2}dx + 2y^2dy$

$\int_{P_1}^{P_2} \mathbf{E} \cdot d\ell = \int_2^8 \sqrt{x/2}dx + \int_1^2 2y^2dy$

$= \frac{2}{3\sqrt{2}}x^{3/2}\Big|_2^8 + \frac{2y^3}{3}\Big|_1^2 = 14$

Along the straight line from $P_1 \to P_2$ $y = \frac{1}{6}x + \frac{2}{3}$

$\int_{P_1}^{P_2} \mathbf{E} \cdot \mathbf{d}\ell = \int_2^8 \left(\frac{1}{6}x + \frac{2}{3}\right) dx + \int_1^2 (6_j - 4)dy$

$= \left(\frac{X^2}{12}\frac{+2X}{3}\right)\Bigg|_2^8 + \left(\frac{6y^2}{2} - 4y\right)\Bigg|_1^2$

$= 9 + 5 = 14$